Name: ███████████████   **Exam Version B**

This exam is multiple choice. Make sure you fill in your answers on the scantron answer sheet provided. Fill in your name, student ID number, and the exam version on the scantron answer sheet. You may use this exam, and the blank pages provided, to work out the problems. You must hand in this exam as well as the scantron answer sheet. To hand in your exam, be prepared to show your picture ID. This is a closed book, closed notes exam. Calculators are allowed but must be equivalent to a TI-83/84; no TI-89s or equivalent are allowed. No cell phones are permitted outside of your bag at anytime during the test.

Use the following to answer questions 1 through 5

The profit from producing and selling x items is given by the equation

$$P(x) = 22x - 3x^2 - 17.$$

1. Which of the following is the best interpretation of $P(5)$?

    (a) $P(5)$ is the approximate profit of producing and selling items 1 through 5.

    (b) $P(5)$ is the approximate profit of producing and selling only item 5.

    (c) $P(5)$ is the exact profit of producing and selling items 1 through 5.

    (d) $P(5)$ is the exact profit of producing and selling only item 5.

    (e) $P(5)$ is the average profit per item of producing and selling items 1 through 5.

2. Which of the following is the best interpretation of $P'(5)$?

    (a) $P'(5)$ is the approximate profit of producing and selling items 1 through 5.

    (b) $P'(5)$ is the approximate profit of producing and selling only item 6.

    (c) $P'(5)$ is the exact profit of producing and selling items 1 through 5.

    (d) $P'(5)$ is the exact profit of producing and selling only item 6.

    (e) $P'(5)$ is the average profit per item of producing and selling items 1 through 5.

3. Which of the following would best approximate the cost of producing the first 6 items?

    (a) $P'(6)$

    (b) $P(5)$

    (c) $P(5) + P'(5)$

    (d) $P'(5)$

    (e) $P(5) + P(1)$

4. Is $P(x)$ increasing or decreasing at $x = 5$?

    (a) increasing

    (b) decreasing

    (c) neither

5. What is the concavity of $P(x)$ at $x = 5$?

    (a) concave up

    (b) concave down

    (c) neither

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Use the following graph to answer questions 6 through 8.
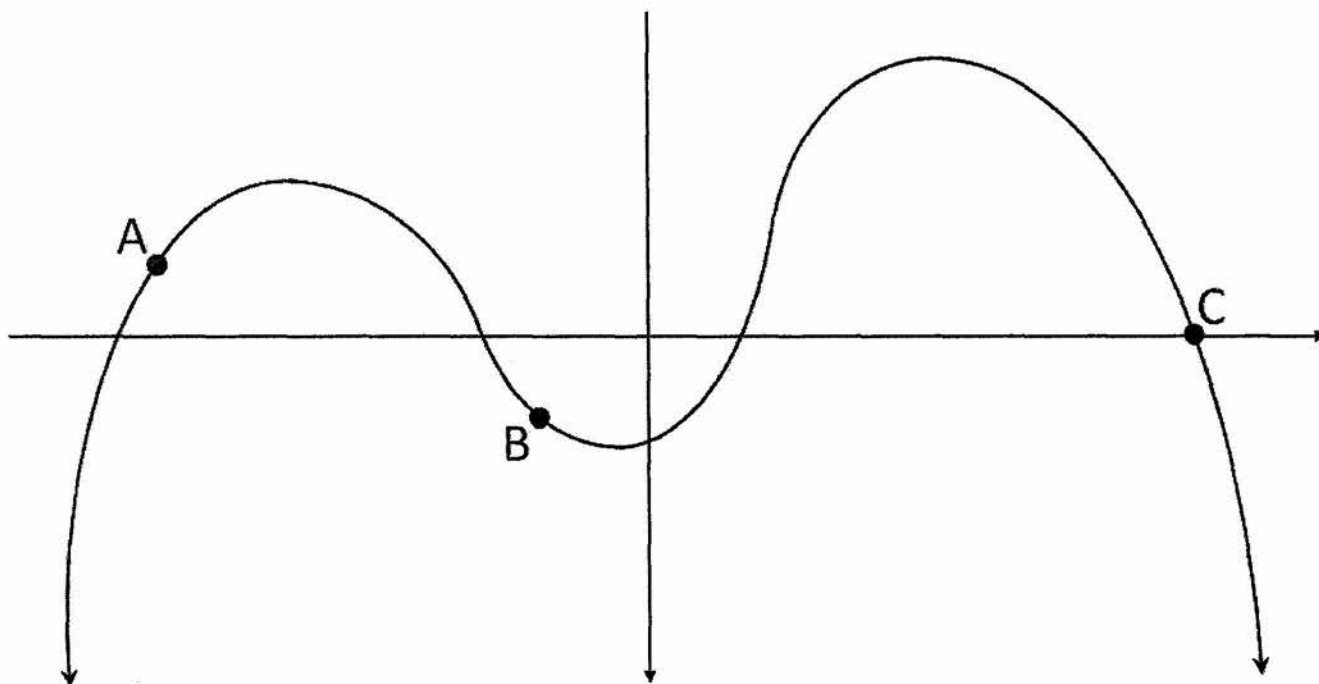
6. At point A, which of the following are true?

    (a) $f(x) > 0$, $f'(x) > 0$, and $f''(x) > 0$.

    (b) $f(x) > 0$, $f'(x) > 0$, and $f''(x) < 0$.

    (c) $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$.

    (d) $f(x) < 0$, $f'(x) < 0$, and $f''(x) < 0$.

    (e) None of the above.

7. At point B, which of the following are true?

    (a) $f(x) > 0$, $f'(x) > 0$, and $f''(x) > 0$.

    (b) $f(x) < 0$, $f'(x) > 0$, and $f''(x) = 0$.

    (c) $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$.

    (d) $f(x) > 0$, $f'(x) > 0$, and $f''(x) < 0$.

    (e) None of the above.

8. At point C, which of the following are true?

    (a) $f(x) > 0$, $f'(x) > 0$, and $f''(x) > 0$.

    (b) $f(x) = 0$, $f'(x) > 0$, and $f''(x) = 0$.

    (c) $f(x) = 0$, $f'(x) > 0$, and $f''(x) > 0$.

    (d) $f(x) = 0$, $f'(x) < 0$, and $f''(x) < 0$.

    (e) None of the above.

9. Which one of the following equations would you use when finding critical values?

   (a) $f''(x) = 0$

   (b) $f'(x) = 0$

   (c) $f(x) = 0$

   (d) $f(x) = f'(x)$

   (e) None of the above.

10. What are the critical values of $g(x) = x^4 - 4x^3$?

   (a) $x = 0$, and $x = 3$

   (b) $x = 0$ only

   (c) $x = 0$ and $x = 4$

   (d) $x = 0$, $x = 4$, and $x = 3$

   (e) None of the above

$$x^4 - 4x^3$$
$$4x^5 - 12x^2$$
$$20x^4 - 24x = 0$$
$$4x(5x^3 - 6)$$

11. Suppose $f(x)$ has only one critical value at $x = 10$. If $f'(9)$ is positive and $f'(12)$ is negative, there is

   (a) an inflection point at $x = 10$

   (b) a relative maximum at $x = 10$

   (c) a relative minimum at $x = 10$

   (d) not enough information to tell

   (e) neither a maximum, a minimum, nor an inflection point at $x = 10$
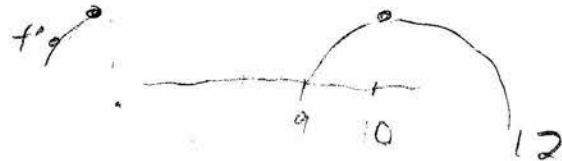
12. If the critical values of $h(x)$ are $x = -2$, $x = 0$, and $x = 5$, and the first derivative of $h(x)$ is $h'(x) = x^3 - 3x^2 - 10x$, classify each critical value as a relative maximum or minimum of $h(x)$.

   (a) $h(x)$ has a local minimum at $x = -2$, a local maximum at $x = 5$, and neither at $x = 0$.

   (b) $h(x)$ has local maxima at $x = -2$, $x = 0$, and $x = 5$, and no local minima.

   (c) $h(x)$ has local minima at $x = -2$ and $x = 5$ and a local maximum at $x = 0$.

   (d) $h(x)$ has a local minimum at $x = 5$, and local maxima at $x = -2$, and $x = 0$.

   (e) None of the above

13. Given $f(4) = 3$, $f'(4) = 0$, and $f''(4) = 10$, there is a

   (a) an inflection point at $x = 4$

   (b) a relative maximum at $x = 4$

   (c) a relative minimum at $x = 4$

   (d) not enough information to tell

   (e) neither a maximum, a minimum, nor an inflection point at $x = 4$

14. What is the absolute minimum of $g(x) = x^3 - 6x^2 + 1$ on the interval $[-1, 2]$?

$g(x) = x^3 - 6x^2 + 1$

$3x^2 - 12x$

$3x(x - 4)$

$x = 0 \quad x = 4$

   (a) 1
   (b) -6
   (c) -15
   (d) -31
   (e) none of the above

15. You have a 12 inch by 12 inch piece of cardboard. You would like to build a box by cutting congruent squares out of the corners and then folding up the sides. What is the length of the side of the square you should cut out in order to maximize the volume of the box?

   (a) 2 inches
   (b) 3 inches
   (c) 4 inches
   (d) 6 inches
   (e) none of the above

$(12 - 2x)(12 - 2x) x = 0$

$(144 - 24x - 24x + 4x^2) x$

$144x - 24x^2 - 24x^2 + 4x^3$

$4x^3 - 48x^2 + 144x = 0$

16. A music venue is trying to decide how much to charge for tickets to a concert. 3000 people show up if the ticket price is $30. For every dollar decrease in ticket price, 100 more people come to the show. Each person will spend an average of $8 on concessions. What ticket price will optimize revenue for the venue?

People

$(3,000 + 100x)(30 - x) + (3,000x + 100) \cdot 8$

$(90,000 + 3,000 - 100x^2)(24,000 + 800)$

   (a) $22
   (b) $26
   (c) $30
   (d) $34
   (e) none of the above

17. The sum of two numbers is 34. What is the largest that their product can be?

   (a) 578
   (b) 289
   (c) 288
   (d) 34
   (e) none of the above

18. Find $\dfrac{dy}{dx}$ if

$$x^2 + 2xy = 3y^2.$$

(a) $\dfrac{dy}{dx} = \dfrac{2x+2}{6y}$

(b) $\dfrac{dy}{dx} = 2x + 2y - 6y$

(c) $\dfrac{dy}{dx} = \dfrac{2x+2y}{6y-2x}$

(d) $\dfrac{dy}{dx} = \dfrac{2x}{6y-2}$

[handwritten: $6y$]

[handwritten:
$2x + 2\left(x\frac{dy}{dx} + y\right) =$
$2x + 2x\frac{dy}{dx} + 2y = 6y$

$\dfrac{2x+2y}{6y-2x}$ *]*

19. What is the slope of the line tangent to the graph of $y^2 + x^2 = 25$ at the point $(4,3)$?

(a) -1

(b) $-\dfrac{3}{4}$

(c) $-\dfrac{4}{3}$

(d) $-\dfrac{9}{16}$

[handwritten: $-\dfrac{9}{16}$]

20. A ladder 30 feet long leans against a vertical wall. If the lower end is being moved away from the wall at the rate of 10 feet per second, how fast is the height at the top decreasing when the lower end is 8 feet from the wall?

(a) 10.0 feet per second

(b) 2.8 feet per second

(c) 15.7 feet per second

(d) 18.0 feet per second

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