NAME: ____________________

Instructor: ____________________

Time your class meets: ____________

Math 160 Calculus for Physical Scientists I
Exam 1
September 18, 2014, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a calculator on this exam. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____
(Signature)

____
(Date)

Please do not write in this space.

1-5. (15pts)	
6. (12pts)	
7. (15pts)	
8. (3pts)	
9. (12pts)	
10. (15pts)	
11. (12pts)	
12. (16pts)	
TOTAL	

Multiple Choice for #1-5 (15pts - 3pts each). Use the function $h(x)$ to answer the following multiple choice questions. **Circle only one answer for each problem**.

$$h(x) = \begin{cases} \cos(x), & x < 0 \\ -2, & x = 0 \\ \cos(x) + 1, & x > 0 \end{cases}$$

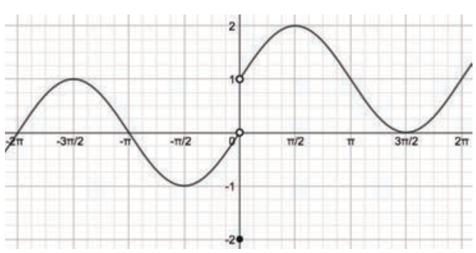
1. $\lim_{x \to 0^-} h(x) =$
   (a) -2
   (b) 0
   (c) 1
   (d) 2
   (e) π
   (f) Does not exist

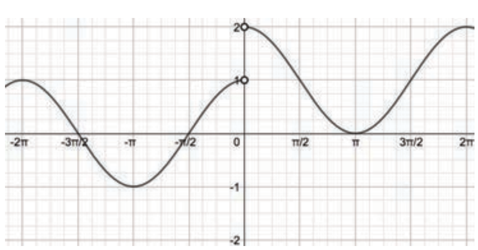
2. $\lim_{x \to 0} h(x) =$
   (a) -2
   (b) 0
   (c) 1
   (d) 2
   (e) π
   (f) Does not exist

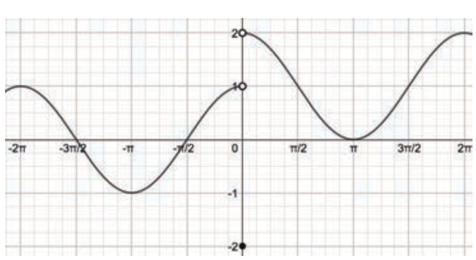
3. $\lim_{x \to 0^+} h(x) =$
   (a) -2
   (b) 0
   (c) 1
   (d) 2
   (e) π
   (f) Does not exist

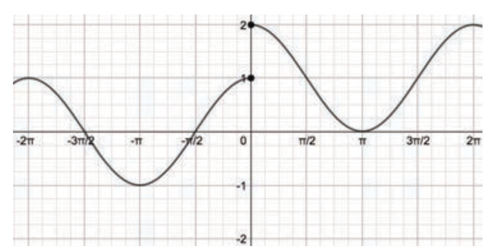
4. $h(0) =$
   (a) -2
   (b) 0
   (c) 1
   (d) 2
   (e) π
   (f) Does not exist

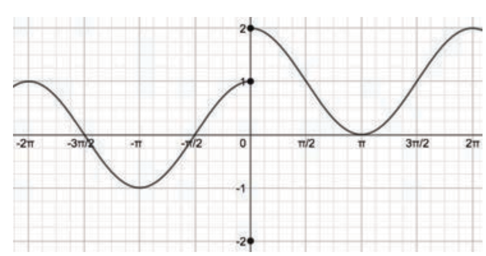
5. Circle the graph that represents the graph of $h(x)$.

(a) 

(b) 

(c) 

(d) 

(e) 

(f) None of the above.

6. (12pts) Who correctly evaluated $\lim_{t\to 0} \frac{8t}{3\sin(t) - t}$?

Below are three different student solutions to the limit. Read over each solution from each student and determine who correctly evaluated the limit and why.

Taylor's Solution:

$$\frac{8 \cdot 0}{3\sin(0) - 0} = \frac{0}{0 - 0} = \frac{0}{0} \qquad \text{The limit does not exist.}$$

Jimminy's Solution:

$$\frac{8t}{3\sin(t) - t} = \frac{8t}{t\left(3 \cdot \frac{\sin(t)}{t} - 1\right)} = \frac{8}{3 \cdot \frac{\sin(t)}{t} - 1} = \frac{8}{3 \cdot 1 - 1} = \frac{8}{2} = 4$$

Margo's Solution:

$$\lim_{t\to 0} \frac{8t}{3\sin(t) - t} = \lim_{t\to 0} \frac{8t}{t\left(3 \cdot \frac{\sin(t)}{t} - 1\right)} = \lim_{t\to 0} \frac{8}{3 \cdot \frac{\sin(t)}{t} - 1} = \frac{8}{3 \cdot 1 - 1} = \frac{8}{2} = 4$$

Taylor **correctly / incorrectly** (circle one) evaluated the limit. If you circled **incorrectly**, state why Taylor's evaluation is incorrect.

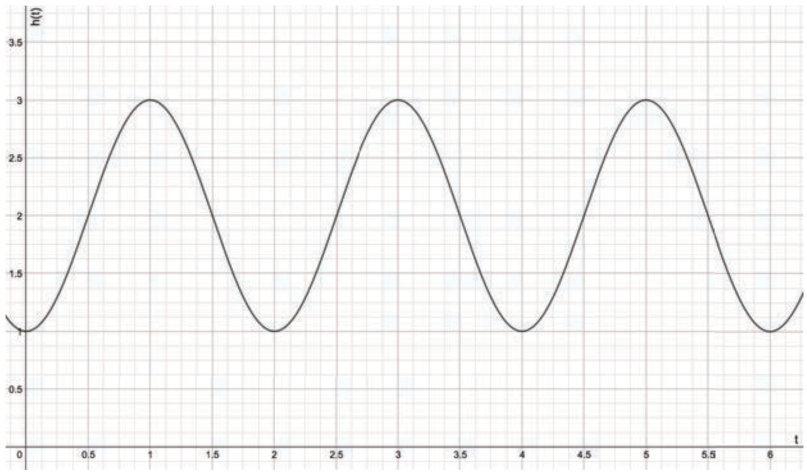
Jimminy **correctly / incorrectly** (circle one) evaluated the limit. If you circled **incorrectly**, state why Jimminy's evaluation is incorrect.

Margo **correctly / incorrectly** (circle one) evaluated the limit. If you circled **incorrectly**, state why Margo's evaluation is incorrect.

7. (15pts) Below is a graph of the position of a mass oscillating up and down on a spring over time. The function that gives the height above the ground, h, of the spring at time, t, is given by

$$h(t) = -\cos(\pi t) + 2$$

where time is measured in seconds, sec, and height is measured in centimeters, cm.



(a) (2pts) At what times during the first 4 seconds is the mass at its highest point?

(b) (9pts) Find the average speed of the spring on the following time intervals. Round values to 4 decimal places.

i. $[2.5, 2.51]$

ii. $[2.5, 2.501]$

iii. $[2.5, 2.5001]$

(c) (4pts) Based on your answers above, how fast would you expect the mass to be moving at exactly t=2.5? Explain.

8. (3pts) From the *mathematical definition* of continuity, we know a function $f(x)$ is continuous at an interior point $x = c$ of its domain if and only if (**circle one**).

   (a) There are no holes, vertical asymptotes, or jumps at $x = c$.
   (b) $\lim_{x \to c} f(x)$ exists and is a real number.
   (c) $\lim_{x \to c} f(x) = f(c)$.
   (d) $f(c)$ exists.

9. (12pts) Consider the function $V(x)$ given by

$$V(x) = \begin{cases} x - 2b, & x < 0 \\ a + 1, & x = 0 \\ x^2 + b, & x > 0 \end{cases}$$

where a and b are constants.

   (a) Find the following. Simplify your results and write your answers in terms of a and b.
      i. (2pts) $V(0) =$

      ii. (3pts) $\lim_{x \to 0^-} V(x) =$

      iii. (3pts) $\lim_{x \to 0^+} V(x) =$

   (b) (4pts) Find a, b such that $V(x)$ is continuous at every point in its domain. Write your answers in the blanks below. Be sure to provide supporting work.

$a =$ ____________ $b =$ ____________

10. (15pts) You are grinding engine cylinders for a company. You receive an order for cylinders that requires a circular cross-sectional area of 9 in^2.

(a) In the blank below, write the function that relates the circular cross-sectional area, A, and the cylinder diameter, d.

$A(d) =$ ______________________

(b) What is the *perfect* diameter? i.e. What diameter will result in a circular cross-sectional area of 9 in^2? Your answer should be written to **4 decimal places**.

$d_0 =$ ______________________

(c) The circular cross-sectional area must be within 0.01 in^2 of 9 in^2. Algebraically determine the interval around d_0 will ensure that corresponding output values are within 0.01 in^2 of 9 in^2.

__________ $< d_0 <$ __________

(d) How much can you deviate from the *perfect* diameter and still have the circular cross-sectional area still be within 0.01 in^2 of 9 in^2? (CIRCLE ALL CORRECT RESPONSES)

i. 0.0015 in

ii. 0.0018 in

iii. 0.0019 in

iv. 0.0022 in

v. None of the above

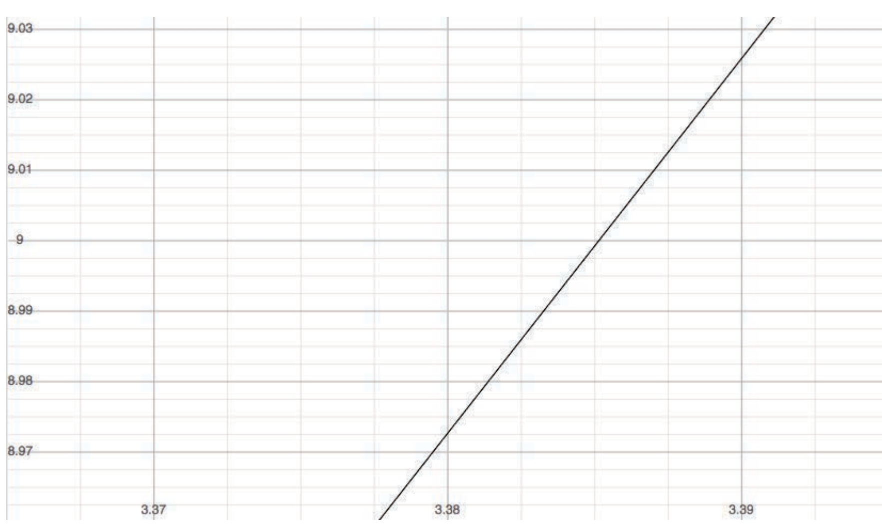
(e) Below is the portion of the graph of $A(d)$ relevant for this problem. Using the graph, label the following:

i. $d = d_0$

ii. $A(d) = 9$ in^2

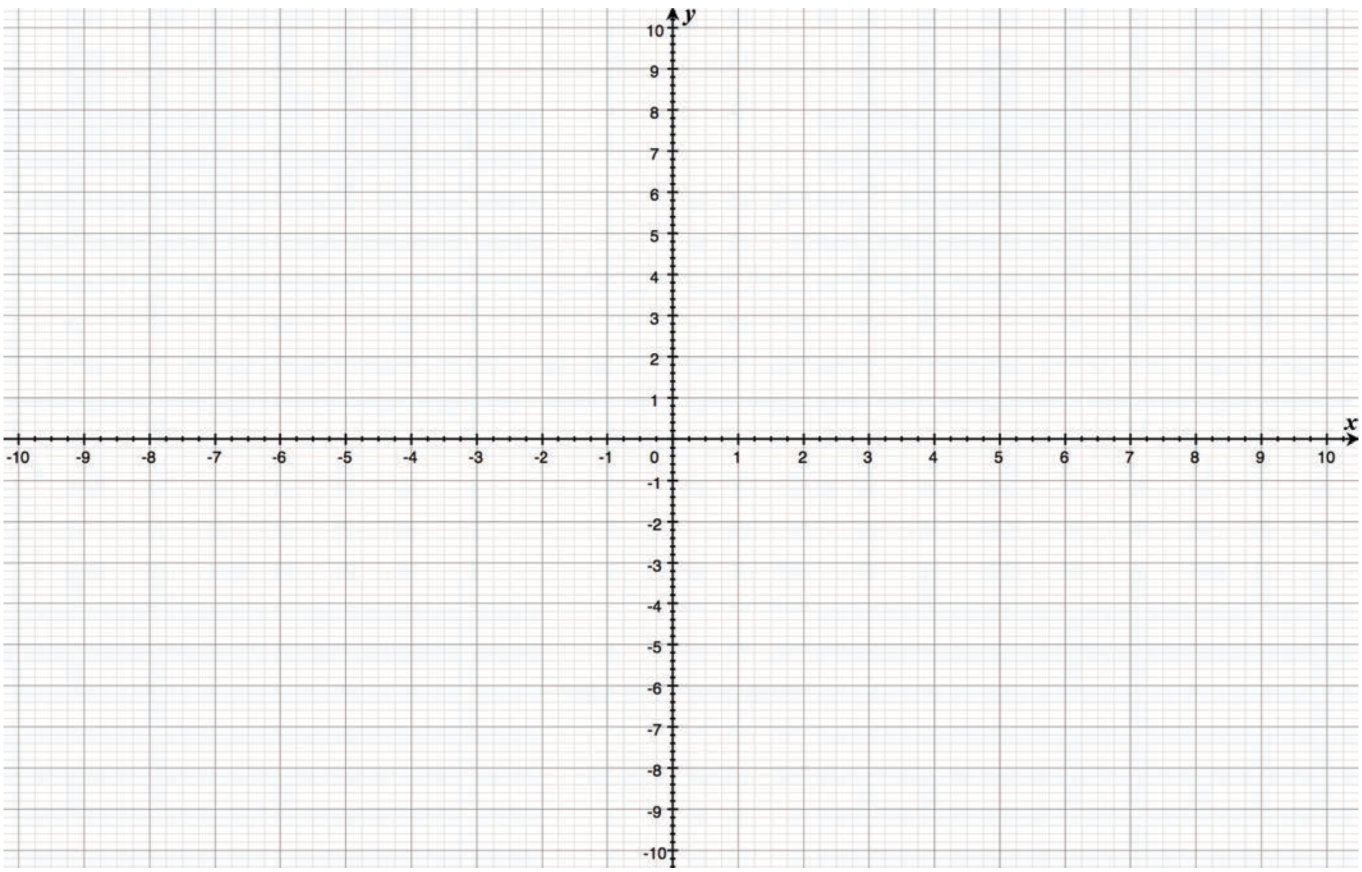
iii. The interval of ± 0.01 in^2 around $A(d) = 9$ in^2.

iv. The interval you found in part (c).



11. (12pts) Sketch the graph of **a function** that has the following properties:

- $\lim_{x \to -\infty} F(x) = \infty$
- $F(-5) = -5$
- $\lim_{x \to -5^-} F(x) = -5$
- $\lim_{x \to -5^+} F(x) = -3$
- $\lim_{x \to 0} F(x) = \infty$
- $F(5) = -3$
- $\lim_{x \to 5^-} F(x) = -3$
- $\lim_{x \to 5^+} F(x) = -5$
- $\lim_{x \to \infty} F(x) = -7$



$F(x)$ has a vertical asymptote at ____________.

$F(x)$ has as a horizontal asymptote at ____________.

12. (16pts - 4pts each) Indicate whether each of the following statements is **True** or **False**. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample. (A counterexample is an example of a function for which the "if" part of the statement is true, but the "then" part is false.) A graph with an explanation can be used as a counterexample.

If you use a term or phrase such as *continuity* or *average rate of change*, be sure to state the definition of the term or phrase that you used.

(a) If $f(\pi) = \pi$ and $f(x)$ is continuous, then $\lim_{x \to \pi^-} f(x) = \pi$.

(b) If the function $f(x)$ has $x = 2$ as a vertical asymptote, then $f(2)$ cannot exist.

(c) If $\lim_{x \to -\infty} g(x) = -3$, then $\lim_{x \to \infty} g(x) = -3$.

(d) If $\lim_{x \to 17} h(x)$ does not exist, then $h(17)$ cannot exist.