

NAME: _____

SECTION: _____ TIME: _____

INSTRUCTOR: _____

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have two hours to work on the exam.

Problem	Points	Score
1	14	
2	14	
3	15	
4	14	
5	14	
6	15	
7	14	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

(Signature)

1. (14 points) Suppose that the population m_t of macaws satisfies the discrete-time dynamical system

$$m_{t+1} = 3(1 - m_t)m_t - hm_t,$$

where $h > 0$ is a positive parameter, and m_t is measured in hundreds of macaws.

- (a) Find all equilibria. For what values of h is there more than one equilibrium that makes biological sense?

- (b) For each equilibrium, use the Stability Theorem/Criterion to determine the values of h for which that equilibrium is stable. Show clearly how you are using the Stability Theorem/Criterion.

2. (14 points) (a) Consider the function $f(x) = x^4 - 4x^2 + 2$. i) Find *all* critical points of $f(x)$. ii) Determine the global maximum and global minimum of $f(x)$ on the interval $[-1, 3]$. Justify your answer and show your work clearly for full credit.

(b) Suppose that the production P of starch in a plant depends on time t in the following manner:

$$P(t) = \frac{10t}{1+t^2} \text{ grams per day.}$$

Find the *positive* critical point of the function $P(t)$, and use either the first or second derivative test to determine if there is either a local maximum or local minimum at that point. Show your work clearly for full credit.

3. (15 points) Evaluate the following definite and indefinite integrals. If necessary, use substitution. Show all of your work.

(a) $\int \frac{7}{x} - 8x^{\frac{9}{4}} + \pi \, dx$

(b) $\int \sec^2(t) + 5 \cos(t)e^{\sin(t)+1} \, dt$

(c) $\int_0^{\sqrt{\pi}} 3t \cos(t^2) \, dt$

(d) Use integration by parts to evaluate $\int 5xe^{3x} \, dx$.

4. (14 points) Suppose that a bacterium is absorbing plutonium from its environment. At time $t = 0$, there is 0.1 mol of plutonium in the bacterium, and plutonium enters the bacterium at a rate of $\frac{1}{2+t^3} \frac{\text{mol}}{\text{hour}}$

(a) Let $p(t)$ represent the amount (mol) of plutonium in the bacterium at time t (hours). Write a pure-time differential equation and an initial condition for the situation described above.

(b) Apply Euler's Method with $\Delta t = 0.25$ to estimate the amount of plutonium in the bacterium at time $t = 0.75$. Show your work clearly using a table. Give your answer to three decimal places.

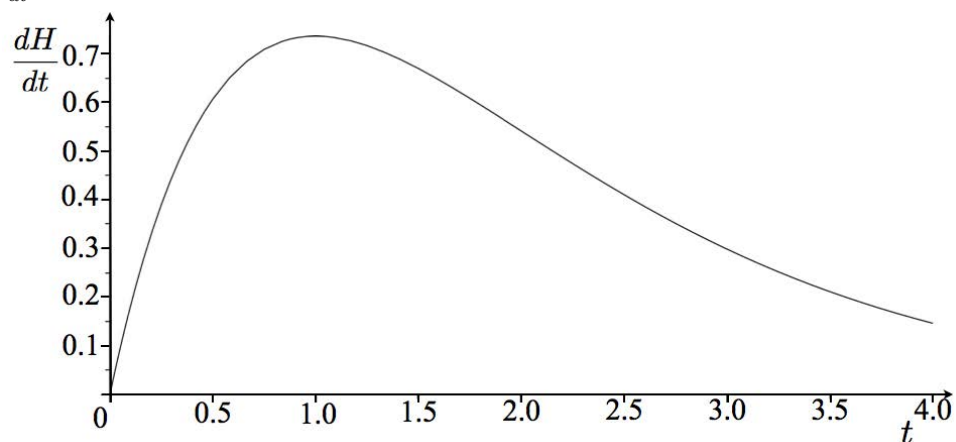
(Recall the formula $\hat{p}_{\text{next}} = \hat{p}_{\text{current}} + \frac{dp}{dt} \Delta t$, or $\hat{p}(t + \Delta t) = \hat{p}(t) + p'(t) \Delta t$).

5. (14 points) After a rainstorm, the growth rate of a bean stalk increases for a while, but eventually the growth rate decreases again as the ground dries up. Suppose that the rate of change of the height of the beanstalk is given by

$$\frac{dH}{dt} = te^{-t},$$

where time t is measured in hours and $H(t)$ is the height (in cm) of the beanstalk t hours after the rainstorm.

- (a) Estimate the total change in $H(t)$ between times $t = 0$ and $t = 2$ using a right-hand Riemann Sum with $\Delta t = 0.5$. Draw your rectangles or step functions on the graph of $\frac{dH}{dt}$ below. Give your answer to three decimal places.



- (b) Find the average value of the function $f(t) = t \sin(t^2 + \pi)$ on the interval $[0, \sqrt{\pi}]$.

6. (15 points)

(a) A parrot starts with a concentration of medicine in his bloodstream equal to 10 milligrams per liter (mg/L). Each day, the parrot uses up 45% of the medicine in his bloodstream. However, at the end of each day the vet gives him enough medication to increase the concentration of medicine in the bloodstream by 5 mg/L. Let M_t = concentration of medicine on day t , and write down a discrete-time dynamical system, together with an initial condition, that describes this situation.

(b) Let $H(t)$ = the height (in meters) of a tree at time t (in years). Suppose that the tree grows at a rate $\frac{dH}{dt} = \frac{2t}{10+t^2}$ meters per year.

i. Use a definite integral to determine the total change in the height of the tree between times $t = 2$ and $t = 12$.

ii. Determine $H(t)$ if $H(0) = 1$. (That is, find a solution to the differential equation $\frac{dH}{dt} = \frac{2t}{10+t^2}$ with initial condition $H(0) = 1$.)

7. (14 points)

(a) A population of striped goldfish obeys the discrete-time dynamical system

$$g_{t+1} = 1.3g_t.$$

(i) Write down the solution to this discrete-time dynamical system if $g_0 = 325$.

(ii) If $g_0 = 325$, at what time will the population reach size 1000?

(b) The density ρ of a very thin rod (measured in grams/cm) varies according to

$$\rho(x) = \frac{3 + 6x}{1 + x + x^2},$$

where x marks a location along the rod, and $x = 0$ at one end of the rod. What is the total mass of the rod if it is 4 cm long? Give units in your answer.

(c) Suppose that a population $b(t)$ of honey badgers satisfies the differential equation

$$\frac{db}{dt} = 1.3b(4 - b).$$

i. Find all equilibria of the differential equation.

ii. Write down an initial condition for which the population will increase in time (at least initially).

$$b(0) = \underline{\hspace{2cm}}.$$