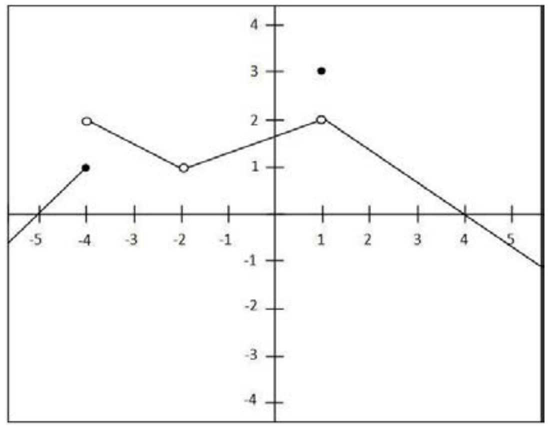
Final Exam – Math 141 Spring 2016

Name: ____________________________ **Exam Version A**

This exam is multiple choice. Make sure you fill in your answers on the scantron answer sheet provided. Fill in your name, student ID number, section number, and the exam version on the scantron answer sheet. You may use this exam to work out the problems. You must hand in this exam as well as the scantron answer sheet. To hand in your exam, be prepared to show your picture ID. This is a closed book, closed notes exam. Calculators are allowed but must be equivalent to a TI-83/84; no TI-89s or equivalent are allowed. No cell phones are permitted outside of your bag at anytime during the test. Note: There are 35 problems in this exam, each one worth 4 points, for a maximum possible score of 140. Your total score will be calculated out of a 128 points (there are 12 possible bonus points).

GOOD LUCK!

Use the following graph to answer questions 1 through 3.



1. $\lim_{x \to -4^+} f(x) =$
   (a) 2
   (b) does not exist (or undefined)
   (c) 1
   (d) none of the above

2. $\lim_{x \to 4} f(x) =$
   (a) 0
   (b) 4
   (c) does not exist (or undefined)
   (d) none of the above

3. Is the function f continuous at $x = 1$?
   (a) Yes.
   (b) No, because $\lim_{x \to 1} f(x)$ does not exist.
   (c) No, because $f(1)$ is not defined.
   (d) No, because $\lim_{x \to 1} f(x) \neq f(1)$.

4. If $f(x) = (x-4)^3(2x^3+x)^2$, then $f'(x) =$

(a) $6(x-4)^2(2x^3+x)(6x^2+1)$
(b) $3(x-4)^2(2x^3+x) - 2(x-4)^3(2x^3+x)(6x^2+1)$
(c) $6(x-4)^2(6x^2+1)$
(d) $3(x-4)^2(2x^3+x)^2 + 2(x-4)^3(2x^3+x)(6x^2+1)$

5. If $y = [\ln(2x)]^2 - 4e^{-x}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{\ln(x)}{x} - 4e^{x}$
(b) $2\dfrac{\ln(2x)}{x} + 4e^{-x}$
(c) $\ln(x) - 4e^{-x}$
(d) $2\dfrac{\ln(2x)}{x} - 4(-x)e^{-x-1}$

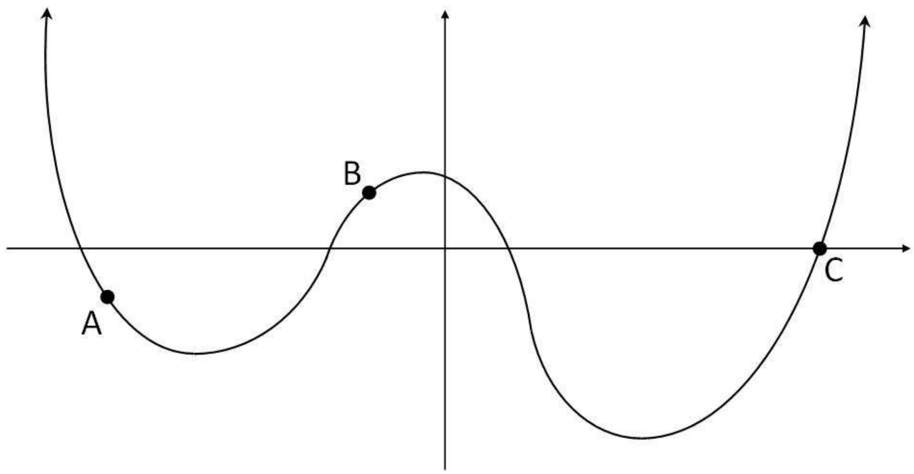
6. $\displaystyle\int \left(e^{-x} + \frac{1}{\pi}\right) dx =$

(a) $e^{-x} + \dfrac{1}{\pi}x + C$
(b) $\dfrac{1}{-x+1}e^{-x+1} + \dfrac{1}{\pi}x + C$
(c) $\dfrac{1}{-x+1}e^{-x+1} + \ln\pi + C$
(d) $-e^{-x} + \dfrac{1}{\pi}x + C$

7. Evaluate $\displaystyle\int \left(2\sqrt[3]{x} - \frac{1}{x} + \sqrt{20}\right) dx$, where $x > 0$.

(a) $2x^{1/3} - \ln(x) + \sqrt{20} + C$
(b) $2x^{4/3} - \ln(x) + \sqrt{20}x + C$
(c) $\dfrac{2}{3}x^{-1/3} - x^{-1} + \sqrt{20}x + C$
(d) $\dfrac{3}{2}x^{4/3} - \ln(x) + \sqrt{20}x + C$

Use the following graph of the function $\mathbf{y = f(x)}$ to answer questions 8 and 9.



8. At point A, which of the following statements is true?
   (a) $f(x)$ is negative, $f'(x)$ is positive, and $f''(x)$ is positive.
   (b) $f(x)$ is positive, $f'(x)$ is negative, and $f''(x)$ is positive.
   (c) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is positive.
   (d) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is negative.

9. At point B, which of the following statements is true?
   (a) $f(x)$ is negative, $f'(x)$ is positive, and $f''(x)$ is positive.
   (b) $f(x)$ is positive, $f'(x)$ is positive, and $f''(x)$ is positive.
   (c) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is positive.
   (d) $f(x)$ is positive, $f'(x)$ is positive, and $f''(x)$ is negative.

10. Sound Software estimates that it will sell N units of a program after spending a thousand dollars on advertising, where
$$N(a) = -a^2 + 300a + 6, \quad 0 \leq a \leq 300.$$
    Find the maximum number of units of the program that can be sold.
    (a) 150,000
    (b) 22,506
    (c) 300,000
    (d) 6,000

11. A carpenter is building a rectangular shed with a fixed perimeter of 54ft. What are the dimensions of the largest shed that can be built?

    (a) 27 ft by 13.5 ft
    (b) 27 ft by 27 ft
    (c) 34 ft by 20 ft
    (d) 13.5 ft by 13.5 ft

12. Suppose that total revenue is $R(x) = 50x - 0.5x^2$ and total cost is $C(x) = 10x + 3$ (both given in dollars), where x represents the number of units of a product that are produced and sold. Find the rate of change of the profit with respect to time when $x = 10$ units and $dx/dt = 5$ units per day.

    (a) \$200 per day
    (b) \$150 per day
    (c) \$80 per day
    (d) \$50 per day

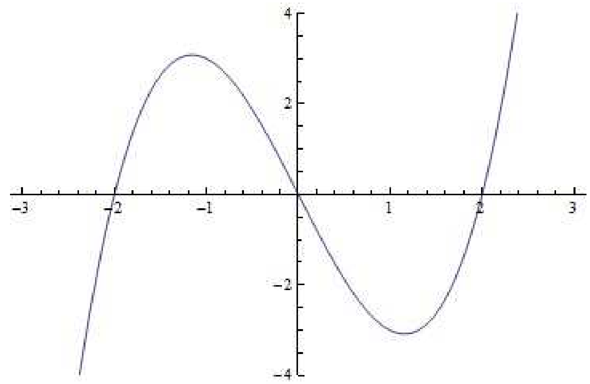
13. Evaluate $\int e^3 dt$.

    (a) $(e^3)t + C$
    (b) $\dfrac{e^4}{4} + C$
    (c) $e^{3t} + C$
    (d) C

14. Find $f(x)$ given $f'(x) = 6x^2 - 4x$ and $f(1) = 2$.

    (a) $f(x) = 2x^3 - 2x^2 + C$
    (b) $f(x) = 2x^3 - 2x^2 + 3$
    (c) $f(x) = 2x^3 - 2x^2 + 2$
    (d) $f(x) = 2x^3 - 2x^2 + 1$

Use the following graph of $\mathbf{y = f(x)}$ to answer questions 15 and 16.



15. $\int_1^2 f(x)dx$ is
    (a) Positive
    (b) Zero
    (c) Negative

16. $\int_{-1}^2 f(x)dx$ is
    (a) Positive
    (b) Zero
    (c) Negative

17. Find the area under the graph of $y = g(x)$ over the interval $[-2, 3]$:

$$g(x) = \begin{cases} x^2 + 4, & x \leq 0, \\ 4 - x, & x > 0 \end{cases}$$

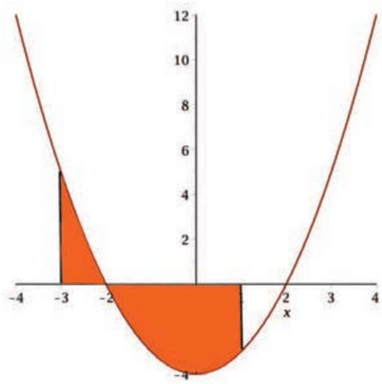
    (a) 109/6
    (b) 18/6
    (c) 95/3
    (d) 35/2

18. Evaluate the integral $\int_{-1}^{1} (x^3 + x)\, dx$. The value of the integral shows that, over the interval $[-1, 1]$, the area bounded by the function $f(x) = x^3 + x$ above the x-axis is
    (a) less than the area bounded by the function below the x-axis.
    (b) equal to the area bounded by the function below the x-axis.
    (c) greater than the area bounded by the function below the x-axis.

19. Find the area of the region bounded by the graphs of the functions $y = x$ and $y = x^4$ in the first quadrant (where $x \geq 0$ and $y \geq 0$).

(a) -0.3
(b) 0.3
(c) 0.5
(d) 0.2

20. Find the area of the shaded region over the interval $[-3, 1]$, where $f(x) = x^2 - 4$.



(a) -34/3
(b) 9/3
(c) 34/3
(d) 7/3

21. Suppose that in a memory experiment the rate of memorizing is given by

$$M'(t) = -0.009t^2 + 0.2t,$$

where $M'(t)$ is the memory rate in words per minute. How many words are memorized in the first 10 minutes?

(a) 5
(b) 7
(c) 8
(d) 10

22. The temperature over a 10-hr period is given by $f(t) = -t^2 + 5t + 40, \quad 0 \le t \le 10$. Find the average temperature, the minimum temperature, and the maximum temperature.

   (a) average temperature: $31.7°$, minimum temperature: $-10°$, maximum temperature: $2.5°$.
   (b) average temperature: $31.7°$, minimum temperature: $-10°$, maximum temperature: $46.25°$.
   (c) average temperature: $316.7°$, no minimum temperature, maximum temperature: $46.25°$.
   (d) average temperature: $31.7°$, no minimum temperature, maximum temperature: $46.25°$.

Use the following information to answer questions 23 through 26.

Let $D(x)$ be the consumer's demand curve, and let $S(x)$ be the producer's supply curve. The equilibrium point in this example is $(14, 26)$.

Figure 1:

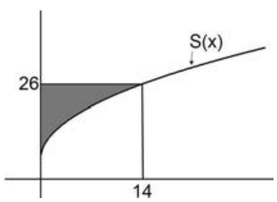


Figure 2:

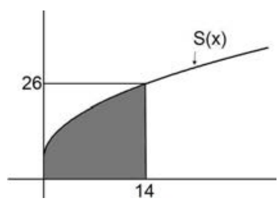


Figure 3:

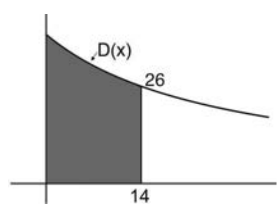
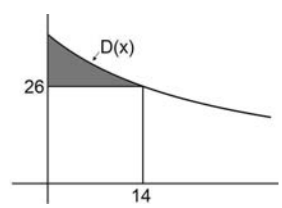


Figure 4:



23. The quantity shaded in Figure 1 is

   (a) $14 * 26$
   (b) $\int_0^{14} D(x)dx$
   (c) $\int_0^{14} S(x)dx$
   (d) $14 * 26 - \int_0^{14} S(x)dx$
   (e) $\int_0^{14} D(x)dx - 14 * 26$

24. The quantity shaded in Figure 2 is

   (a) $14 * 26$
   (b) $\int_0^{14} D(x)dx$
   (c) $\int_0^{14} S(x)dx$
   (d) $14 * 26 - \int_0^{14} S(x)dx$
   (e) $\int_0^{14} D(x)dx - 14 * 26$

25. The quantity shaded in Figure 3 is

   (a) $14 * 26$
   (b) $\int_0^{14} D(x)dx$
   (c) $\int_0^{14} S(x)dx$
   (d) $14 * 26 - \int_0^{14} S(x)dx$
   (e) $\int_0^{14} D(x)dx - 14 * 26$

26. The quantity shaded in Figure 4 is

   (a) $14 * 26$
   (b) $\int_0^{14} D(x)dx$
   (c) $\int_0^{14} S(x)dx$
   (d) $14 * 26 - \int_0^{14} S(x)dx$
   (e) $\int_0^{14} D(x)dx - 14 * 26$

27. Find the equilibrium point for the supply curve $S(x) = x^2 + 4x + 11$ and the demand curve $D(x) = 81 - x^2$.

(a) (5,5)
(b) (56,5)
(c) (5,56)
(d) (56,56)

28. For the supply curve $S(x) = x^2 + 4x + 11$ and the demand curve $D(x) = 81 - x^2$, determine the producer surplus at the equilibrium point.

(a) \$146.67
(b) \$133.33
(c) \$-133.33
(d) \$83.33

29. The equilibrium point for the supply curve $S(x) = 1 + 0.2x$ and the demand curve $D(x) = 12/(x+1)$ is (5,2). Determine the consumer surplus at the equilibrium point.

(a) \$-2.50
(b) \$2.50
(c) \$21.50
(d) \$11.50

30. Evaluate $\int (2x^2 - 3x + 7)^3 (4x - 3) dx$.

(a) $\frac{1}{4}(4x - 3)^4 + C$
(b) $\frac{1}{4}(2x^2 - 3x + 7)^4(\frac{2}{3}x^3 - \frac{3}{2}x^2 + 7x)(2x^2 - 3x) + C$
(c) $3(2x^2 - 3x + 7)^2(4x - 3)^2 + (2x^2 - 3x + 7)^2(4) + C$
(d) $\frac{1}{4}(2x^2 - 3x + 7)^4 + C$

31. Evaluate $\int \frac{12x^2}{4x^3 - 7} dx$, where $x > 2$.

(a) $-(4x^3 - 7)^{-2} + C$
(b) $\dfrac{(4x^3 - 7)(12x) - 12x^2(12x^2)}{(4x^3 - 7)^2} + C$
(c) $\ln(4x^3 - 7) + C$
(d) $3\ln(x) - \dfrac{4}{7}x^3 + C$

32. Which of the following integrals can be solved by using the technique of u-substitution **only**, with $u(x) = x^2$?

   (a) $\int 2xe^{x^2}\,dx$

   (b) $\int x^2 e^{x^2}\,dx$

   (c) $\int \frac{x^2}{(1+x^3)^2}dx$

33. If we let $u = 3 + e^t$, then the integral $\int \frac{e^t}{3+e^t}dt$ can be re-written as

   (a) $\int \frac{du}{u}$

   (b) $\int u du$

   (c) $\int \frac{1}{3+u}du$

34. Which of the following is **false**?

   (a) $\int \ln(t)dt = t\ln(t) - t + C$

   (b) $\int \frac{1}{4}dx = \ln(4) + C$

   (c) $\int e^0 dx = x + C$

35. Which of the following is **true**?

   (a) $\int \ln(3e^x)dx = \int \ln(3x)dx.$

   (b) If f is continuous on $[a, b]$, and $a < c < b$, then $\int_a^b f(t)dt = \int_a^c f(t)dt - \int_c^b f(t)dt.$

   (c) If f is continuous on $[a, b]$, and F is an antiderivative of f, then $\int_a^b f(x)dx = F(b) - F(a).$