

NAME: \_\_\_\_\_

SECTION: \_\_\_\_\_ TIME: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

**Instructions:** The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have two hours to work on the exam.

Problem	Points	Score
1	14	
2	14	
3	15	
4	14	
5	14	
6	15	
7	14	
Total	100	

## CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

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(Signature)

1. (14 points) Suppose that the population  $q_t$  of quails satisfies the discrete-time dynamical system

$$q_{t+1} = q_t(4 - q_t) - hq_t,$$

where  $h > 0$  is a positive parameter, and  $q_t$  is measured in thousands of quails.

- (a) Find all equilibria. For what values of  $h$  is there more than one nonnegative equilibrium?

- (b) For each equilibrium, use the Stability Theorem/Criterion to determine the values of  $h$  for which that equilibrium is stable. Show clearly how you are using the Stability Theorem/Criterion.

2. (14 points) (a) Consider the function  $f(x) = x^3 - \pi x + 3$ . i) Find *all* critical points of  $f(x)$ .  
ii) Determine the global maximum and global minimum of  $f(x)$  on the interval  $[0, 3]$ . Justify your answer and show your work clearly for full credit.

(b) Suppose that the production of a pharmaceutical drug,  $D$ , depends on the population (measured in thousands) of some fungus  $p$ , in the following manner:

$$D(p) = 3p^2e^{-0.001p}.$$

Find the *positive* critical point of the function  $D(p)$ , and use either the first or second derivative test to determine if there is either a local maximum or local minimum at that point. Show your work clearly for full credit.

3. (15 points) Evaluate the following definite and indefinite integrals. If necessary, use substitution. Show all of your work.

(a)  $\int 4t^{\frac{3}{2}} - \frac{6}{t} + \pi dt$

(b)  $\int 5 \sin(x)e^{\cos(x)+\pi} dx$

(c)  $\int_0^1 \frac{4x}{x^2+2} dx$

(d) Use integration by parts to evaluate  $\int 2x \sin(7x) dx$ .

4. (14 points) Suppose that a bacterium is absorbing copper from its environment. At time  $t = 0$ , there is 0.6 mol of copper in the bacterium, and copper enters the bacterium at a rate of  $\frac{1}{2+t^2} \frac{\text{mol}}{\text{min}}$

(a) Let  $c(t)$  represent the amount (mol) of copper in the bacterium at time  $t$  (minutes). Write a pure-time differential equation and an initial condition for the situation described above.

(b) Apply Euler's Method with  $\Delta t = 0.5$  to estimate the amount of copper in the bacterium at time  $t = 1.5$ . Show your work clearly using a table. Give your answer to three decimal places.

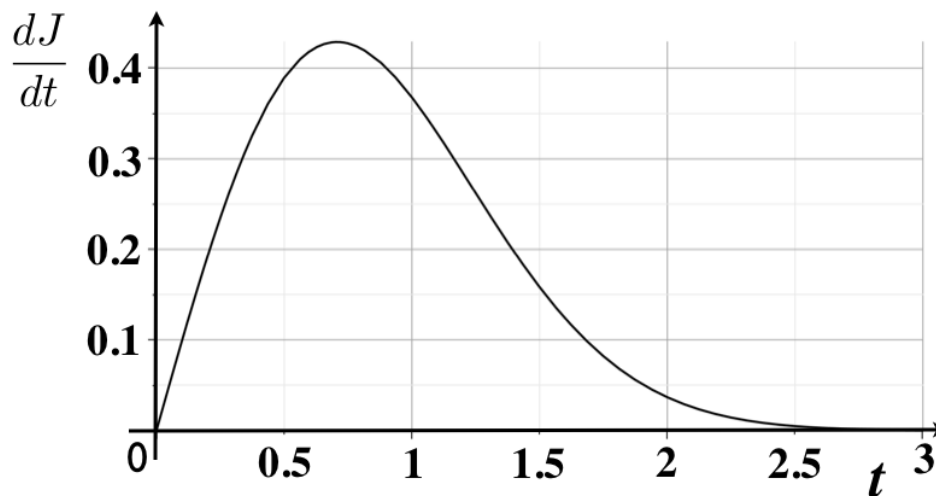
(Recall the formula  $\hat{c}_{\text{next}} = \hat{c}_{\text{current}} + \frac{dc}{dt} \Delta t$ , or  $\hat{c}(t + \Delta t) = \hat{c}(t) + c'(t) \Delta t$ ).

5. (14 points) Your pancreas produces insulin depending on the time since you ate green and gold ice cream. Assume that the rate of insulin production is

$$\frac{dJ}{dt} = te^{-t^2},$$

where time  $t$  is measured in hours and  $J(t)$  is the amount of insulin (in moles) that the pancreas has produced  $t$  hours after you ate the ice cream.

- (a) Estimate the total change in  $J(t)$  between times  $t = 0$  and  $t = 2$  using a left-hand Riemann Sum with  $\Delta t = 0.5$ . Draw your rectangles or step functions on the graph of  $\frac{dJ}{dt}$  below. Give your answer to three decimal places.



- (b) Find the exact area under the curve  $te^{-t^2}$  between times  $t = 0$  and  $t = 2$ . What does this area represent biologically?

6. (15 points)

(a) To celebrate completion of Math 155, you plant an elm tree on the oval. The tree is only 260 cm tall when planted, but it grows 80 cm per year after being planted. Let  $h_t$  = the height of the tree (in cm)  $t$  years after being planted, and write down a discrete-time dynamical system, together with an initial condition, that describes this situation.

(b) Let  $L(t)$  = the length (in cm) of a fish at time  $t$  (in years). Suppose that the fish grows at a rate  $\frac{dL}{dt} = 5.0e^{-0.2t}$ .

i. Use a definite integral to determine the total change in length of the fish between times  $t = 5$  and  $t = 10$ .

ii. Determine  $L(t)$  if  $L(0) = 2$ . (That is, find a solution to the differential equation  $\frac{dL}{dt} = 5.0e^{-0.2t}$  with initial condition  $L(0) = 2$ .)

7. (14 points)

- (a) Use the tangent-line approximation of the function  $f(x) = \sqrt{x}$  to approximate  $\sqrt{16.1}$ . Give your answer to four decimal places.

- (b) The density  $\rho$  of a very thin rod (measured in grams/cm) varies according to

$$\rho(x) = 1 + \frac{x}{1 + x^2},$$

where  $x$  marks a location along the rod, and  $x = 0$  at one end of the rod. What is the total mass of the rod if it is 10 cm long? Give units in your answer.

- (c) Suppose that the population  $w(t)$  of warblers satisfies the differential equation

$$\frac{dw}{dt} = 2w(8 - w).$$

- i. Find all equilibria of the differential equation.
  
  
  
  
  
  
  
  
  
  
- ii. Write down an initial condition for which the population will increase in time (at least initially).

$$w(0) = \underline{\hspace{2cm}}.$$