Math 155 **Exam 1** **Spring 2016**

NAME: ____

SECTION: ____________________ TIME: ____________________

INSTRUCTOR: ____

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have one hour and 50 minutes to work on the exam.

Problem	Points	Score
1	15	
2	14	
3	14	
4	14	
5	14	
6	14	
7	15	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

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(Signature)

1. (15 points) Suppose that the population P of prairie chickens at time t is modeled by the equation

$$P(t) = P_0 e^{0.17t},$$

where P_0 is the initial population size, and t is measured in years.

(a) Find the time that it takes for the population to double in size.

(b) Suppose that at time $t = 3$, the population of prairie chickens is $5,000$. What is the initial population P_0?

(c) Consider the discrete-time dynamical system

$$d_{t+1} = 1.09 d_t,$$

where d_t is the population of dolphins at time t, measured in thousands. Find the (explicit) solution to the discrete-time dynamical system, given the inital condition $d_0 = 3$. Express your solution in terms of an exponential function in base e.

2. (14 points) Three baby elephant seals splash into their salt-water pool, spilling 40 L of water. Their pool usually holds 5000 L of water. If one replaces the 40 L of water with salt water that has a concentration of 2 mol/L, the concentration of salt may change.

(a) Fill in the blank boxes below to model the situation above. Let s_t represent the concentration of salt in the pool after the elephant seals have splashed t times, measured in mol/L. Remember that concentration is equal to the amount of salt (mol) divided by the volume (L).

Step	Volume (L)	Total Salt (mol)	Salt Concentration (mol/L)
$H_2 0$ in pool before seals jump in	5000		s_t
Water lost	40		s_t
$H_2 0$ in pool after seals jump in	4960		s_t
Water replaced	40		
$H_2 0$ in pool after replacing water			

(b) Write down the discrete-time dynamical system derived from the chart in part (a):

$$s_{t+1} =$$

(c) Suppose that 14 L of water with a salt concentration of C_1 moles/L is mixed with 21 L of water with a salt concentration of C_2 moles/L . Express the salt concentration of the resulting mixture in terms of a weighted average.

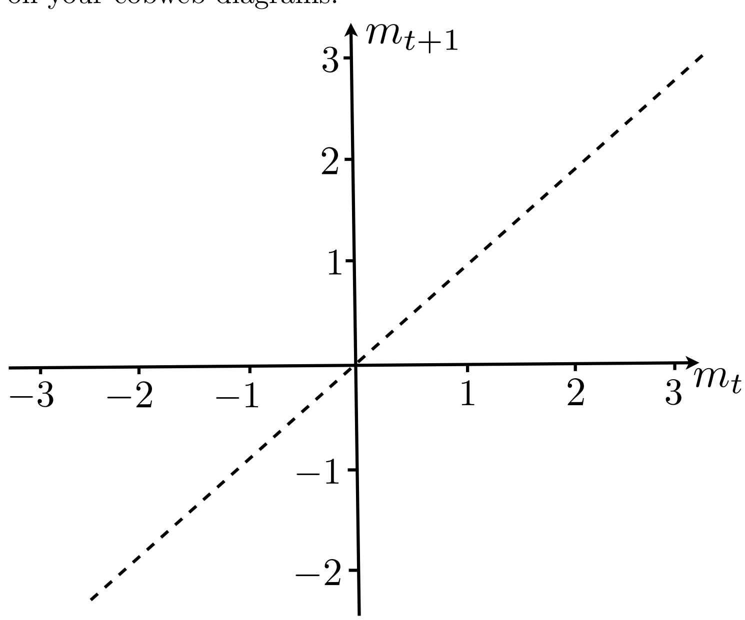
3. (14 points) (a) Write down a discrete-time dynamical system and an initial condition to describe the following situation: Every day, one of Mark's unwell dogs uses up 14% of the medicine in its bloodstream. However, at the end of each day, Mark gives the dog enough medication to increase the concentration of medicine in its bloodstream by 0.6 milligrams per liter. The dog starts with a concentration of medicine in its bloodstream of 1.34 milligrams per liter. (Let M_t = the concentration of medicine in the dog's bloodstream on day t, in milligrams per liter.)

(b) Find all equilibria of the discrete-time dynamical system

$$v_{t+1} = \frac{cv_t}{10v_t + 4}$$

where c is a parameter. For what values of c is there a positive equilibrium?

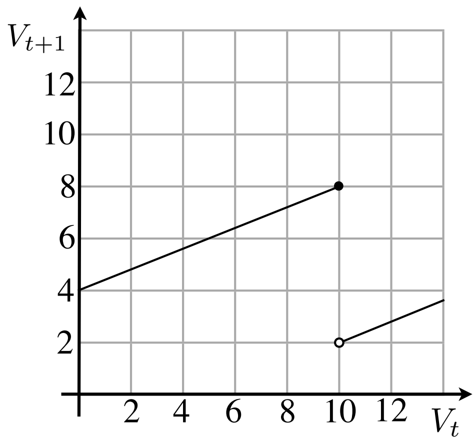
(c) Consider the discrete-time dynamical system $m_{t+1} = 3m_t - 2$. i) *Graph* the updating function on the axes below (the diagonal $m_{t+1} = m_t$ is already graphed). ii) *Circle* all of the equilibria, and use *cobwebbing* to help you label each of the equilibria as stable or unstable. Include arrows on your cobweb diagrams.

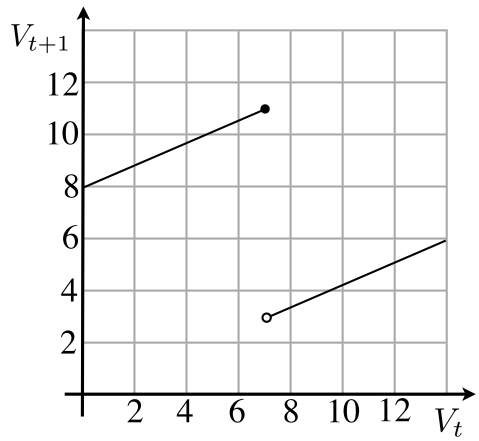


4. (14 points) Let V_t represent the voltage at the AV node in the heart model

$$V_{t+1} = \begin{cases} e^{-\alpha\tau} V_t + u, & \text{if } V_t \leq e^{\alpha\tau} V_c \\ e^{-\alpha\tau} V_t, & \text{if } V_t > e^{\alpha\tau} V_c \end{cases}$$

a) For each of the following two graphs of the updating function, cobweb starting from an initial value of $V_0 = 12$, and determine if the heart i) is healthy, ii) has a 2:1 block, or iii) has the Wenckebach phenomenon. (Include arrows on your cobweb diagram.)





b) Now let $e^{-\alpha\tau} = \frac{3}{4}$, $u = 6$, and $V_c = 7$. Does the system have an equilibrium? Justify your answer algebraically (without drawing a graph), and find the equilibrium if there is one.

5. (14 points) (a) Find the following limits, if they exist. Show all of your work, and justify your answers to receive full credit. If a limit does not exist, write "DNE," and explain why it does not exist.

i) $\lim\limits_{r \to -5} \dfrac{r^2 - 25}{r + 5}$

ii) $\lim\limits_{t \to 5} \dfrac{10}{(t-5)^9}$

iii) $\lim\limits_{x \to 2} f(x)$, where $f(x) = \begin{cases} 2 - x, & \text{if } x < 2 \\ 2\ln(x-1), & \text{if } x \geq 2 \end{cases}$

iv) $\lim\limits_{\Delta t \to 0} \dfrac{-2(t + \Delta t)^2 + 2t^2}{\Delta t}$

6. (14 points) (a) We are interested in the density of a substance at a temperature of absolute zero (which is 0 Kelvin). However, we cannot measure the density directly at 0 Kelvin because it is impossible to reach absolute 0. Instead, we measure density for small values of the temperature.

Suppose that density D is a function of temperature T (measured in Kelvin) according to $D(T) = \frac{100}{1+20T}$.

i) What is $\displaystyle\lim_{T\to 0^+} \frac{100}{1+20T}$?

ii) How close to 0 Kelvin would the temperature have to be for the density to be within 1% of the limit?

(b) Consider the function $f(V)$ defined by

$$f(V) = \begin{cases} \frac{1}{4}V + 10, & \text{if } V \leq 32 \\ \frac{1}{4}V, & \text{if } V > 32 \end{cases}$$

i) Find the following limits, if they exist. If a limit does not exist, write "DNE," and explain why it does not exist.

$\displaystyle\lim_{V\to 32^-} f(V)$

$\displaystyle\lim_{V\to 32^+} f(V)$

$\displaystyle\lim_{V\to 32} f(V)$

ii) Is $f(V)$ a continuous function? Fully justify your answer using the definition of continuity. (A phrase like "the graph can be drawn without lifting the pencil" does not justify your answer).

7. (15 points) Vanessa throws a ball up into the air from the top of a tower. Suppose that the height $h(t)$ (in meters) of the ball as a function of time t (in seconds) is given by $h(t) = -5t^2 + 6t + 40$.

(a) Find a formula for the slope of the secant line that passes through the points $(2, h(2))$ and $(2 + \Delta t, h(2 + \Delta t))$. Simplify your answer.

(b) Find the average rate of change in h between time $t = 2$ and time $t = 2.5$.

(c) Find the instantaneous rate of change of h at $t = 2$ *using the limit-definition of the derivative/instantaneous rate of change.* Is the height of the ball increasing or decreasing at time $t = 2$?

(d) Find the equation for the tangent line to the graph of $h(t)$ at the point $(2, h(2))$.