

Math 155

Final Exam

Spring 2009

NAME: \_\_\_\_\_

SECTION: \_\_\_\_\_ TIME: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

**Instructions:** The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have two hours to work on the exam.

Problem	Points	Score
1	16	
2	10	
3	10	
4	16	
5	12	
6	14	
7	12	
8	12	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person.

\_\_\_\_\_  
(Signature)

1. (16 pts) Find the following limits. Show all your work. If you use L'Hospital's rule, justify why it can be applied each time you use it. If you use knowledge of leading behaviors, justify your work.

a)  $\lim_{x \rightarrow 0} \frac{2 - 2e^{-2x}}{x^2 + 3x}$

b)  $\lim_{x \rightarrow \infty} \frac{10x - 1}{10 \ln x}$

c)  $\lim_{x \rightarrow 1} \frac{\sin(\pi/2)}{\cos \pi x}$

d) Find  $f_\infty(x)$ , the leading behavior of  $f(x)$  as  $x \rightarrow \infty$  for

$$f(x) = \frac{6x^2 + \ln x + 2x + 100}{e^{-x} + 2x + 12}$$

e) Find  $f_0(x)$ , the leading behavior of  $f(x)$  as  $x \rightarrow 0$ , for

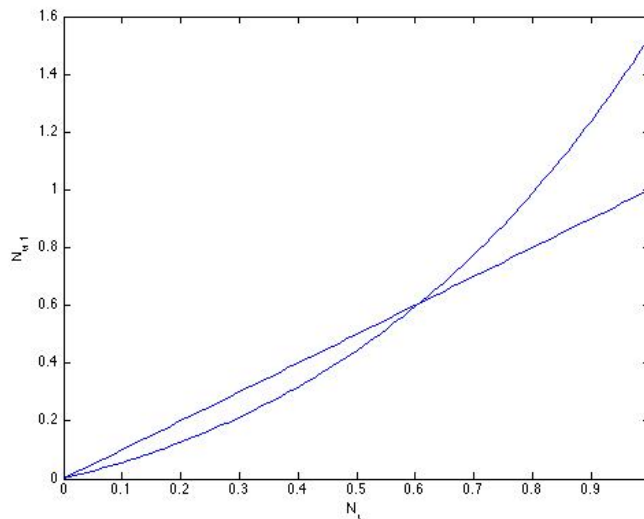
$$f(x) = \frac{e^{x^2} + 12x + x^{-1}}{x^2 + x + 7}$$

2. (10 pts) The following updating function describes the number of fish  $N_t$  of a certain fish population.

$$N_{t+1} = 0.6N_t e^{N_t} - hN_t$$

The term  $hN_t$  is the harvest and  $h$  is the harvesting effort.

a) The graph below corresponds to  $h = 0.1$ . Draw a cobweb diagram on the graph, starting at  $N_t = 0.5$ . Circle all equilibria on the graph. Determine the stability of each equilibrium from your cobweb diagram.



b) Use the stability criterion to check your answer to part a for the nonzero equilibrium  $N^* = \ln(1.833)$ . (Here  $h = 0.1$ , as in part a).

3. (10 pts) a) Find the equation of the tangent line at  $t = 1$  to the function  $f(t) = e^{2-t^3}$ . Write your answer in the form  $y = mt + b$ .

b) Use Newton's method to approximate the value of the zero of the function  $g(x) = x^2 - \sin(2x) - 3$ . Take an initial guess of  $x_0 = 2$ . Do two iterations (find  $x_2$ ). The formula for Newton's method is

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$$

3. (16 pts) Evaluate the following definite and indefinite integrals. If necessary, use substitution. Show all of your work to receive full credit.

a)  $\int (x^{-1} - \pi - \cos x) dx$

b)  $\int_1^2 x^2 \sqrt{x^3 - 1} dx$

c)  $\int_1^\infty (e^{-3x} - x^{-3/2}) dx$

d)  $\int (\sec^2(2t) - e^t - t) dt$

4. (14 pts) Suppose the rate at which a chemical product is formed in a reaction is

$$\frac{dP}{dt} = 42.4e^{-0.3t} \text{ moles/s}$$

a) If there is no product at time  $t = 0$  (so  $P(0) = 0$ ), how much is present at time  $t$ ? (Solve the differential equation.)

b) Use a definite integral to represent and find the net change in product between times  $t = 10$  and  $t = 20$ .

c) What is the limiting value of the rate  $\frac{dP}{dt}$  as  $t \rightarrow \infty$ ? (Take a limit.)

d) What is the limiting value of the product  $P(t)$  as  $t \rightarrow \infty$ ? (Take a limit.)

5. a) (8 pts) The following differential equation is a model for the rate of change of the volume of blood in liters in the liver with respect to time  $t$  in seconds. Solve the initial value problem for the volume of blood  $V(t)$ .

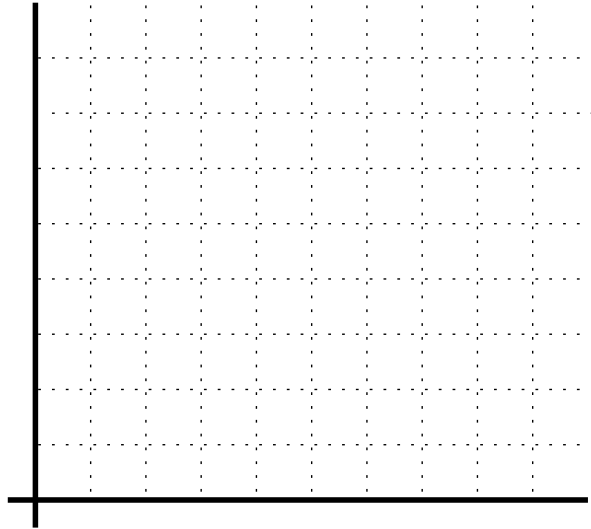
$$\frac{dV}{dt} = 0.8 + \sin(2\pi t - \pi), \quad V(0) = 0.75$$

b) (6 pts) Find the total change in a population between times  $t = 10$  and  $t = 15$  hours if the population  $P(t)$  is governed by the differential equation

$$\frac{dP}{dt} = (3t + 1)^2$$

6. a) (6 pts) Sketch the updating function for  $V_t$  below. Label your axes. Does the updating function have an equilibrium for positive values of  $V_t$ ? Why or why not?

$$V_{t+1} = \begin{cases} 0.75V_t + 10 & \text{if } V_t \leq 25 \\ 0.75V_t & \text{if } V_t > 25 \end{cases}$$



b) (6 pts) Use integration by parts to evaluate the indefinite integral.

$$\int 3xe^{2x} dx$$



7. (12 pts) Suppose the total food collected by a bee is given by

$$F(t) = \frac{t}{2+t}$$

and the rate at which nectar is collected is given by

$$R(t) = \frac{F(t)}{t+1} = \frac{t}{(2+t)(t+1)}$$

Find the time  $t$  that will maximize the rate at which nectar is collected on the time interval  $[0, 3]$ . Verify that you have found a maximum by using either the first or second derivative test and determine if it is a global maximum on the interval.