

Math 141 Exam 2 Study Guide

1. Algebraic Rewrite Rules
 - A. Radicals or Roots (Square Roots)
 - B. X on the Bottom of a Fraction
 - C. Both Rewrites at Once
2. Derivative Rules
 - A. Power Rule
 - B. Product Rule
 - C. Quotient Rule
 - D. Chain Rule
 - E. Rules Inside Rules
3. Max's and Min's
 - A. Relative Max's and Min's
 - B. Absolute Max's and Min's
4. Concept Questions
 - A. Marginal Revenue, Cost, Profit
 - B. Meanings of $f(x)$, $f'(x)$, and $f''(x)$

1. Algebraic Rewrite Rules

Before you start taking a derivative using your derivative rules, you **ALWAYS** need to look to rewrite the equation to make sure that it is setup properly for you. There are two rewrite rules from Algebra that you will need to look to apply. Below are those two rules plus the situation where you would need to use both rules at once.

A. Radicals or Roots (Square Roots)

Any time you have an **x** underneath a radical (root) you will need to rewrite the radical so that it is a power.

Example	$root \sqrt{x^{power}}$	$r \sqrt{x^p}$	$\sqrt[3]{x^5}$
Rewrite: Change to an exponent with the power written over the root .	$x^{\frac{power}{root}}$	$x^{\frac{p}{r}}$	$x^{\frac{5}{3}}$
One of the most common radicals to have to rewrite is a square root. However, the square root has its power = 1 and its root = 2 unwritten.	\sqrt{x}	$\sqrt[2]{x^1}$	$x^{\frac{1}{2}}$

B. X on the Bottom of a Fraction

Whenever there is a single **x** to a power on the bottom of a fraction (denominator) with a single number on the top of the fraction (numerator), you will want to rewrite the fraction to move the **x** from the bottom to the top.

Example	$\frac{number}{x^{power}}$	$\frac{n}{x^p}$	$\frac{5}{x^8}$
Rewrite: Bring the x from the bottom to the top and change the power to a negative.	$number \cdot x^{-power}$	$n \cdot x^{-p}$	$5 \cdot x^{-8}$

C. Both at Once

The final rewrite you need to look out for is just a combination of the first two rewrites. The situation is when you have a radical (root) on the bottom of a fraction.

Example	$\frac{number}{root \sqrt{x^{power}}}$	$\frac{n}{r \sqrt{x^p}}$	$\frac{5}{\sqrt[3]{x^5}}$
Step 1: Do the Radical Rewrite	$\frac{number}{x^{\frac{power}{root}}}$	$\frac{n}{x^{\frac{p}{r}}}$	$\frac{5}{x^{\frac{5}{3}}}$
Step 2: Do the X on the Bottom Rewrite	$number \cdot x^{-\frac{power}{root}}$	$n \cdot x^{-\frac{p}{r}}$	$5 \cdot x^{-\frac{5}{3}}$

2. Derivative Rules

There are 4 main derivative rules; the Power Rule, the Product Rule, the Quotient Rule and the Chain Rule.

When we find a derivative we are finding an equation that allows us to find the slope of the tangent line or *instantaneous* rate of change at a single point. In your past Algebra life you found the slope of a line (the **m** in $y=mx+b$), which is the *average* rate of change or the rate of change between two points.

Notation:

There are 3 main ways to notate a derivative. They all mean the same thing, they are just different ways of stating a derivative.

- 1) If you start with $f(x) =$, then the derivative would be notated $f'(x) =$, and read **f prime of x**.
- 2) If you start with $y =$, then the derivative would be notated as $y' =$, and read **y prime of x**.
- 3) Another option if you start with $y =$, would have the derivative notated and read **d-y-d-x**.

A. Power Rule

The Power Rule is the main rule that you will use when taking a derivative. All other rules are just methods for taking something big and crazy looking and breaking it back down into the Power Rule.

The **POWER RULE** Saying

Bring the power down, then subtract 1 from the power.

Example 1	Original Equation	Derivative	
Derivative of a Constant	$f(x)=5$	$f'(x)=0$	The derivative of a number by itself (constant) is always 0 because it has no change .

Example 2	Original Equation	Derivative	
Derivative of just an x	$f(x)=x$	$f'(x)=1$	You use the Power Rule .
More details	$f(x)=x^1$	$f'(x)=1x^{1-1}$	An x by itself has an unwritten 1 so when we apply the power rule we bring that power down and then subtract 1 from the power.
Simplify		$f'(x)=1x^0$	Doing the math in the exponent leaves us with 0 .
Final Result		$f'(x)=1$	Anything to the 0 power is always just 1 .

Example 3	Original Equation	Derivative	
Derivative of a single term equation.	$f(x) = 3x^5$	$f'(x) = 15x^4$	You use the Power Rule .
More details	$f(x) = 3x^5$	$f'(x) = 5 \cdot 3x^{5-1}$	Bring the Power down and subtract 1 from the Power .
Simplify		$f'(x) = 15x^4$	

Example 4	Original Equation	Derivative	
Derivative of a multiple term equation	$f(x) = 3x^5 + 7x^4 - 2x - 9$	$f'(x) = 15x^4 + 28x^3 - 2$	You use the Power Rule on each individual term.
Term 1	$f(x) = 3x^5$	$f'(x) = 5 \cdot 3x^{5-1} = 15x^4$	Bring the Power down and subtract 1 from the Power .
Term 2	$f(x) = 7x^4$	$f'(x) = 4 \cdot 7x^{4-1} = 28x^3$	Bring the Power down and subtract 1 from the Power .
Term 3	$f(x) = -2x$	$f'(x) = -2$	Derivative of just an x , the x drops off.
Term 4	$f(x) = -9$	$f'(x) = 0$	Derivative of a Constant is always zero .

Example 5	Original Equation	Derivative	
Derivative of an equation with rewrites.	$f(x) = \sqrt{x} + \sqrt[5]{x^7} - \frac{3}{x^8} - \frac{7}{\sqrt[3]{x^4}}$	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{7}{5}x^{\frac{2}{5}} + 24x^{-9} + \frac{28}{3}x^{-\frac{7}{3}}$	You use the Power Rule on each individual term.
Term 1 Square Root Rewrite	$f(x) = \sqrt{x} = x^{\frac{1}{2}}$	$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$	Bring the Power down and subtract 1 from the Power .
Term 2 Radical Rewrite	$f(x) = \sqrt[5]{x^7} = x^{\frac{7}{5}}$	$f'(x) = \frac{7}{5}x^{\frac{7}{5}-1} = \frac{7}{5}x^{\frac{2}{5}}$	Bring the Power down and subtract 1 from the Power .
Term 3 X on the bottom Rewrite	$f(x) = -\frac{3}{x^8} = -3x^{-8}$	$f'(x) = -8 \cdot -3x^{-8-1} = 24x^{-9}$	Derivative of just an x , the x drops off.
Term 4 Both Rewrites	$f(x) = -\frac{7}{\sqrt[3]{x^4}} = -\frac{7}{x^{\frac{4}{3}}} = -7x^{-\frac{4}{3}}$	$f'(x) = -\frac{4}{3} \cdot -7x^{-\frac{4}{3}-1} = \frac{28}{3}x^{-\frac{7}{3}}$	Derivative of a Constant is always zero .

B. Product Rule

The Product Rule is used whenever you have two or more equations being **multiplied**. It is really just a method for taking a large crazy looking equation and breaking it down into small bite size pieces that just require the *Power Rule*.

The **PRODUCT RULE** Saying

F **G-Prime** Plus **G** **F-Prime**

$$f \cdot g' + g \cdot f'$$

Example	
The Problem	Differentiate (i.e. Take the derivative) $H(x) = (3x^5 - 9x^3 - 7x)(4x^2 - 2x^{-3} + 5)$
Break Equation into two smaller equations. Label one f and one g .	$f = (3x^5 - 9x^3 - 7x)$ $g = (4x^2 - 2x^{-3} + 5)$
Take the derivative of each one individually, applying the Power Rule .	$f' = (15x^4 - 27x^2 - 7)$ $g' = (8x + 6x^{-4})$
Bring the Pieces Together: Use the Product Rule saying.	F G-Prime Plus G F-Prime $f \cdot g' + g \cdot f'$
The Final Answer <i>Do Not Simplify</i>	$H'(x) = (3x^5 - 9x^3 - 7x)(8x + 6x^{-4}) + (4x^2 - 2x^{-3} + 5)(15x^4 - 27x^2 - 7)$

C. Quotient Rule

The Quotient Rule is used whenever you have two equations being **divided**. It is really just a method for taking a large crazy looking equation involving division and breaking it down into small bite size pieces that just require the *Power Rule*.

The **QUOTIENT RULE** Saying

LOW D-Hi Minus **Hi D-LOW** All Over **LOW Squared**

$$\frac{(Low)(D-Hi) - (Hi)(D-Low)}{(Low)^2}$$

Example	
The Problem	Differentiate (i.e. Take the derivative) $H(x) = \frac{(3x^5 - 9x^3 - 7x)}{(4x^2 - 2x^{-3} + 5)}$
Break Equation into two smaller equations. Label the one from the top Hi and one from the bottom Low .	$Hi = (3x^5 - 9x^3 - 7x)$ $Low = (4x^2 - 2x^{-3} + 5)$
Take the derivative of each one individually, applying the Power Rule .	$D-Hi = (15x^4 - 27x^2 - 7)$ $D-Low = (8x + 6x^{-4})$
Bring the Pieces Together: Use the Quotient Rule saying.	<p>LOW D-Hi Minus Hi D-LOW All Over LOW Squared</p> $\frac{(Low)(D-Hi) - (Hi)(D-Low)}{(Low)^2}$
The Final Answer <i>Do Not Simplify</i>	$H'(x) = \frac{(4x^2 - 2x^{-3} + 5)(15x^4 - 27x^2 - 7) - (3x^5 - 9x^3 - 7x)(8x + 6x^{-4})}{(4x^2 - 2x^{-3} + 5)^2}$

D. Chain Rule

The Chain Rule is used whenever you have one equations **inside** another equation. People often refer to these types of equations as composite equations because it is one equation composed (or made up) of multiple equations. It is really just a method for taking a large crazy looking equation involving division and breaking it down into small bite size pieces that just require the *Power Rule*.

The **CHAIN RULE** Saying

D-IN Times D-OUT or **Derivative of the Inside Times Derivative of the Outside**

$$(D-IN) \cdot (D-OUT)$$

Example	
The Problem	Differentiate (i.e. Take the derivative) $f(x) = (3x^5 - 9x^3 - 7x)^5$
Break Equation into two smaller equations. Label the one inside as In and the one on the outside as Out .	$IN = 3x^5 - 9x^3 - 7x$ $OUT = ()^5$
Take the derivative of each one individually, applying the Power Rule .	$D-IN = 15x^4 - 27x^2 - 7$ $D-OUT = 5(---)^4$
Bring the Pieces Together: Use the Chain Rule saying.	D-IN Times D-OUT $(D-IN) \cdot (D-OUT)$ $f'(x) = (15x^4 - 27x^2 - 7) \cdot 5(---)^4$
The Final Answer: Put the In back inside the D-Out and you are done. Do Not Simplify	$f'(x) = (15x^4 - 27x^2 - 7) \cdot 5(3x^5 - 9x^3 - 7x)^4$

E. Rules Inside Rules

The next step in the process of finding derivatives involves the taking of the derivative of functions where multiple derivative rules are required to get the final answer.

The key when doing these types of problems is to first identify the **primary rule**. Which of your three main derivative rules (Product, Quotient, or Chain) is the first rule you have to go through? Once you have identified the **primary rule**, then break down the pieces of that rule to find the derivatives of those smaller pieces.

So, the **primary rule** might be the Quotient Rule, but inside that rule might be a Chain Rule problem as either the D-Hi or D-Low. So, when you look at the Hi piece as a smaller piece, the D-Hi might require you to do a Chain Rule.

Primary Rules

Product Rule	$(Something) \cdot (Something)$
Quotient Rule	$\frac{(Something)}{(Something)}$
Chain Rule	$(Something)^{(Something)}$

Example 1

The Problem	<p>Differentiate (i.e. Take the derivative)</p> $f(x) = \frac{(3x^5 - 9x^3 - 7x)^5}{(7x^3 - 3x^2 - x + 7)^9}$	
<p>Identify the Primary Rule: In this example the primary rule is the Quotient Rule. First and foremost this is two equations being divided.</p>	$f(x) = \frac{(3x^5 - 9x^3 - 7x)^5}{(7x^3 - 3x^2 - x + 7)^9}$	
<p>Apply the Quotient Rule: When we look at the smaller pieces as individual problems what we see is that both the Hi and Low are examples of a Chain Rule problem with equations inside equations.</p>	$Hi = (3x^5 - 9x^3 - 7x)^5$	$Low = (7x^3 - 3x^2 - x + 7)^9$
<p>Find the DHi and DLow: We will need to do two Chain Rule processes. So we need to find our D-In and D-Out's.</p>	$IN = 3x^5 - 9x^3 - 7x$ $D-IN = 15x^4 - 27x^2 - 7$ $OUT = (---)^5$ $D-OUT = 5(---)^4$	$IN = 7x^3 - 3x^2 - x + 7$ $D-IN = 21x^2 - 6x - 1$ $OUT = (---)^9$ $D-OUT = 9(---)^8$
<p>Bring the Pieces Together: Both the DHi and the DLow use the Chain Rule saying: D-IN Times D-OUT</p>	$DHi = (15x^4 - 27x^2 - 7) \cdot 5(3x^5 - 9x^3 - 7x)^4$	$DLow = (21x^2 - 6x - 1) \cdot 9(7x^3 - 3x^2 - x + 7)^8$
<p>The Final Answer: Bring all the pieces back together to complete the original Quotient Rule process.</p>	$\frac{(Low)(D-Hi) - (Hi)(D-Low)}{(Low)^2}$	

$$\frac{(7x^3 - 3x^2 - x + 7)^9 (15x^4 - 27x^2 - 7) \cdot 5(3x^5 - 9x^3 - 7x)^4 - (3x^5 - 9x^3 - 7x)^5 (21x^2 - 6x - 1) \cdot 9(7x^3 - 3x^2 - x + 7)^8}{((7x^3 - 3x^2 - x + 7)^9)^2}$$

Example 2

<p>The Problem</p>	<p style="text-align: center;">Differentiate (i.e. Take the derivative)</p> $f(x) = (3x^5 - 9x^3 - 7x)^5 (7x^3 - 3x^2 - x + 7)^9$	
<p>Identify the Primary Rule: In this example the primary rule is the Product Rule. First and foremost this is two equations being multiplied.</p>	$f(x) = (3x^5 - 9x^3 - 7x)^5 (7x^3 - 3x^2 - x + 7)^9$	
<p>Apply the Product Rule: Look at the smaller pieces as individual problems and what we see is that both the f and g are examples of a Chain Rule problem with equations inside equations.</p>	$f = (3x^5 - 9x^3 - 7x)^5$	$g = (7x^3 - 3x^2 - x + 7)^9$
<p>Find the f' and g': We will need to do two Chain Rule processes. So we need to find our D-In and D-Out's.</p>	$\begin{aligned} IN &= 3x^5 - 9x^3 - 7x \\ D-IN &= 15x^4 - 27x^2 - 7 \\ OUT &= (\text{---})^5 \\ D-OUT &= 5(\text{---})^4 \end{aligned}$	$\begin{aligned} IN &= 7x^3 - 3x^2 - x + 7 \\ D-IN &= 21x^4 - 6x - 1 \\ OUT &= (\text{---})^9 \\ D-OUT &= 9(\text{---})^8 \end{aligned}$
<p>Bring the Pieces Together: The f' and the g' use the Chain Rule saying: D-IN Times D-OUT</p>	$f' = (15x^4 - 27x^2 - 7) \cdot 5(3x^5 - 9x^3 - 7x)^4$	$g' = (21x^4 - 6x - 1) \cdot 9(7x^3 - 3x^2 - x + 7)^8$
<p>The Final Answer: Bring all the pieces back together to complete the original Product Rule process.</p>	$f \cdot g' + g \cdot f'$	

$$f'(x) = (3x^5 - 9x^3 - 7x)^5 (21x^4 - 6x - 1) \cdot 9(7x^3 - 3x^2 - x + 7)^8 + (7x^3 - 3x^2 - x + 7)^9 (15x^4 - 27x^2 - 7) \cdot 5(3x^5 - 9x^3 - 7x)^4$$

Example 3

The Problem	Differentiate (i.e. Take the derivative) $f(x) = \left(\frac{3x^5 - 9x^3 - 7x}{7x^3 - 3x^2 - x + 7} \right)^8$	
Identify the <i>Primary Rule</i>: In this example the primary rule is the <i>Chain Rule</i> . First and foremost this is an equation inside another equation. Take note of how this is different than Example 1.	$f(x) = \left(\frac{3x^5 - 9x^3 - 7x}{7x^3 - 3x^2 - x + 7} \right)^8$	
Apply the <i>Product Rule</i>: When we look at the smaller pieces as individual problems what we see is that the <i>In</i> requires a <i>Quotient Rule</i> and the <i>Out</i> is a <i>Power Rule</i> .	$IN = \frac{3x^5 - 9x^3 - 7x}{7x^3 - 3x^2 - x + 7}$	$OUT = (---)^8$
Find the <i>D-In</i>: Do the <i>Quotient Rule</i> processes, and find our <i>DHi</i> and <i>DLow</i> . Find the <i>D-Out</i>: Do a <i>Power Rule</i> .	$Hi = 3x^5 - 9x^3 - 7x^5$ $DHi = 15x^4 - 27x^2 - 7$ $Low = 7x^3 - 3x^2 - x + 7$ $DLow = 21x^4 - 6x - 1$	$OUT = (---)^8$ $D-OUT = 8(---)^7$
Bring the Pieces Together: For both the <i>D-In</i> and the <i>D-Out</i> .	$D-IN = \frac{(7x^3 - 3x^2 - x + 7)(15x^4 - 27x^2 - 7) - (3x^5 - 9x^3 - 7x^5)(21x^4 - 6x - 1)}{(7x^3 - 3x^2 - x + 7)^7}$	$D-OUT = 8 \left(\frac{3x^5 - 9x^3 - 7x}{7x^3 - 3x^2 - x + 7} \right)^7$
The Final Answer: Bring all the pieces back together to complete the original <i>Chain Rule</i> process.	$(D-IN) \cdot (D-OUT)$	
	$f'(x) = \frac{(7x^3 - 3x^2 - x + 7)(15x^4 - 27x^2 - 7) - (3x^5 - 9x^3 - 7x^5)(21x^4 - 6x - 1)}{(7x^3 - 3x^2 - x + 7)^7} \cdot 8 \left(\frac{3x^5 - 9x^3 - 7x}{7x^3 - 3x^2 - x + 7} \right)^7$	

3. Max's and Min's

One of the reasons that you may want to use a derivative is to find the max's and the min's of an equation. While it is great that derivatives can do this, even better at accomplishing this feat is your calculator. Since you are taking Math 141, you won't really need to use derivatives to find your results as you will be taking a multiple choice exam that only requires you to get the right answer, and it does not care how you arrived at that answer.

There are two types of Max's and Min's that you will be required to find in this class. The first is called a Relative Max or Min, or Relative Extrema (meaning find both if they exist). These types of max's and mins are really just the peaks and the valleys that an equation creates. The second type of max's and min's that you will need to be able to find are Absolute Max/Min's or Absolute Extrema (meaning find both). Absolute Max/Mins are the largest and smallest **y-values** in the interval given no matter where they occur.

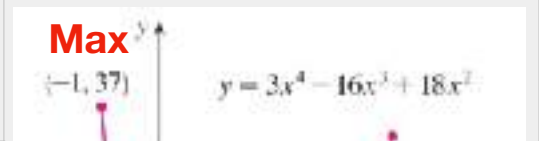

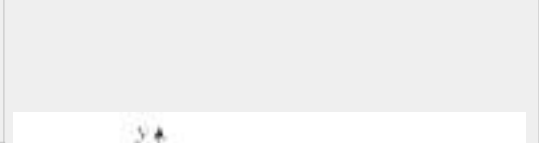

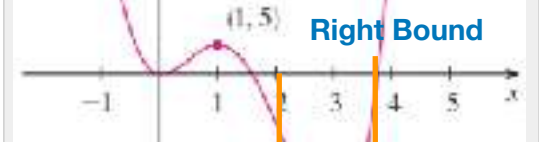

A. Relative Max's and Min's

To find Relative Max's and Min's in the simplest way possible will require you to use a TI-83/84. We are going to graph the equation we are given, adjust our window to see all the peaks and valleys, and then use the calculate maximum or minimum feature on our calculator depending on which of those we are trying to find. You are not always going to have both. Some problems might have just a Relative Max, some just a Relative Min, and some have both and could even have multiple different Relative Max's and Relative Mins. They exist **anywhere** you see a peak or a valley.

Example		
The Problem	Find the Relative Maximum and Minimum (Relative Extrema) of: $f(x) = -x^3 + 3x^2 - 2$	
Step 1:	Plug the equation into Y1 in the Y= area of your calculator.	
Step 2:	Graph the Equation and then adjust your window so that you can see ALL the peaks (Max's) and valleys (Min's).	
Step 3:	<p><u>Calculate the Max:</u> Hit 2nd—> Calc—> #4Maximum</p>	
	<p><u>Find a Left Bound:</u> Move the cursor until you are to the LEFT of the maximum value.</p>	
	<p><u>Find a Right Bound:</u> Move the cursor until you are to the RIGHT of the maximum value.</p>	
	<p><u>Guess:</u> Just hit ENTER. The Guess does not matter.</p>	
	Maximum is the y-value .	
		Y = 2
Step 4:	<p><u>Calculate the Min:</u> Hit 2nd—> Calc—> #3 Minimum</p>	
	<p><u>Find a Left Bound:</u> Move the cursor until you are to the LEFT of the minimum value.</p>	
	<p><u>Find a Right Bound:</u> Move the cursor until you are to the RIGHT of the minimum value.</p>	
	<p><u>Guess:</u> Just hit ENTER. The Guess does not matter.</p>	
	Minimum is the y-value .	
		Y = -2
Step 5:	Final Answer	<p>Relative Max of Y = 2 at x = 2 Relative Min of Y = -2 at x = 0</p>

B. Absolute Max's and Min's

Absolute Max/Min problems vary from Relative Max/Min problems in that the Absolute Max/Min problem just wants to know what the Max Y-value and the Min Y-value is a specific interval (window). The Absolute Max and Absolute Min **do not** have to occur at peaks and valleys. They can occur at a peak, a valley, or one of the two end points of the interval given. You will **ALWAYS** have both an Absolute Max and an Absolute Min.

Example		
The Problem	Find the Absolute Maximum and Minimum (Absolute Extrema) of: $f(x) = 3x^4 - 16x^3 + 18x^2$ on interval $[-1, 4]$	
Step 1:	Plug the equation into Y1 in the Y= area of your calculator.	
Step 2:	Graph the Equation and then adjust your window so that X-Min = -1 the left endpoint of the interval given. X-Max = 4 the left endpoint of the interval given.	
Step 3:	<u>Calculate the Absolute Max:</u> Identify the location of the Largest Y-value in the window.	
	In this example the Largest Y-value occurs at the left end point of the interval, $x = -1$. <i>Note that at $x=1$ there is a peak, but that is not the largest y-value in our window so we ignore it in an Absolute Max situation.</i>	
	<u>Calculator Instructions</u> Since the location of the max is an end point we are going to Calculate a Value. Hit: 2nd → Calc → #1 Value and enter $x = -1$.	
	Absolute Maximum is the y-value .	
Step 4:	<u>Calculate the Absolute Min:</u> Identify the location of the Smallest Y-value in the window.	
	In this example the Largest Y-value occurs at the valley located at $x = 3$.	
	<u>Calculate the Min:</u> Hit: 2nd → Calc → #3 Minimum	
	<u>Find a Left Bound:</u> Move the cursor until you are to the LEFT of the minimum value.	
	<u>Find a Right Bound:</u> Move the cursor until you are to the RIGHT of the minimum value.	
	<u>Guess:</u> Just hit ENTER . The Guess does not matter.	
	Minimum is the y-value .	
Step 5:	Final Answer	Absolute Max of $Y = 37$ at $x = -1$ Absolute Min of $Y = -27$ at $x = 3$

3. Concept Questions

Exam 2's concept questions revolve around the meanings of Marginal Revenue, Marginal Cost, and Marginal Profit. Also included are some similar concept questions as Exam 1, which include the meaning of $f(x)$, $f'(x)$, and $f''(x)$.

A. Marginal Revenue, Cost, and Profit

The concept questions on this exam that involve Marginal Revenue, Cost, and Profit center on the meaning of these terms as well as the meaning of Revenue, Cost, and Profit.

Term	Notation	Meaning	Example
Marginal Revenue	$R'(x)$	The approximate revenue from selling the $x+1$ item. Or the revenue of one more .	$R'(10)$ Represents the approximate revenue from selling the 11th item.
Revenue	$R(x)$	The exact revenue from selling the first x items , or items 0 thru x all together.	$R(10)$ Represents the exact revenue from selling the first 10 items, or the total revenue of items 0 thru 10 .
Marginal Cost	$C'(x)$	The approximate cost from selling the $x+1$ item. Or the cost of one more .	$C'(10)$ Represents the approximate cost from selling the 11th item.
Cost	$C(x)$	The exact cost from selling the first x items , or items 0 thru x all together.	$C(10)$ Represents the exact cost from selling the first 10 items, or the total cost of items 0 thru 10 .
Marginal Profit	$P'(x)$	The approximate profit from selling the $x+1$ item. Or the profit of one more .	$P'(10)$ Represents the approximate profit from selling the 11th item.
Profit	$P(x)$	The exact profit from selling the first x items , or items 0 thru x all together.	$P(10)$ Represents the exact profit from selling the first 10 items, or the total profit of items 0 thru 10 .

B. Meanings of $f(x)$, $f'(x)$, and $f''(x)$

Exam 2's Graphical Concept questions revolve around understanding what $f(x)$ represents graphically, what $f'(x)$ represents graphically, and what $f''(x)$ represents graphically. The first two pieces were concept questions from Exam 1. Exam 2 keeps those two and then adds $f''(x)$.

- $f(x)$ is asking you about the y-values.
- $f'(x)$ is asking you about the instantaneous rate of change, the slope of the tangent, or in common language is the graph increasing, decreasing, or neither.
- $f''(x)$ is asking you about the concavity of the graph. So the graph is either concave up like a cup, or concave down like a frown.

<p>Concept</p> <ul style="list-style-type: none"> - If the graph is above the x-axis, then the y-value is positive and $f(x) > 0$. - If the graph is below the x-axis, then the y-value is negative and $f(x) < 0$. - If the graph is on the x-axis, then the y-value is zero and $f(x) = 0$. 	<p style="text-align: center;">$f(x) = y\text{-values}$</p>
<p>Concept</p> <ul style="list-style-type: none"> - If the graph is increasing, then $f'(x)$ is positive or $f'(x) > 0$. - If the graph is decreasing, then $f'(x)$ is negative or $f'(x) < 0$. - If the graph is at a Max, Min, or goes horizontal, then $f'(x)$ is zero or $f'(x) = 0$. <p>Note: ALWAYS READ the GRAPH Left to Right.</p>	<p style="text-align: center;">$f'(x) = \text{Slopes of Tangent Lines}$</p>
<p>Concept</p> <ul style="list-style-type: none"> - If the graph is concave up like a cup, then $f''(x)$ is positive or $f''(x) > 0$. - If the graph is concave down like a frown, then $f''(x)$ is negative or $f''(x) < 0$. - If the graph is changing from one type of concavity to another, then $f''(x)$ is zero or $f''(x) = 0$, and we call it an inflection point. 	<p style="text-align: center;">$f''(x) = \text{Concavity}$</p>