Math 141, Exam 2
Spring 2025

Name: _____

Student ID: _____

Version: **A**

Instructions:

- Do NOT open exam booklet until instructed.
- Write your Name and Student ID Number on the lines above.
- Write your Name and Student ID Number on the answer sheet.
- Fill in version (A or B) on your answer sheet.
- No calculators, personal devices (phones, computers, tablets, etc.), or reference materials may be used during the exam.
- You may use any blank space on this exam booklet for your scratch work or ask for a blank sheet for scratch work. DO NOT USE YOUR OWN SCRATCH PAPER!
- Indicate your answer to each question on the answer sheet by fully filling in the appropriate bubble.
- The exam booklet and answer sheet will be collected at the end of the exam. Only the answer sheet will be graded.

Page intentionally left blank for scratch work.

1. Which of the following statements are true about critical points?

(a) Every critical point is either a maximum or minimum.

(b) Critical points occur at x-values with $f''(x) = 0$.

(c) Critical points are candidates for inflection points.

(d) Critical points are candidates for where a relative maximum or minimum occurs.

2. Suppose $f(x)$ is continuous and has only one critical value at $x = 10$. If $f'(0) = 5$ and $f'(15) = -2$, then

(a) $f(x)$ has a relative minimum at $x = 10$.

(b) $f(x)$ has a relative maximum at $x = 10$.

(c) $f(x)$ has neither a relative minimum nor a relative maximum at $x = 10$.

(d) there is not enough information to tell whether $f(x)$ has a relative minimum, relative maximum, or neither at $x = 10$.

3. Suppose $f(x)$ is continuous and has only one critical value at $x = 8$. If $f'(7) > 0$ and $f'(9) > 0$, then

(a) $f(x)$ has a relative minimum at $x = 8$.

(b) $f(x)$ has a relative maximum at $x = 8$.

(c) $f(x)$ has neither a relative minimum nor a relative maximum at $x = 8$.

(d) there is not enough information to tell whether $f(x)$ has a relative minimum, relative maximum, or neither at $x = 8$.

4. If $f''(x) = 6x + 6$, what can you say about $x = -1$?

(a) f has a critical point at $x = -1$ since $f''(-1) = 0$.

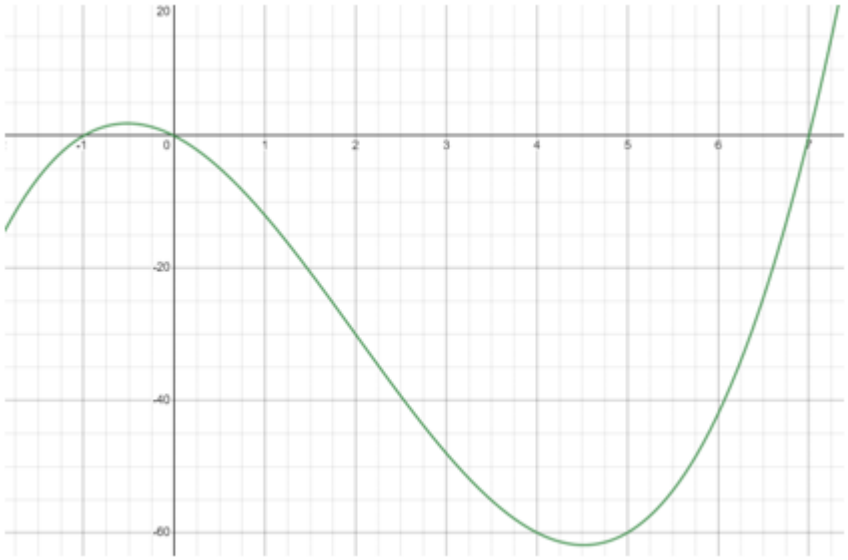
(b) f has an inflection point at $x = -1$ since $f''(-1) = 0$.

(c) f has an inflection point at $x = -1$ since f changes from concave down to concave up at $x = -1$.

(d) f does not have an inflection point at $x = -1$.

(e) There is not enough information to conclude whether f has an inflection point at $x = -1$.

Let $f(x)$ be a function whose derivative $f'(x)$ is graphed below. Use the graph to answer the next **two** questions.



5. What are the critical values of $f(x)$?

(a) $x = -1$, $x = 0$, and $x = 7$

(b) $x = -\frac{1}{2}$ and $x = 4.5$.

6. Which of the following statements is true about $f(x)$?

(a) There is a relative minimum at $x = -1$.

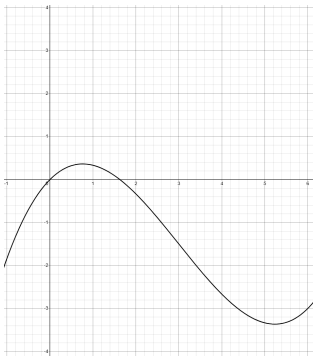
(b) There is a relative minimum at $x = 0$.

(c) There is a relative maximum at $x = 7$.

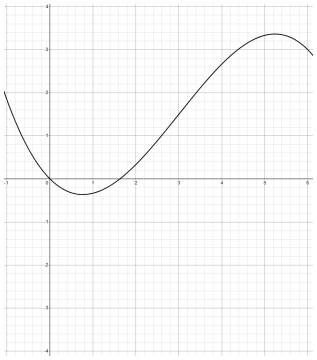
(d) There is a relative maximum at $x = -\frac{1}{2}$.

7. Which of the following graphs satisfy $f'(1) > 0$, $f''(1) < 0$?

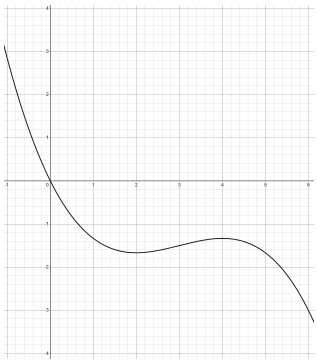
(a)



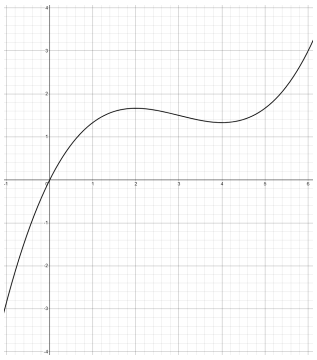
(b)



(c)



(d)



8. Let $f(x) = 2x^3 - 6x + 1$. If you want to determine where the absolute maximum or absolute minimum occurs over the interval $[-3, 2]$, which x-values should you consider?

(a) -1 and 1 because an absolute maximum or absolute minimum over a closed interval can only occur at a critical value.

(b) -3 and -1 because they are the smallest.

(c) $-3, -1, 1, 2$ because the endpoints of $[-3, 2]$ might be critical values.

(d) $-3, -1, 1, 2$ because the largest or smallest value of $f(x)$ over the interval $[-3, 2]$ might occur at the critical values or at the endpoints of the interval.

9. Let $f(x) = 2x^3 - 6x + 1$. What is the absolute maximum over the interval $[-3, 2]$?

(a) The absolute maximum is 5.

(b) The absolute maximum occurs at $x = 5$.

(c) There is no absolute maximum, but there is an absolute minimum at $x = -3$.

(d) More than one x-value has an output of 5, so there is not an absolute maximum.

10. Let $f(x)$ be a continuous function which is differentiable on $[0, 10]$ and has only one critical value at $x = 4$ over the interval $[0, 10]$. If $f''(4) = 7$, what can you conclude about $f(x)$?

(a) $f(4)$ is the absolute maximum of $f(x)$ over the interval $[0, 10]$.

(b) $f(4)$ is a maximum, but not necessarily the absolute maximum, of $f(x)$ over the interval $[0, 10]$.

(c) $f(4)$ is the absolute minimum of $f(x)$ over the interval $[0, 10]$.

(d) $f(4)$ is a minimum, but not necessarily the absolute maximum, of $f(x)$ over the interval $[0, 10]$.

11. What is the linearization of $f(x)$ at $x = a$?

(a) It is another way of writing the differential of x and differential of y.

(b) It is the equation of the tangent line to $f(x)$ at $x = a$.

(c) It is **not** the equation of the tangent line to f at $x = a$, but it is a good approximation for the tangent line.

(d) It is $f'(x) = \frac{dy}{dx}$.

12. What is the linearization of $f(x) = \sqrt{x}$ at $x = 16$?

(a) $\Delta y = f'(16)\Delta x$

(b) $y = f'(16) - f(16)(x - 16)$

(c) $dy = \frac{1}{2\sqrt{16}}dx$

(d) $y = \sqrt{16} + \frac{1}{2\sqrt{16}}(x - 16)$

(e) $\frac{dy}{dx} = f'(16)$

13. Use the linearization of $f(x) = \sqrt{x}$ at $x = 16$ to approximate $\sqrt{18}$.

(a) 4

(b) 4.125

(c) 4.25

(d) 4.24

14. You want to fence a rectangular area on the side of your house. Since your house will serve as one side of the rectangle you only need to fence three sides to make an enclosure. You have 12 feet of fencing. What is the maximum area possible for the rectangle? (The area of a rectangle with length ℓ and width w is $\ell \cdot w$.)

(a) 16 square feet

(b) 18 square feet

(c) 36 square feet

(d) 9 square feet

15. What is the maximum value of $Q = x^2y$ where x and y are positive numbers such that $x + y = 6$.

(a) 32

(b) 16

(c) 30

(d) 10

16. Differentiate $y^4 + x^2y - x = -6$ implicitly to find $\frac{dy}{dx}$.

(a) $\frac{dy}{dx} = \frac{1-2xy-4y^3}{x^2}$

(b) $\frac{dy}{dx} = \frac{1-2xy}{4y^3+x^2}$

(c) $\frac{dy}{dx} = \frac{1}{4y^3+2x}$

(d) $\frac{dy}{dx} = 4y^3 + 2x - 1$

(e) A function of more that one variable cannot be differentiated.

17. Find the slope of the tangent line to the curve $y^4 + x^2y - x = -6$ at the point $(6, 0)$.

(a) $\frac{1}{12}$

(b) $\frac{1}{36}$

(c) $\frac{1}{4}$

(d) The slope is undefined.

In the next **two** problems consider the following scenario. A circular oil spill on a flat surface is expanding over time. The area A of the spill is given by the formula $A = \pi r^2$, where r is the radius of the spill in meters and A is the area in square meters. The radius of the spill is increasing at a rate of $\frac{dr}{dt} = 0.2\ m/s$.

18. At the moment when the radius is $r = 5\ m$, how fast is the area of the oil spill changing?

(a) $0.4\pi\ m^2/s$

(b) $10\pi\ m^2/s$

(c) $2\pi\ m^2/s$

(d) $4\pi\ m^2/s$

19. Which of the following best describes the relationship between the rate at which the radius is increasing and the rate at which the area is increasing?

(a) The area increases more quickly than the radius.

(b) The radius increases more quickly than the area.

(c) The area increases at the same rate as the radius.

(d) The rates are unrelated.

20. Which of the following statements about derivatives is true.

(a) A derivative can only be computed for continuous functions.

(b) A derivative represents the slope of a secant line to a function.

(c) A derivative represents the rate of change of a function at a given point.

(d) The derivative of a function is always positive.