Exam II - Math 141 Spring 2016

Name: ____________________________ **Exam Version A**

This exam is multiple choice. There are 21 problems, each one worth 5 points, for a maximum possible total of 105 points (5 possible bonus points). Make sure you fill in your answers on the scantron answer sheet provided. Fill in your name, student ID number, and the exam version on the scantron answer sheet. You may use this exam to work out the problems. You must hand in this exam as well as the scantron answer sheet. To hand in your exam, be prepared to show your picture ID. This is a closed book, closed notes exam. Calculators are allowed but must be equivalent to a TI-83/84; no TI-89s or equivalent are allowed. No cell phones are permitted outside of your bag at anytime during the test.

GOOD LUCK!

1. If $f(x) = (5+x^3)^2 - (3+x^2)^4$, then $f'(x) =$

   (a) $2(3x^2) - 4(2x)^3$
   (b) $2(5+x^3) - 4(3+x^2)^3$
   (c) $2(5+x^3)(3x^2) - 4(3+x^2)^3(2x)$
   (d) $2(5+x^3)(x^3) - 4(3+x^2)^3(x^2)$

2. If $f(x) = (3x-1)^3(x+3)^2$, then $f'(x) =$

   (a) $3(3x-1)^2(2)(x+3)$
   (b) $3(3x-1)^2(3)(2)(x+3)$
   (c) $3(3x-1)^2(3)(x+3)^2 - (3x-1)^3(2)(x+3)$
   (d) $3(3x-1)^2(3)(x+3)^2 + (3x-1)^3(2)(x+3)$

3. If $y = \left(\dfrac{2x}{x^2+1}\right)^3$, then $\dfrac{dy}{dx} =$

   (a) $\left(\dfrac{2x}{x^2+1}\right)^3 \left(\dfrac{2(x^2+1)-4x^2}{(x^2+1)^2}\right)$
   (b) $3\left(\dfrac{2x}{x^2+1}\right)^2 \left(\dfrac{2(x^2+1)-4x^2}{(x^2+1)^2}\right)$
   (c) $3\left(\dfrac{2x}{x^2+1}\right)^2$
   (d) $3\left(\dfrac{2x}{x^2+1}\right)^2 \left(\dfrac{1}{x}\right)$

4. By the chain rule, $\dfrac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$. Hence, $\dfrac{d}{dx}[f(3x^2 - 5x + 1/2)] =$

   (a) $f'(3x^2 - 5x + 1/2)$
   (b) $f(3x^2 - 5x + 1/2) \cdot (6x-5)$
   (c) $f'(3x^2 - 5x + 1/2) \cdot (6x-5)$
   (d) $f'(3x^2 - 5x + 1/2) \cdot (3x^2 - 5x + 1/2)$

5. If $f(x) = x^4 - \dfrac{1}{x} + 2$, then $f''(2) =$

   (a) 0
   (b) 32.25
   (c) 47.75
   (d) 48.25

6. If the revenue (in dollars), $R(x)$, and cost (in dollars), $C(x)$, for the production and sale of x units of a product are given by

$$R(x) = 50x - 0.5x^2, \quad C(x) = 4x + 10,$$

   then the profit (in dollars), $P(x)$, equals

   (a) $-0.5x^2 - 46x - 10$
   (b) $-0.5x^2 + 46x - 10$
   (c) $-0.5x^2 - 46x + 10$
   (d) $0.5x^2 - 46x + 10$

Use the function $P(x)$ from problem 6 to answer questions 7 through 9.

7. Which of the following is the best interpretation of $P(10)$?

   (a) The exact profit from producing and selling the first 10 units is \$400.
   (b) The exact profit from producing and selling the 10th unit is \$400.
   (c) The approximate profit from producing and selling the first 10 units is \$400.
   (d) The approximate profit from producing and selling the 10th unit is \$400.

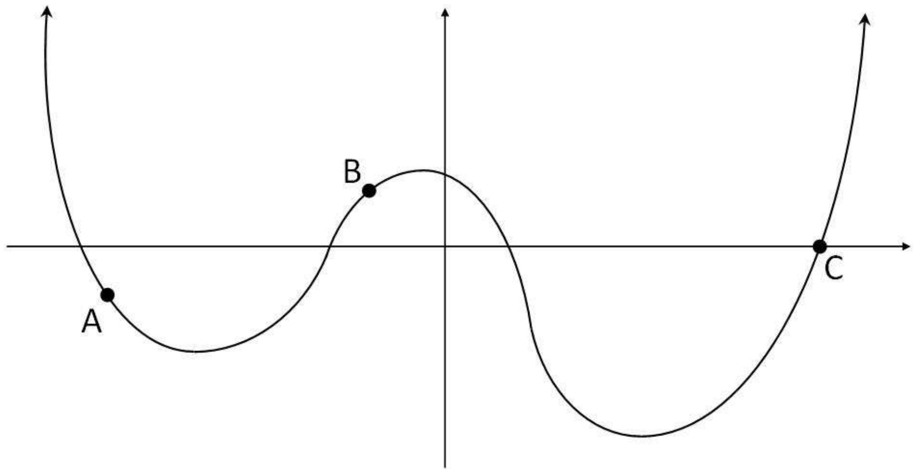
8. Which of the following is the best interpretation of $P'(10)$?

   (a) The exact profit from producing and selling the first 10 units is \$36.
   (b) The approximate profit from producing and selling the 10th unit is \$36.
   (c) The exact profit from producing and selling the first 11 units is \$36.
   (d) The approximate profit from producing and selling the 11th unit is \$36.

9. Given that $P'(20) = 26$ dollars per unit, which of the following statements is **false**?

   (a) $R'(20) - C'(20) = 26$.
   (b) When 20 units are produced and sold, the marginal profit is 26 dollars per unit.
   (c) When 20 units are produced and sold, the marginal profit is increasing at 26 dollars per unit.
   (d) When 20 units are produced and sold, the profit is increasing at 26 dollars per unit.

Use the following graph of the function $y = f(x)$ to answer questions 10 through 12.



10. At point A, which of the following statements is true?

    (a) $f(x)$ is positive, $f'(x)$ is positive, and $f''(x)$ is positive.
    (b) $f(x)$ is negative, $f'(x)$ is positive, and $f''(x)$ is positive.
    (c) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is positive.
    (d) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is negative.

11. At point B, which of the following statements is true?

    (a) $f(x)$ equals zero, $f'(x)$ is positive, and $f''(x)$ is positive.
    (b) $f(x)$ is positive, $f'(x)$ is positive, and $f''(x)$ is negative.
    (c) $f(x)$ is negative, $f'(x)$ is zero, and $f''(x)$ is positive.
    (d) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is negative.

12. Which of the following statements is **false**?

    (a) Between points A and C, there are two points of inflection.
    (b) Between points A and C, there are two relative minima and one relative maximum.
    (c) At point B, $f''(x) < 0$.
    (d) For all x between points B and C, $f'(x)$ is positive.

13. Suppose that the total cost, in dollars, for the production of x items is $C(x) = 1000 + \sqrt{x}$. Then, the average cost, in dollars per item, is

   (a) $\dfrac{1}{2\sqrt{x}}$
   (b) $1000 + \dfrac{1}{2\sqrt{x}}$
   (c) $\dfrac{1000 + \sqrt{x}}{x}$
   (d) $\dfrac{1000}{x} + \sqrt{x}$

14. What are the critical values of $g(x) = x^3 - x^4$?

   (a) $x = 0$ and $x = 1$
   (b) $x = 1$ only
   (c) $x = 0$ and $x = \frac{4}{3}$
   (d) $x = 0$ and $x = \frac{3}{4}$

15. Which of the following is true?

   (a) If $f'(c) = 0$, then f has a relative extreme value at $x = c$.
   (b) If f has a point of inflection at $x = c$, then either $f''(c) = 0$ or $f''(c)$ does not exist.
   (c) If f has a relative maximum at $x = c$, then $f'(c) = 0$.

16. Suppose that $y = f(x)$ is differentiable over all real numbers and has only one critical value at $x = 10$. If $f'(5)$ is negative and $f'(15)$ is positive, there is

   (a) an inflection point at $x = 10$
   (b) a relative maximum at $x = 10$
   (c) a relative minimum at $x = 10$

17. Find the relative extrema of the function $f(x) = x^3 - x^4$.

   (a) The relative minimum is 0 and the relative maximum is $\frac{3}{4}$.
   (b) The relative minimum is 0 and the relative maximum is $\frac{27}{256}$.
   (c) The relative maximum is $\frac{3}{4}$. There is no relative minimum.
   (d) The relative maximum is $\frac{27}{256}$. There is no relative minimum.

18. Find the absolute extrema of $f(x) = \dfrac{-2x}{x^2 + 1}$ over the interval $[0, 3]$.

   (a) The absolute maximum is 0 and the absolute minimum is -1.
   (b) The absolute maximum is -0.6 and the absolute minimum is -1.
   (c) The absolute minimum is -1. There is no absolute maximum.
   (d) The absolute maximum is 0. There is no absolute minimum.

19. Sound Software estimates that it will sell N units of a program after spending a thousand dollars on advertising, where

$$N(a) = -a^2 + 300a + 6, \quad 0 \leq a \leq 300.$$

Find the amount that must be spent on advertising in order to achieve the maximum number of units that can be sold.

(a) $22,506
(b) $150,000
(c) $300,000
(d) $6,000

20. Of all the numbers whose difference is 6, find the two that have the minimum product. What is the minimum product?

(a) 0
(b) -9
(c) -6
(d) 9

21. A carpenter is building a rectangular shed with a fixed perimeter of 54ft. What are the dimensions of the largest shed that can be built?

(a) 27 ft by 13.5 ft
(b) 27 ft by 27 ft
(c) 13.5 ft by 13.5 ft
(d) 34 ft by 20 ft