Math 155 Final Exam Spring 2016

NAME: ______________________________________

SECTION: ___________________ TIME: ___________________

INSTRUCTOR: ______________________________________

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have two hours to work on the exam.

Problem	Points	Score
1	14	
2	14	
3	15	
4	14	
5	14	
6	15	
7	14	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

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(Signature)

1. (14 points) Suppose that the population h_t of wild horses satisfies the discrete-time dynamical system

$$h_{t+1} = \frac{6}{5} h_t (k - h_t),$$

where $k > 0$ is a positive parameter.

(a) Find all equilibria. For what values of k is there more than one equilibrium that makes biological sense?

(b) For each equilibrium, use the Stability Theorem/Criterion to determine the values of k for which that equilibrium is stable. Show clearly how you are using the Stability Theorem/Criterion.

2. (14 points)

(a) Consider the function $f(x) = x^2 e^{-x}$.

i) Find *all* critical points of $f(x)$.

ii) Determine the global maximum and global minimum of $f(x)$ on the interval $[-1, 5]$. Justify your answer and show your work clearly for full credit.

(b) Mary jumps from a diving board. Her height (in meters) above the water at time t (in seconds) is given by $h(t) = -5t^2 + 8t + 5$, and she jumps at time $t = 0$.

i) Find Mary's velocity $v(t)$ and acceleration $a(t)$ at time $t = 1$.

ii) Use ***Calculus*** to determine the time at which Mary reaches her maximum height above the water. What is this height? Verify, ***using Calculus***, that this is a local maximum.

3. (15 points) Evaluate the following definite and indefinite integrals. If necessary, use substitution. Show all of your work.

(a) $\int \frac{t^7-3t^2}{t^4} + 2\pi^2 \, dt$

(b) $\int 2x^4 \sin(\pi + 3x^5) \, dx$

(c) $\int_0^2 \frac{t}{(t^2+4)^2} \, dt$

(d) Use integration by parts to evaluate $\int -2x \sin(9x) \, dx$.

4. (14 points) Suppose that a bacterium is absorbing lactose from its environment. At time $t = 0$, there is 0.2 mol of lactose in the bacterium, and lactose enters the bacterium at a rate of $0.1 \sin^2(t) \frac{\text{mol}}{\text{hour}}$

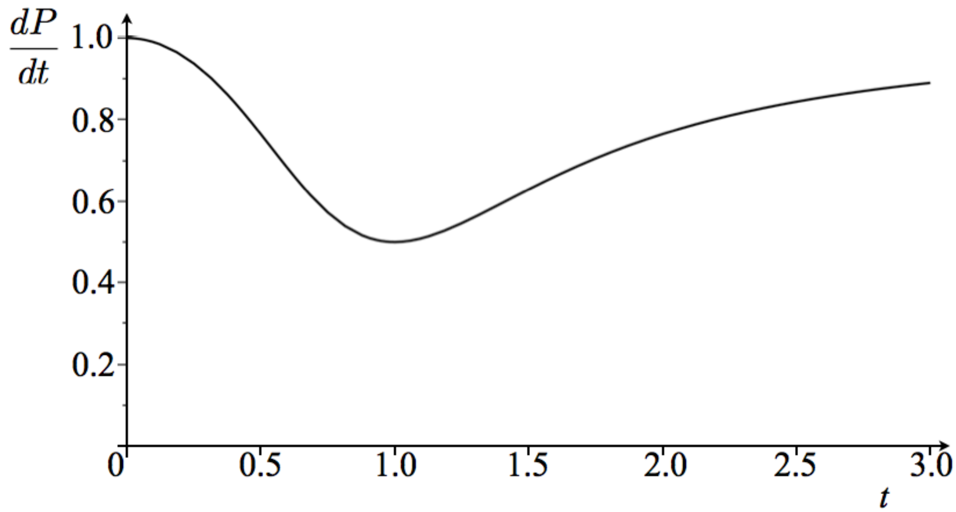
(a) Let $L(t)$ represent the amount (mol) of lactose in the bacterium at time t (hours). Write a pure-time differential equation and an initial condition for the situation described above.

(b) Apply Euler's Method with $\Delta t = 0.5$ to estimate the amount of lactose in the bacterium at time $t = 1.5$. Show your work clearly. Give your answer to three decimal places. (Recall the formula $\hat{L}_{\text{next}} = \hat{L}_{\text{current}} + \frac{dL}{dt}\Delta t$, or $\hat{L}(t + \Delta t) = \hat{L}(t) + L'(t)\Delta t$).

5. (14 points) A western wood pewee slows down a bit to catch a fly, and then increases its speed again as it flies on. Denoting the position (in meters) of the pewee at time t (in seconds) by $P(t)$, suppose that the pewee's velocity is given by

$$\frac{dP}{dt} = 1 - \frac{t^2}{1+t^4}.$$

(a) Estimate the total change in $P(t)$ between times $t = 0.5$ and $t = 2$ using a right-hand Riemann Sum with $\Delta t = 0.5$. Draw your rectangles or step functions on the graph below.



(b) Find the average value of the function $\frac{4t}{3+t^2}$ between $t = 0$ and $t = 2$.

6. (15 points)

(a) Andy's horse starts with a concentration of medicine in her bloodstream equal to 3 milligrams per liter (mg/L). Each day, the horse uses up 23% of the medicine in her bloodstream. However, at the end of each day the vet gives her enough medication to increase the concentration of medicine in the bloodstream by 2 mg/L. Let $M_t =$ concentration of medicine on day t, and write down a discrete-time dynamical system, together with an initial condition, that describes this situation.

(b) Let $V(t) =$ the volume (in liters) of blood at time t (in seconds) in Dean's liver. Suppose that

$$\frac{dV}{dt} = 0.2\cos(\pi t - \pi/2).$$

i. Use a definite integral to determine the total change in the volume of blood in Dean's liver between times $t = 1$ and $t = 4$.

ii. Determine $V(t)$ if $V(0) = 0.47$. (That is, find a solution to the differential equation $\frac{dV}{dt} = 0.2\cos(\pi t - \pi/2)$ with initial condition $V(0) = 0.47$.)

7. (14 points)

(a) A population p_t of puffins on an island obeys the discrete-time dynamical system

$$p_{t+1} = 1.12 p_t.$$

i. Write down the solution to this discrete-time dynamical system if $p_0 = 1234$.

ii. If $p_0 = 1234$, at what time will the population reach size 2000?

(b) The density ρ of a very thin rod (measured in grams/cm) varies according to

$$\rho(x) = 4x\mathrm{e}^{-2x^2},$$

where x marks a location along the rod, and $x = 0$ at one end of the rod. What is the total mass of the rod if it is 5 cm long? Give units in your answer.

(c) Suppose that a population $p(t)$ of porcupines satisfies the differential equation

$$\frac{dp}{dt} = 1.11p(10 - p).$$

i. Find all equilibria of the differential equation.

ii. Write down an initial condition for which the population will increase in time (at least initially).

$$p(0) = \underline{\hspace{3em}}.$$