# CALCULUS AB <br> SECTION II, Part A <br> Time- $\mathbf{3 0}$ minutes <br> Number of problems-2 

## A graphing calculator is required for these problems.

| $t$ (minutes) | 0 | 4 | 9 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $W(t)$ (degrees Fahrenheit) | 55.0 | 57.1 | 61.8 | 67.9 | 71.0 |

1. The temperature of water in a tub at time $t$ is modeled by a strictly increasing, twice-differentiable function $W$, where $W(t)$ is measured in degrees Fahrenheit and $t$ is measured in minutes. At time $t=0$, the temperature of the water is $55^{\circ} \mathrm{F}$. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times $t$ for the first 20 minutes are given in the table above.
(a) Use the data in the table to estimate $W^{\prime}(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
(b) Use the data in the table to evaluate $\int_{0}^{20} W^{\prime}(t) d t$. Using correct units, interpret the meaning of $\int_{0}^{20} W^{\prime}(t) d t$ in the context of this problem.
(c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_{0}^{20} W(t) d t$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_{0}^{20} W(t) d t$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
(d) For $20 \leq t \leq 25$, the function $W$ that models the water temperature has first derivative given by $W^{\prime}(t)=0.4 \sqrt{t} \cos (0.06 t)$. Based on the model, what is the temperature of the water at time $t=25$ ?

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2. Let $R$ be the region in the first quadrant bounded by the $x$-axis and the graphs of $y=\ln x$ and $y=5-x$, as shown in the figure above.
(a) Find the area of $R$.

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(b) Region $R$ is the base of a solid. For the solid, each cross section perpendicular to the $x$-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
(c) The horizontal line $y=k$ divides $R$ into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of $k$.

