NAME: ______________________________

Instructor: ____________________________

Time your class meets: __________________

Math 160 Calculus for Physical Scientists I
Exam 3
November 13, 2014, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a calculator on this exam. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____
(Signature)

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(Date)

Please do not write in this space.

1-3. (12pts)	
4. (14pts)	
5. (12pts)	
6. (15pts)	
7. (20pts)	
8-10. (12pts)	
11. (15pts)	
TOTAL	

Multiple Choice for #1-4. Circle only one answer for each problem unless it indicates otherwise.

1. Let $f(x)$ be a differentiable function on a closed interval where $x = c$ is one of the endpoints of the interval and $f'(c) > 0$.

   (a) f could have an absolute maximum or an absolute minimum at $x = c$.

   (b) f cannot have an absolute maximum at $x = c$.

   (c) f must have an absolute minimum at $x = c$.

   (d) f must have an absolute maximum at $x = c$.

2. If f is an antiderivative of g, and g is an antiderivative of h, then

   (a) h is an antiderivative of f.

   (b) h is the second derivative of f.

   (c) h is the derivative of f''.

3. Let $f(x) = \begin{cases} x^2, & x \leq 3 \\ 7x - 12, & x > 3 \end{cases}$

   Which of the following statements is true?

   (a) $\int_0^4 f(x)\,dx > 0$.

   (b) $\int_0^4 f(x)\,dx < 0$.

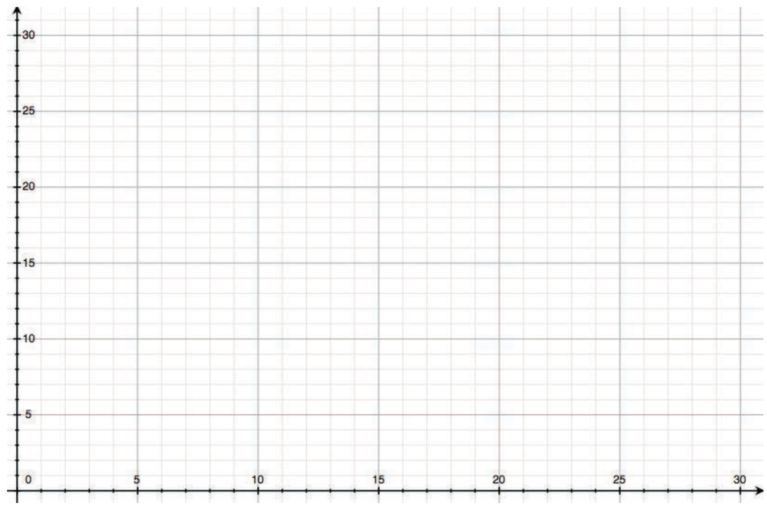
   (c) $\int_0^4 f(x)\,dx = 0$.

   (d) $\int_0^4 f(x)\,dx$ is undefined.

4. Water is flowing into a boat through a hole at the bottom at a rate of $r(t)$. Water is flowing in at increasing rates for the first 10 minutes and then at decreasing rates after the first 10 minutes. You do not know the function, $r(t)$, but you do have values of $r(t)$ at particular time values. Time and rate information is given in the table below:

t minutes	0	5	10	15	20	25	30
$r(t)$ liters/minute	12	20	24	16	10	4	1

(a) (3pts) Plot the values of $r(t)$ for each of the above time values in the axes below



(b) (5pts) Using the values given in the table with subintervals of 5 minutes, compute an upper estimate for the area under the curve $r(t)$. If it is helpful to you, you may draw rectangles in the plot from part (a).

(c) (3pts) In the context of this problem, write one to two sentences describing what the value you found in part (b) represents. Be sure to include appropriate units in your explanation as needed.

(d) (3pts) What does $\int_0^{30} r(t)\,dt$ represent in the context of this problem? Be sure to include appropriate units in your explanation as needed.

5. (8pts) The velocity of a particle moving back and forth along a number line is given by the equation: $v(t) = \sqrt[3]{t} - \frac{1}{2}\sin(t)$.

(a) Determine the function that gives the position, $s(t)$, of the particle at time t if $s(0) = 0$. Evaluate all trigonometric functions exactly.

$s(t) =$

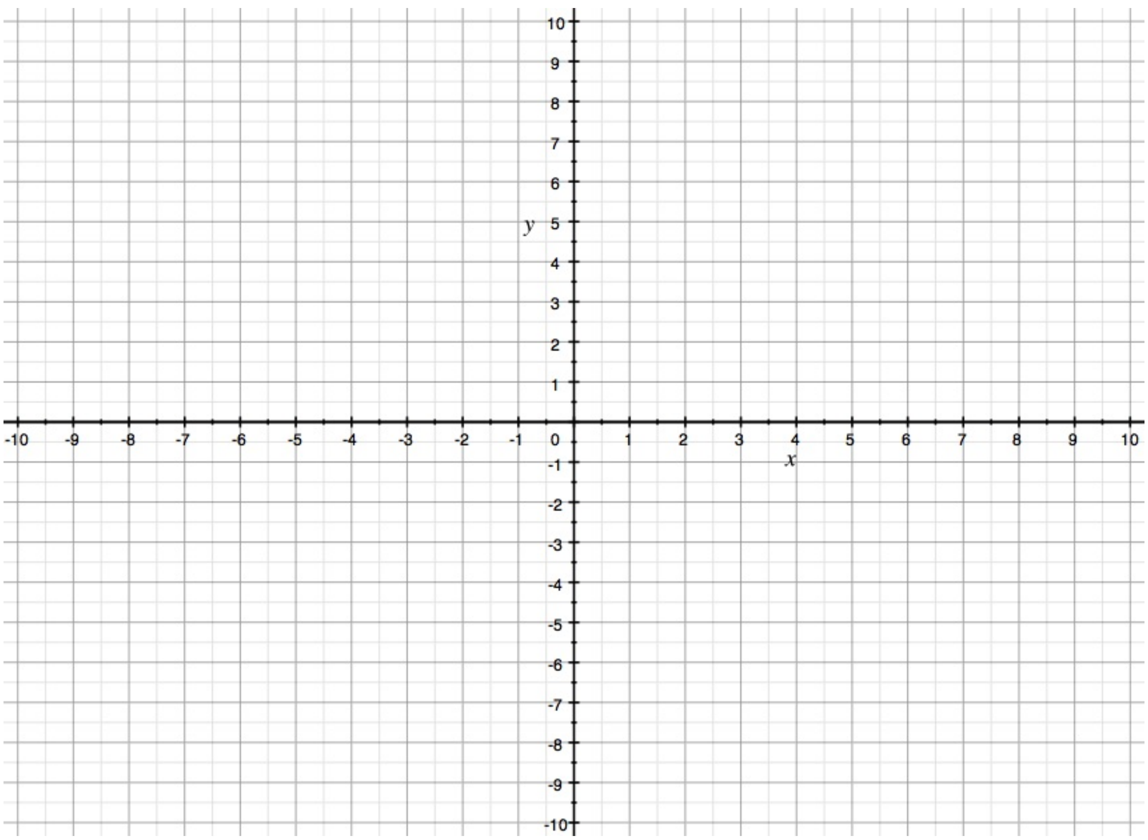
(b) (4pts) Using the function you found in (a), determine the position of the particle when $t = \frac{\pi}{2}$. Evaluate all trigonometric functions exactly. If you express your final answer in decimal form, round to at least 4 decimal places.

6. (15pts) In the axes provided, sketch the graph of a function that has the properties listed in the table as well as the integral property.

Function and Derivative Properties:

	f(x)	***f'(x)***	***f''(x)***
$x < -4$		$f'(x) > 0$	$f''(x) > 0$
$x = -4$	$\lim_{x \to -4} f(x) = \infty$		
$-4 < x < -2$		$f'(x) < 0$	$f''(x) > 0$
$-2 < x < 0$		$f'(x) > 0$	$f''(x) > 0$
$x = 0$	4		
$0 < x < 3$		$f'(x) > 0$	$f''(x) < 0$
$x > 3$	$\lim_{x \to \infty} f(x) = 0$		

Integral Property: $\int_{-3}^{-1} f(x)\, dx < 0$



7. You have been provided with 100 feet of fencing to create an enclosed garden with maximal area. One of your neighbors, Douglas, suggested you split the fencing around two areas, one circular and one square. Your other neighbor, Katherine, insists that you only need one of those (though she doesn't specify which).

Let s represent the side length of the square. Let r represent the radius of the circle.

(a) (2pts) Write an equation that expresses the combined perimeter of both shapes in terms of s and r.

Combined Perimeter Equation: ________________________

(b) (2pts) Write an equation that expresses the combined total area of both shapes in terms of s and r.

Combined Area Equation: ________________________

(c) (2pts) Using interval notation, provide the domain of values for the side length of the square, s.

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(d) (2pts) Using interval notation, provide the domain of values for the radius of the circle, r. Keep values in exact form (no decimals).

____

(e) (12pts) Use **calculus** to determine the values of s and r which result in maximal area for your garden. Be sure to demonstrate that the values you found result in a maximum area. If you use decimals, do not round until your final answers and round to at least 4 decimal places. *[Hint: As it turns out, Katherine is correct, but calculus must be used to show this!]*

8. (4pts) Sam evaluated the following integral. If Sam correctly evaluated the integral, draw a smiley face.

   If Sam did not correctly evaluate the integral, explain the error(s) that Sam made.

$$\int \cos(3x)\,dx = \frac{1}{3}\sin(3x) + C$$

9. (4pts) Wes evaluated the following integral. If Wes correctly evaluated the integral, draw a smiley face.

   If Wes did not correctly evaluate the integral, explain the error(s) that Wes made.

$$\int \pi^2\,dx = \frac{\pi^3}{3} + C$$

10. (4pts) Hilary evaluated the following integral. If Hilary correctly evaluated the integral, draw a smiley face.

    If Hilary did not correctly evaluate the integral, explain the error(s) that Hilary made.

$$\int \sqrt{v}\,dv = \frac{3}{2}v^{3/2} + C$$

11. (15pts - 5pts each) Indicate whether each of the following statements is **True** or **False**. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample. (A counterexample is an example of a function for which the "if" part of the statement is true, but the "then" part is false.) A graph with an explanation can be used as a counterexample.

(a) An antiderivative of a product of functions, fg, is an antiderivative of f times an antiderivative of g.

(b) If $p'(x) = q'(x)$, then $p(x) = q(x)$.

(c) If $f(x)$ is increasing on $[2, 3]$, then $\int_2^3 f(x)\,dx > 0$.