Math 155 **Exam 2** **Fall 2012**

NAME: ____

SECTION: ____________________ TIME: ____________________

INSTRUCTOR: ____

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have one hour and 45 minutes to work on the exam.

Problem	Points	Score
1	15	
2	15	
3	8	
4	5	
5	15	
6	15	
7	14	
8	13	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

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(Signature)

1. (15 points) Compute the derivatives of the following functions. You do NOT need to simplify your answer. Use parentheses to indicate multiplication where appropriate, and make sure your notation is correct.

   (a) $f(t) = \ln(10) - 2t^4 + \sqrt[5]{t^4} + 9t^{-\frac{1}{3}}$

   (b) $g(x) = 2^x(x^3 + \pi)$

   (c) $r(x) = 3e^{x\sin(x)+2}$

   (d) $q(u) = \frac{(u^2+1)^{12}}{\cos(u)}$

   (e) $p(t) = \tan(\ln(t) + k)$, where k is a constant.

2. (15 points) Suppose that the population b_t of boreal owls satisfies the discrete-time dynamical system

$$b_{t+1} = \frac{3b_t}{r + 2b_t},$$

where r is a positive parameter.

(a) Find all equilibria of the discrete-time dynamical system. For what range of values of r is there a positive equilibrium?

(b) Show that the derivative of the updating function is $\frac{3r}{(r+2b)^2}$.

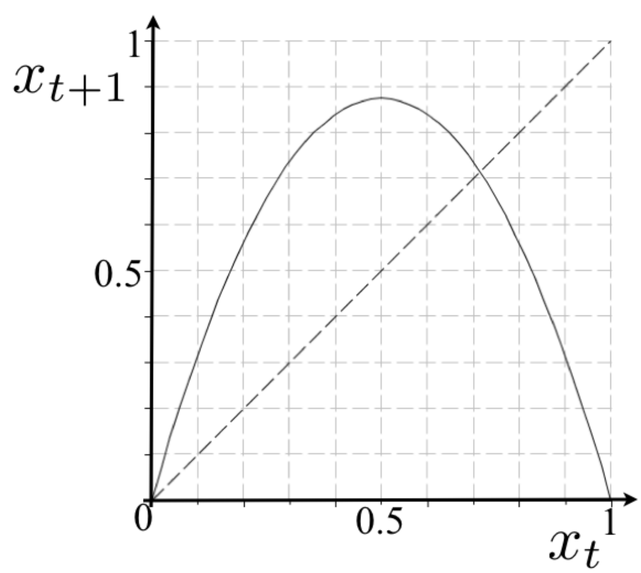
(c) For each equilibrium, use the Stability Criterion/Stability Theorem to find the range of r for which that equilibrium is stable.

3. (8 points) Jumpie the Monkey jumps from a tree branch down to the ground. Her height (in meters) above the ground at time t (in seconds) is given by $h(t) = -4.9t^2 + 3t + 8$, and she jumps at time $t = 0$.

   (a) Find the velocity $v(t)$ and the acceleration $a(t)$ of the monkey at time $t = 1$.

   (b) At what time does the monkey reach her maximum height above the ground? What is this height? Use Calculus to justify your answer.

4. (5 points) The updating function for the discrete-time dynamical system $x_{t+1} = 3.52x_t(1-x_t)$ is graphed below, along with the diagonal. Cobweb for at least 8 steps starting from $x_0 = 0.5$. What is the long-term behavior of this system?



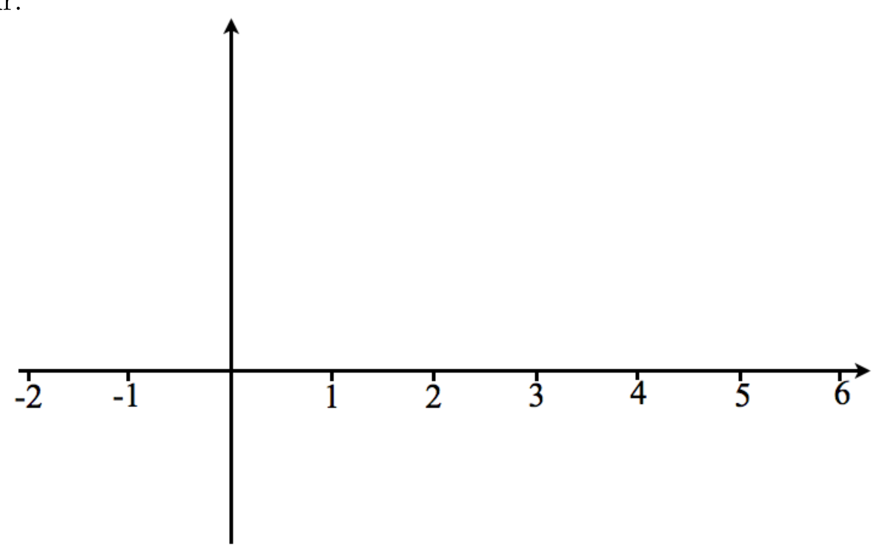
5. (15 points) Consider the function $f(x) = x^3 - 3x^2 - 9x + 2$ on the interval $[-2, 6]$.

(a) Calculate $f'(x)$, and use this to find all the critical points of $f(x)$.

(b) Calculate $f''(x)$, and use this to find regions where $f(x)$ is concave up or concave down.

(c) For each critical point, determine if $f(x)$ has a local maximum or a local minimum there. Justify your answer using the first or second derivative test.

(d) Use the information found above to sketch a graph of the function $f(x)$ on the interval $[-2, 6]$. Indicate where any local maxima, local minima, global maxima, or global minima occur.



6. (15 points) Consider the discrete-time dynamical system

$$M_{t+1} = M_t(10 - h - M_t) - hM_t$$

describing a population M_t of mussels in a lake being harvested at rate h. In this model, the activity related to harvesting affects the carrying capacity of the population.

(a) Find the nonzero equilibrium population M^* as a function of h. What is the largest value of h consistent with a nonnegative equilibrium?

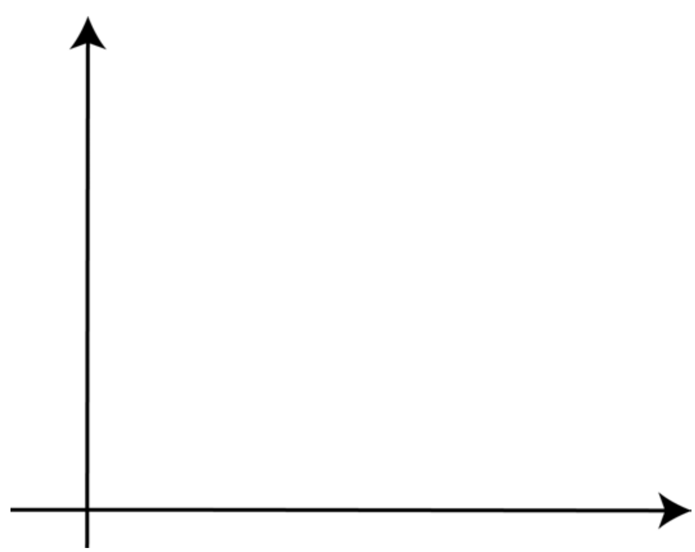
(b) The equilibrium harvest is given by $P(h) = hM^*$, where M^* is the equilibrium you found in part (a). Find the value of h that maximizes $P(h)$. Use the first or second derivative test to justify that this value of h gives a local maximum.

7. (14 points) For the following functions, $f(x)$, find $f_\infty(x)$, the leading behavior of $f(x)$ as $x \to \infty$, and $f_0(x)$, the leading behavior of $f(x)$ as $x \to 0$.

(a) $f(x) = \mathrm{e}^{-5x} + 100x^4 + 30x^2 + \mathrm{e}^{2x} + x^{-1}$

(b) $f(x) = \dfrac{5x^3 + x^{-2}}{10 + x^{-1} + x^2}$

(c) For the function in part (b), use the method of matched leading behaviors to sketch the graph of $f(x)$ for $x \geq 0$. Graph and indicate where you have graphed $f_\infty(x)$, $f_0(x)$, and $f(x)$.



8. (13 points) Evaluate the following limits. Show all of your work. If you use leading behavior, justify your answer by explaining all of your steps. If you use L'Hopital's Rule, justify why it can be applied each time you use it.

(a) $\lim_{x \to \infty} \frac{2e^{-x} + \ln(x) + 5}{e^{-2x} + x^7}$

(b) $\lim_{x \to 0} \frac{e^x - 1}{\sin(4x)}$

(c) $\lim_{x \to \infty} \frac{\ln(3x+1)}{\ln(10x^2+2)}$