

NAME: \_\_\_\_\_

SECTION: \_\_\_\_\_ TIME: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

**Instructions:** The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have one hour and 45 minutes to work on the exam.

Problem	Points	Score
1	15	
2	14	
3	15	
4	15	
5	15	
6	14	
7	12	
Total	100	

## CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

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(Signature)

1. (15 points) Compute the derivatives of the following functions. You do NOT need to simplify your answer. Use parentheses to indicate multiplication where appropriate, and make sure your notation is correct.

(a)  $f(x) = 4x^2e^{-2x+1}$

(b)  $g(x) = \frac{x^3}{\ln(x)}$

(c)  $h(x) = \tan(x + 3\pi x^2)$

(d)  $m(x) = k(3x - 2)^4 + \cos(4x - 2)$ , where  $k$  is a constant.

(e)  $p(x) = e^{\sin(3x)+2}$

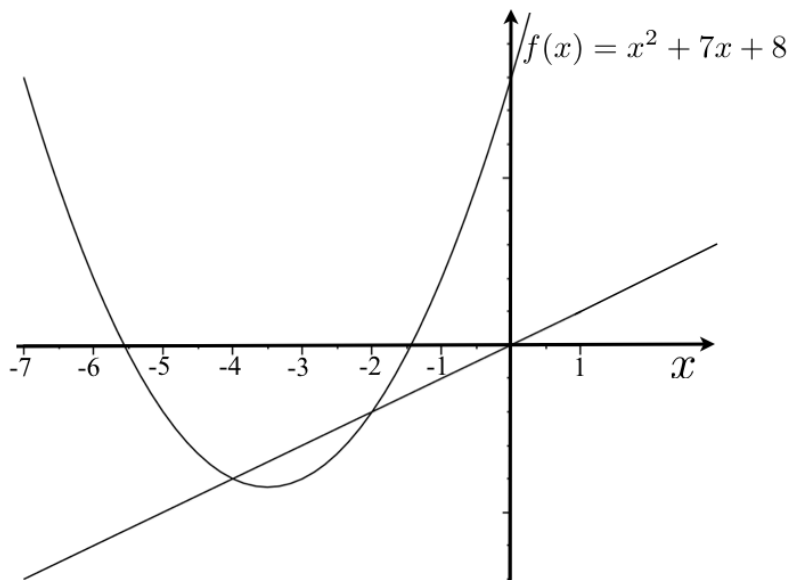
2. (14 points) Consider the following discrete-time dynamical system:

$$x_{t+1} = x_t^2 + 7x_t + 8$$

(a) Find the equilibria algebraically.

(b) Apply the Stability Test/Slope Criterion to each of the equilibria you found in (a). What can you conclude?

(c) The updating function is graphed below, along with the diagonal. Cobweb for at least 3 steps starting from the initial condition  $x_0 = -3.5$ . Describe in detail the long-term behavior of this solution.



3. (15 points) Suppose that the population  $x_t$  of octopuses satisfies the discrete-time dynamical system

$$x_{t+1} = \frac{(p-5)x_t}{1+2x_t},$$

where  $p$  is a parameter.

- (a) Verify algebraically that the equilibria are  $x^* = 0$  and  $x^* = \frac{p-6}{2}$ .

- (b) Show that the derivative of the updating function is  $\frac{p-5}{(1+2x)^2}$ .

- (c) Use the Stability Criterion/Stability Theorem to find the range of  $p$  for which  $x^* = 0$  is stable.

- (d) Use the Stability Criterion/Stability Theorem to find the range of  $p$  for which  $x^* = \frac{p-6}{2}$  is *unstable*.

4. (15 points) A diver jumps from a diving board. Her height (in feet) above the water at time  $t$  (in seconds) is given by  $h(t) = -16t^2 + 16t + 32$ , and she jumps at time  $t = 0$ .

(a) How high is the diving board from the water?

(b) At what time  $t$  does the diver hit the water? (That is, at what time is the height equal to 0?)

(c) Find the velocity  $v(t)$  and the acceleration  $a(t)$  of the diver at time  $t = 1$ .

(d) At what time does the diver reach her maximum height above the water? What is this height?

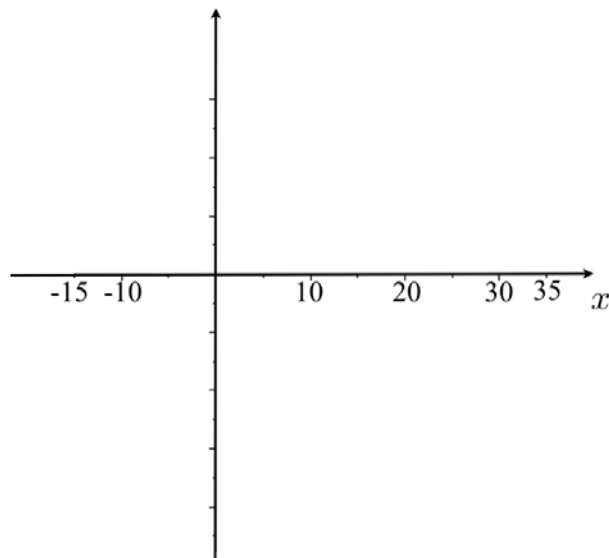
5. (15 points) Consider the function  $f(x) = x^3 - 30x^2 + 600$  on the interval  $[-15, 35]$ .

(a) Calculate  $f'(x)$ , and use this to find all the critical points of  $f(x)$ .

(b) Calculate  $f''(x)$ , and use this to find regions where  $f(x)$  is concave up or concave down.

(c) For each critical point, determine if  $f(x)$  has a local maximum or a local minimum there. Justify your answer using the first or second derivative test.

(d) Use the information found above to sketch a graph of the function  $f(x)$  on the interval  $[-15, 35]$ . Indicate where any local maxima, local minima, global maxima, or global minima occur.



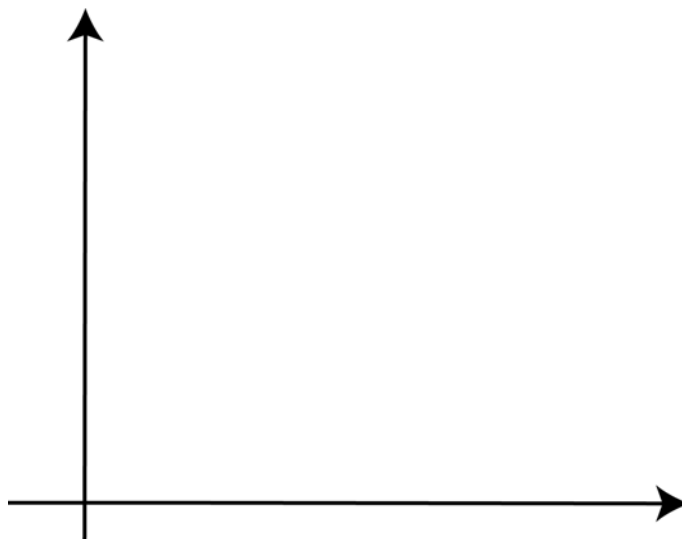
6. (14 points) For the following functions,  $f(x)$ , find  $f_\infty(x)$ , the leading behavior of  $f(x)$  as  $x \rightarrow \infty$ , and  $f_0(x)$ , the leading behavior of  $f(x)$  as  $x \rightarrow 0$ .

(a)  $f(x) = 17e^{-4x} + 4x^{-2} + 57x^{-8}$

(b)  $f(x) = \frac{13e^{-7x} + 17x^2 + x}{e^{3x} + x^2 - x}$

(c)  $f(x) = \frac{15x}{3+x}$

(d) For the function in part (c), use the method of matched leading behaviors to sketch the graph of  $f(x)$  for  $x \geq 0$ . Graph and indicate where you have graphed  $f_\infty(x)$ ,  $f_0(x)$ , and  $f(x)$ .



7. (12 points) Evaluate the following limits. Show all of your work. If you use leading behavior, justify your answer by explaining all of your steps. If you use L'Hopital's Rule, justify why it can be applied each time you use it.

$$(a) \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$$

$$(b) \lim_{x \rightarrow \infty} \frac{\ln(3x^2)}{x^2 + x}$$

$$(c) \lim_{x \rightarrow \infty} \frac{3e^{2x} + x^2 + 100}{\ln(x) + e^x}$$

$$(d) \lim_{x \rightarrow 0} \frac{3e^{2x} + 1}{x + 1}$$