NAME: ______________________

Instructor: ______________________

Time your class meets: ______________

Math 160 Calculus for Physical Scientists I
Exam 2
March 12, 2015, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" -*Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a calculator on this exam. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____
(Signature)

____
(Date)

Please do not write in this space.

1-4. (18pts)	
5. (12pts)	
6. (8pts)	
7. (15pts)	
8. (15pts)	
9. (12pts)	
10. (15pts)	
TOTAL	

Multiple Choice: Circle only one answer for each problem unless stated otherwise. Answers will be graded as right or wrong with no partial credit.

1. (3pts) If $f'(7)$ exists and $f'(7) = 4$, then $\lim_{x\to 7} f(x)$

   (a) need not exist.

   (b) is 4.

   (c) is 0.

   (d) is $f(7)$.

   (e) None of the above.

2. (3pts) A train travels along a straight track. The distance it has traveled after x hours is given by a function $f(x)$. An engineer is walking along the top of the box cars at the rate of 2.25 mi/hr in the same direction as the train is moving. The speed of the man relative to the ground is

   (a) $f(x) + 2.25$

   (b) $f(x) - 2.25$

   (c) $f'(x) + 2.25$

   (d) $f'(x) - 2.25$

   (e) None of the above.

3. (8pts) Which of the following statements are true?
   (CIRCLE ALL CORRECT RESPONSES)

   (a) If $f(x)$ is differentiable at $x = 42$, then $f(x)$ is also continuous at $x = 42$.

   (b) If $f(x)$ is continuous at $x = 42$, then $f(x)$ is also differentiable at $x = 42$.

   (c) If $f'(x)$ is continuous at $x = 42$, then $f(42)$ exists.

   (d) If $f(42) = \lim_{x\to 42} f(x)$, then $f'(42)$ exists.

   (e) If $\lim_{h\to 0} \dfrac{f(42+h) - f(42)}{h} = -2$, then $f'(42)$ exists.

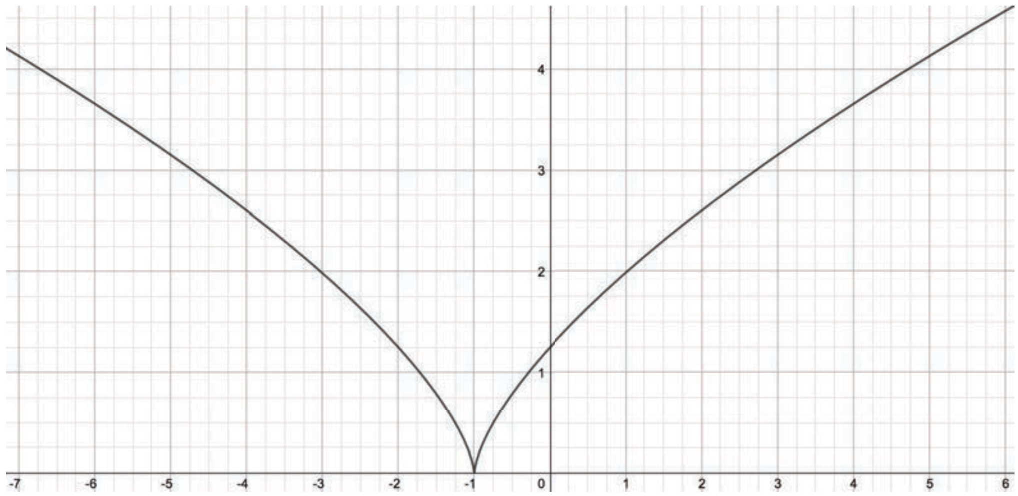
   (f) If $\lim_{h\to 0} \dfrac{f(42+h) - f(42)}{h} = -2$, then $f(42)$ exists.

   (g) If $\lim_{x\to 42} f(x)$ exists, then $f(42)$ exists.

   (h) None of the above statements are true.

4. (4pts) The graph of $f(x) = (x+1)^{2/3}$ below has a cusp at $x = -1$. Which of of the following mathematical statements are consistent with the behavior of the function at $x = -1$?

(CIRCLE ALL CORRECT RESPONSES)



(a) $\lim_{h \to 0^-} \frac{f(-1+h) - f(-1)}{h} = -1$

(b) $\lim_{h \to 0^+} \frac{f(-1+h) - f(-1)}{h} = 1$

(c) $\lim_{h \to 0^-} \frac{f(-1+h) - f(-1)}{h} = -\infty$

(d) $\lim_{h \to 0^+} \frac{f(-1+h) - f(-1)}{h} = \infty$

(e) $\lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$ does not exist.

(f) $f'(-1)$ does not exist.

(g) $f'(-1)$ does exist.

(h) None of the above.

5. (12pts) Suppose that $f(x)$ denotes a function defined for all real numbers. The statement below is true sometimes. Give an example of a function for which it holds true and an example of a function for which it does not hold true. Explain your reasoning. Provide your answers by filling in the table below:

$$\boxed{\text{A function that is not continuous has an absolute maximum.}}$$

Example of True	Example of False
Why is the statement true for your example?	Why is the statement false for your example?

6. (8pts) The function of $f(x)$ is represented by the graph below.

Explain in complete sentences in terms of the graph what the equation $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ means. (Be sure to talk about secant lines and the tangent line for the function).

7. (15pts - 5pts each) Use the information provided in the table to compute the following derivatives. Be sure to show all of your work. An answer with no supporting work will receive no credit.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	2	3	4
2	$\frac{1}{2\pi}$	7	51	-1

$h(x) = f(x) \cdot \sin(\pi x)$

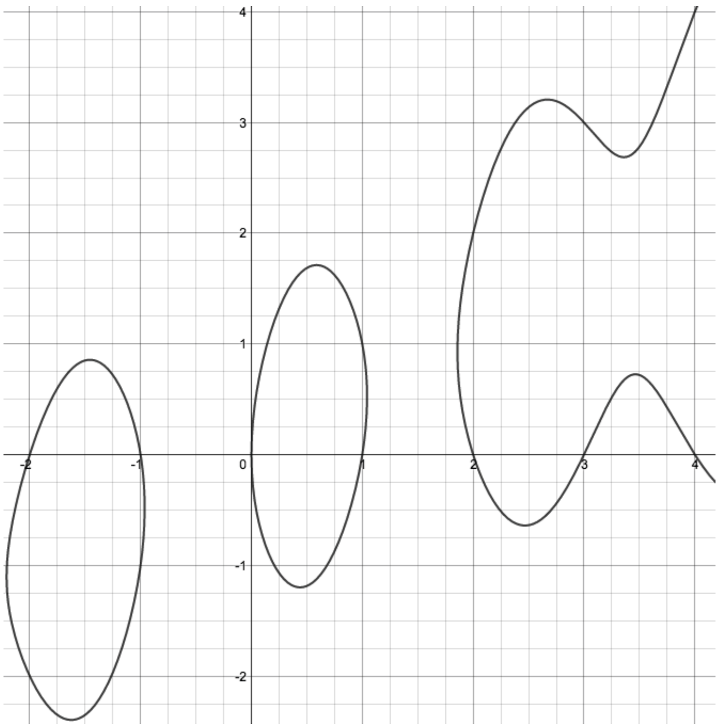
$k(x) = g(\tan(x))$

(a) $h'(2)$

(b) $k'(0)$

(c) $\left.\frac{d}{dx}\left(\frac{x}{g(x)}\right)\right|_{x=0}$

8. (15pts) The graph below is defined by the equation $y^2 = 2\sin(\pi x) + xy$



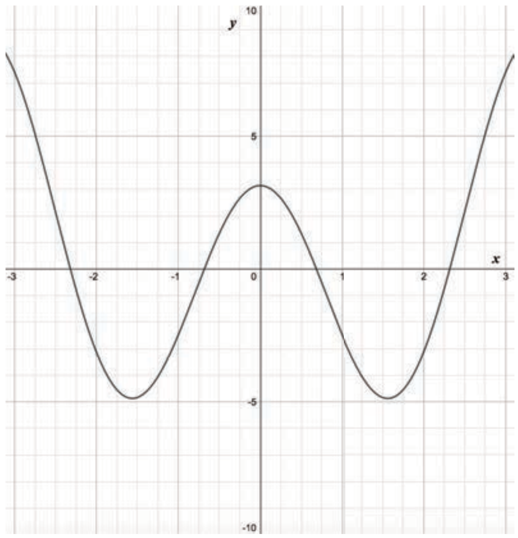
(a) Algebraically verify that the point $(-1, -1)$ lies on the graph above.

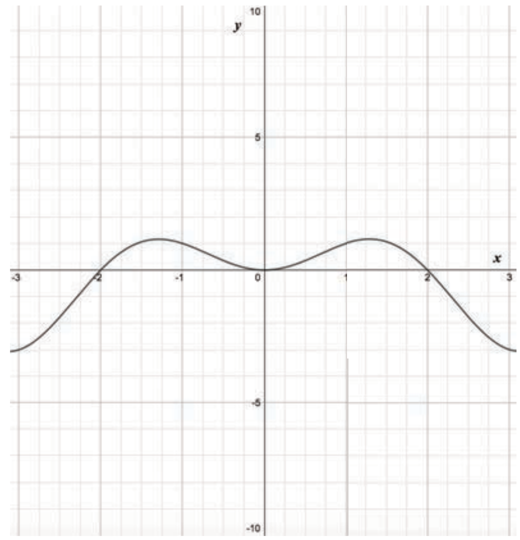
(b) In the graph above, draw the line tangent to the curve at the point $(-1, -1)$.

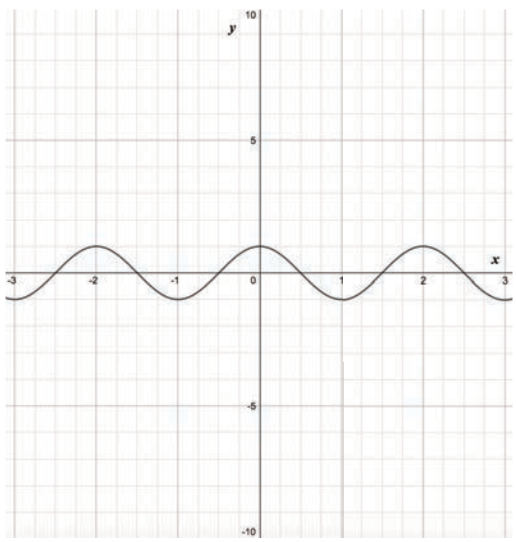
(c) Using calculus, determine $\dfrac{dy}{dx}$.

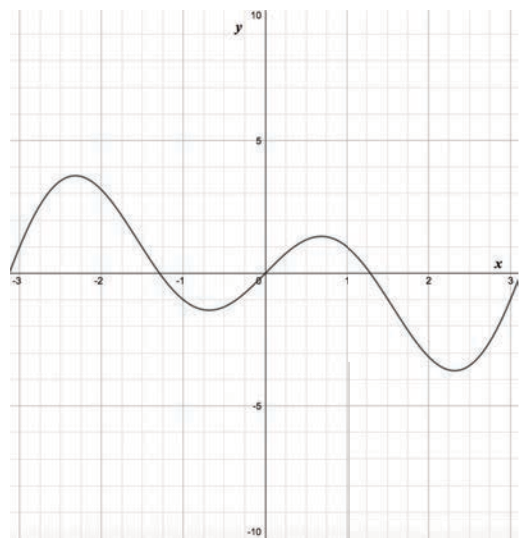
(d) Find the equation of the line tangent to the curve above at the point $(-1, -1)$. If you choose to use decimals, round all values to at least 6 decimal places. Exact form is also acceptable.

9. (a) (12pts) The graphs of a function and its derivatives ($f(x)$, $f'(x)$, and $f''(x)$) are shown below. A fourth, a random graph, is also included. Identify each graph as $f(x)$, $f'(x)$, $f''(x)$ or "Random" by filling in the blanks with the correct label.

(1) 

(2) 

(3) 

(4) 

Graph 1 = __________ Graph 2 = __________ Graph 3 = __________ Graph 4 = __________

(b) In 2-4 sentences, explain why the graph you chose as "Random" does not relate to the other graphs (i.e. was not chosen to be $f(x)$, $f'(x)$ or $f''(x)$). Your explanation should include a discussion of slopes.

10. (15pts) Below is a complete statement of the Mean Value Theorem from Thomas' Calculus (CSU edition):

Suppose that $y = f(x)$ is continuous over a closed interval $[a, b]$ and differentiable on the interval's interior (a, b). Then there is at least one point c in (a, b) at which

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Now consider the following problem.

Problem Statement: It took 14 seconds for a mercury thermometer to rise from $-19°C$ to $100°C$ when it was taken from a freezer and placed in boiling water. Show that at some point in time the mercury was rising at a rate of $8.5°C/sec$.

(a) In 2-4 sentences, explain why the Mean Value Theorem applies in this context.

(b) Evaluate the following expression in terms of the problem statement: $\dfrac{f(b) - f(a)}{b - a}$

(c) What does the Mean Value Theorem conclude in the context of the problem statement?