Math 141 Exam 1 Study Guide

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1. 3 Types of Limits

There are 3 different limits that you need to know how to find.
A left hand limit, a right hand limit, and an overall limit.

A. Left Hand Limit (LHL)

A left hand limit is asking you what **y-value** it looks like you are heading towards as you approach the specific **x-value** from the left direction. So, going from the left side of the graph moving right. ($\longrightarrow$). You know that they are talking about the left hand limit when the exponent of the x-value you are looking for is a negative symbol (**–**).

Example:

General Form	$\lim_{x \to a^-} f(x) = y$	**Read:** Limit of **f(x)** as x approaches a from the left equals y.
Specific Example	$\lim_{x \to 5^-} f(x) = 7$	**Read:** Limit of **f(x)** as x approaches 5 from the left equals 7.

B. Right Hand Limit (RHL)

A right hand limit is asking you what **y-value** it looks like you are heading towards as you approach the specific **x-value** from the right direction. So, going from the right side of the graph moving left. ($\longleftarrow$). You know that they are talking about the left hand limit when the exponent of the x-value you are looking for is a positive symbol (**+**).

Example:

General Example	$\lim_{x \to a^+} f(x) = y$	**Read:** Limit of **f(x)** as x approaches a from the right equals y.
Specific Example	$\lim_{x \to 5^+} f(x) = 7$	**Read:** Limit of **f(x)** as x approaches 5 from the right equals 7.

C. Overall Limit

An overall limit is asking you what **y-value** it looks like you are heading towards as you approach the specific **x-value** from the both directions. Essentially, the overall limit is asking you if the results of the Left Hand Limit and the Right Hand Limit are the same **y-value**, and therefore the answer to the overall limit is the same **y-value**. **OR** Are the results of the Left Hand Limit and the Right Hand Limit different **y-values**, and therefore the answer to the overall limit is **Does Not Exist (DNE)**. You know that they are talking about the overall limit when there is no exponent on the **x-value** you are looking for.

Example:

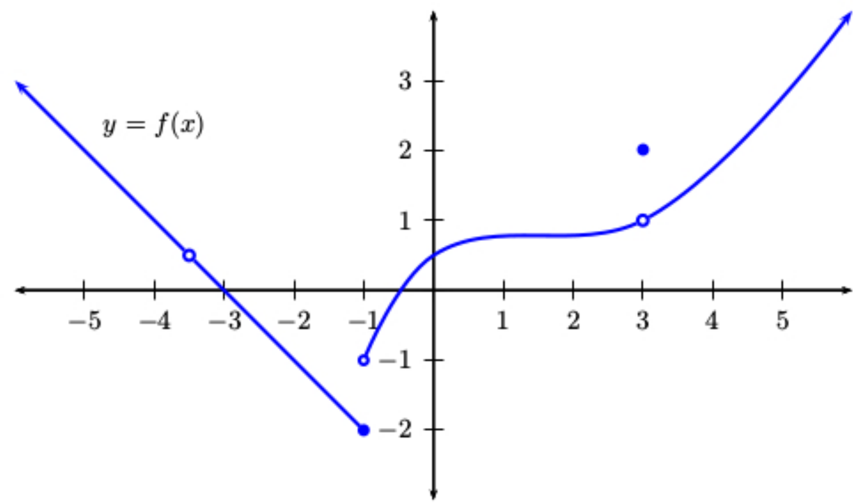
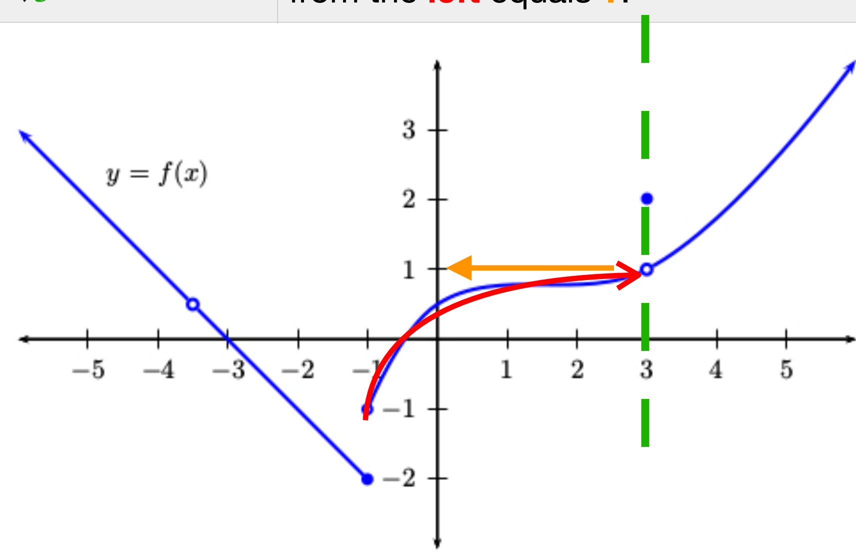
General Example	$\lim_{x \to a} f(x) = y$	**Read:** Limit of **f(x)** as x approaches a equals y.
Specific Example	$\lim_{x \to 5} f(x) = 7$	**Read:** Limit of **f(x)** as x approaches 5 equals 7.

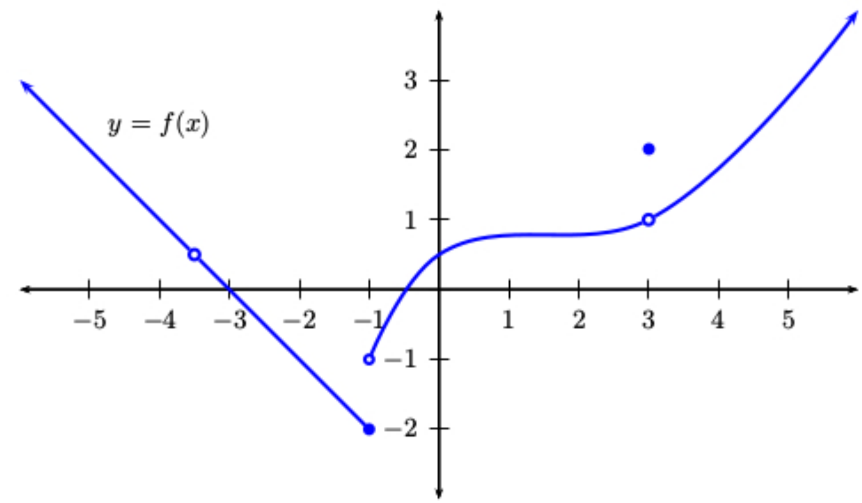
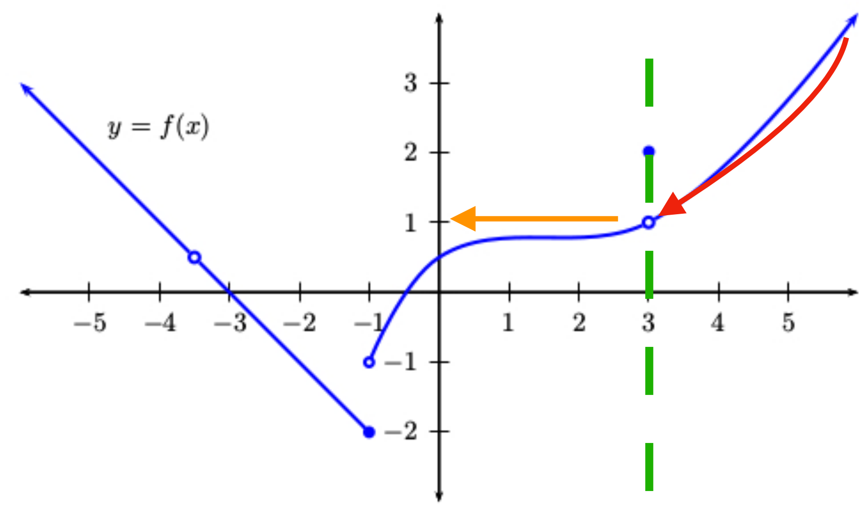
2. Solving Limits Graphically & Using Equations

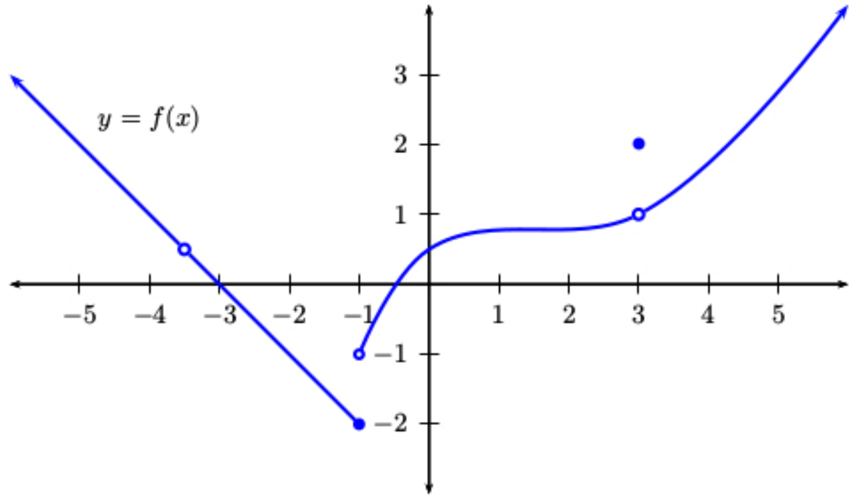
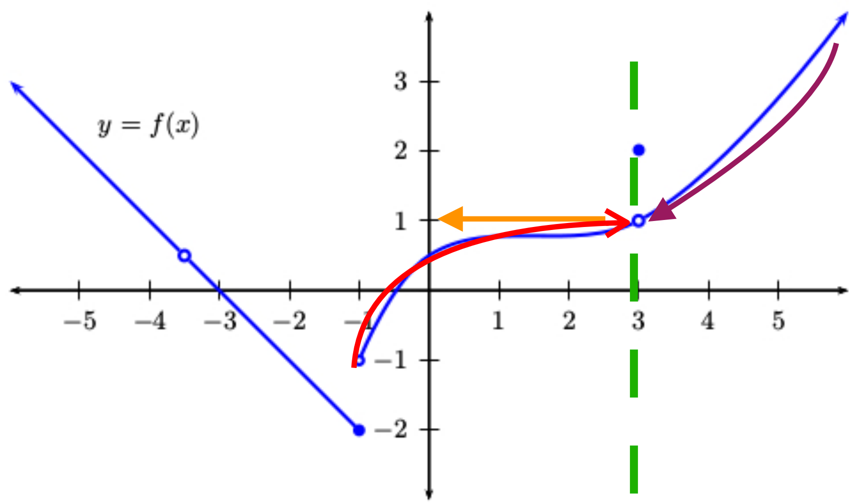
There are two standard styles of limit questions that you will want to be able to handle. The first is graphically, meaning they give you a graph to work from to find the answer. The second is based on an equation, meaning they will give you an equation and they will want you to find a specific limit at a specific point using that equation.

A. Graphically

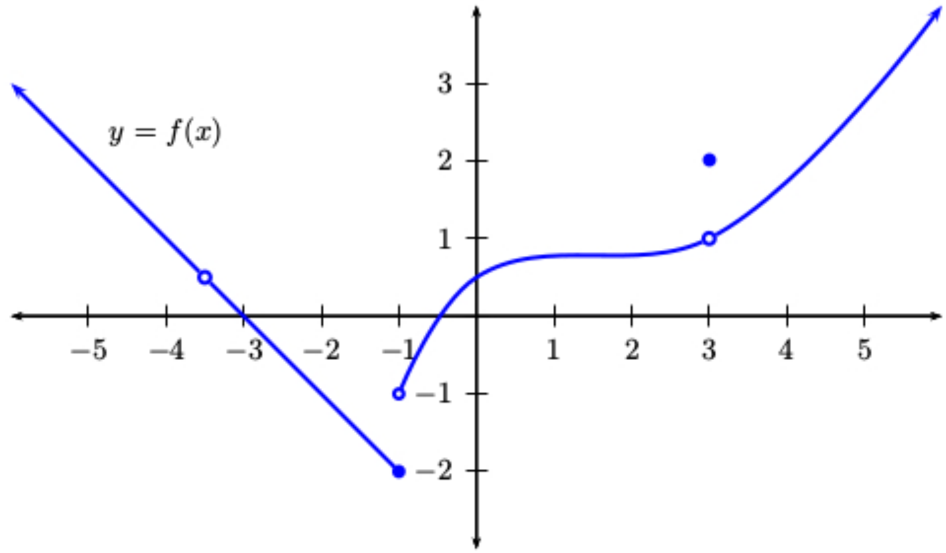
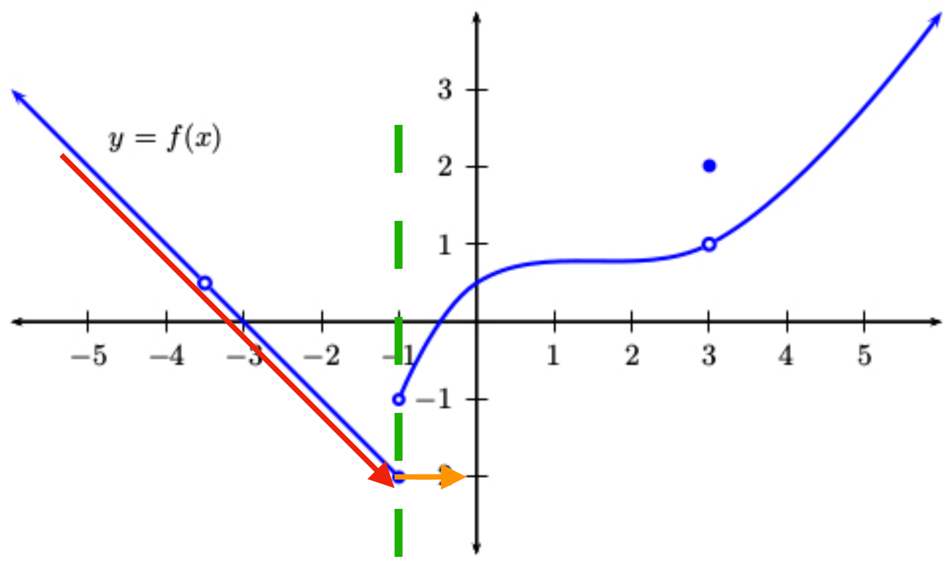
a. Example 1

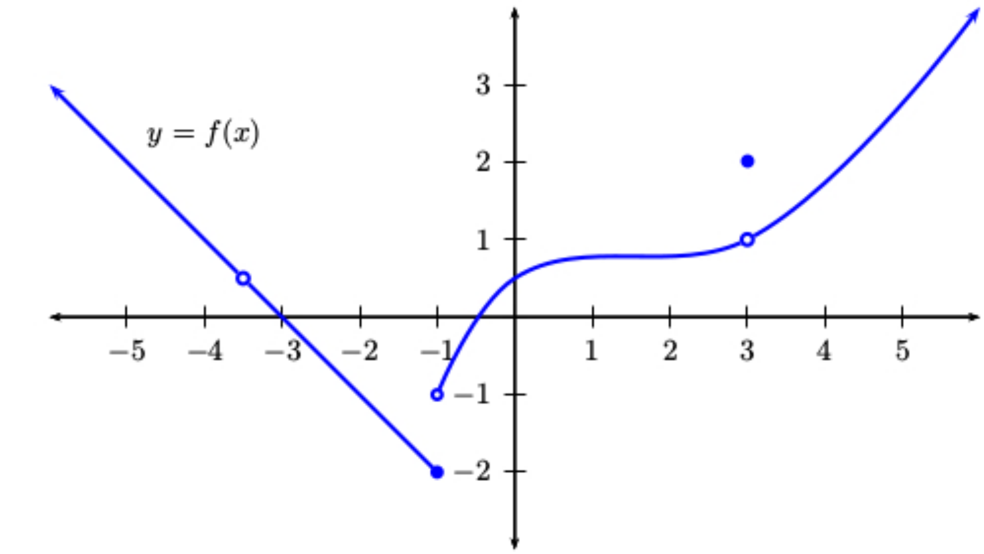
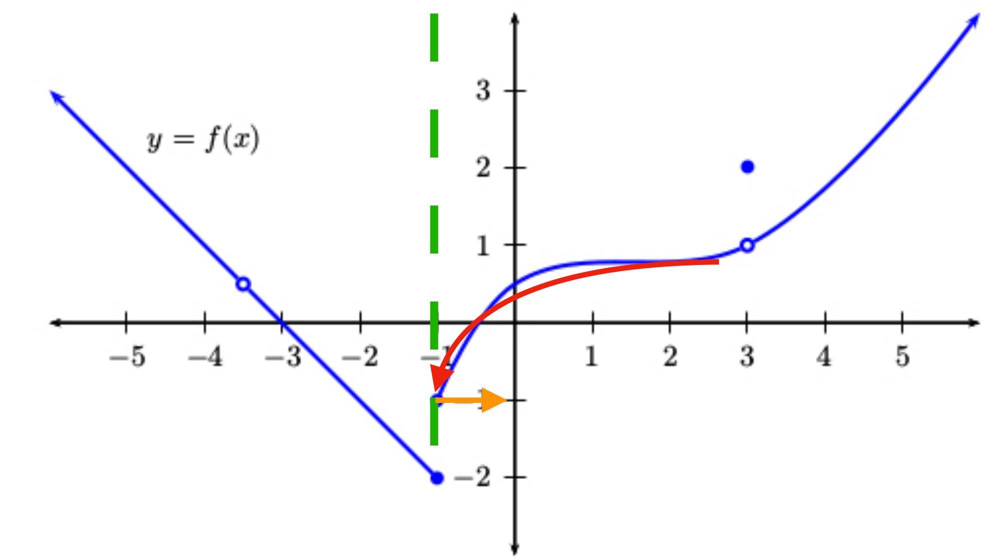
Question 1a		
Left Hand Limit	$\lim_{x \to 3^-} f(x) =$	**Read:** Limit of **f(x)** as **x** approaches **3** from the **left** equals **?**.
Solution 1a		
Step 1	Follow along the graph moving from **Left** to Right and head toward the **x-value** of **3**.	
Step 2	The **y-value** that it looks like you are headed towards is the answer. *NOTE: The Limit Does Not Care if there is a Dot or an Open Circle. It only cares what it looks like it is headed towards.*	
Answer	$\lim_{x \to 3^-} f(x) = 1$	**Read:** Limit of **f(x)** as **x** approaches **3** from the **left** equals **1**.

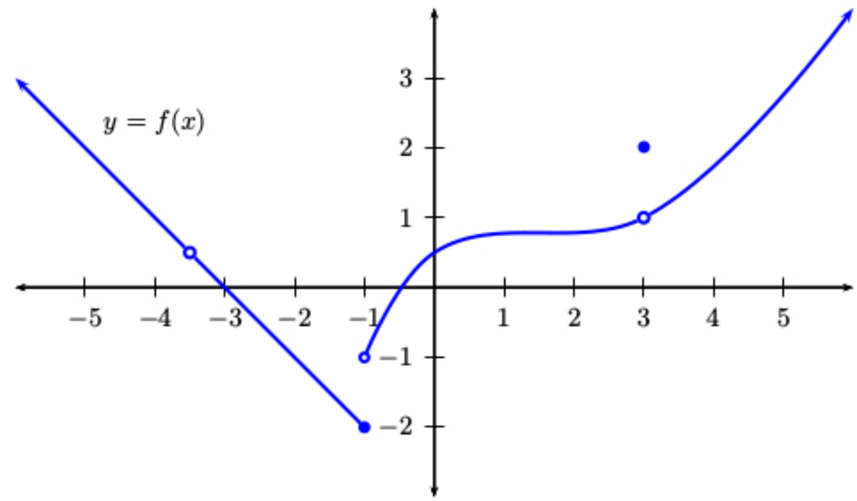
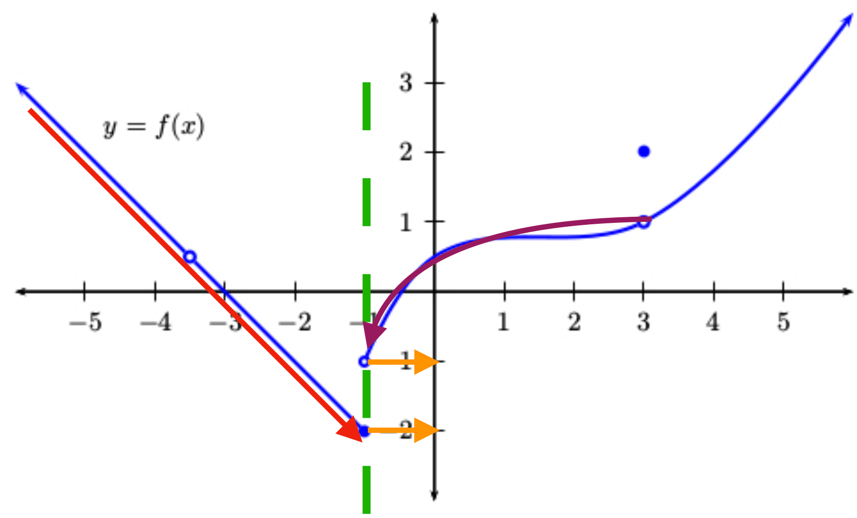
Question 1b		
Right Hand Limit	$\lim\limits_{x \to 3^+} f(x) =$	**Read:** Limit of **f(x)** as **x** approaches **3** from the **right** equals **?**.
Solution 1b		
Step 1	Follow along the graph moving from **Right** to Left and head toward the **x-value** of **3**.	
Step 2	The **y-value** that it looks like you are headed towards is the answer. *NOTE: The Limit Does Not Care if there is a Dot or an Open Circle. It only cares what it looks like it is headed towards.*	
Answer	$\lim\limits_{x \to 3^+} f(x) = 1$	**Read:** Limit of **f(x)** as **x** approaches **3** from the **right** equals **1**.

Question 1c		
Overall Limit	$\lim_{x \to 3} f(x) =$	**Read:** Limit of **f(x)** as **x** approaches **3** equals **?**.
Solution 1c		
Note	When finding an **Overall Limit**, we have to answer the question, Does the **LHL**=**RHL**? If they do equal each other, then the overall limit is the same. If they do not equal each other, then the overall limit Does Note Exist (DNE).	
Step 1	Find the **Left Hand Limit (LHL)** as **x** approaches **3**. $\lim_{x \to 3^-} f(x) = 1$ *Note: We found this result in Question 1a above.*	
Step 2	Find the **Right Hand Limit (RHL)** as **x** approaches **3**. $\lim_{x \to 3^+} f(x) = 1$ *Note: We found this result in Question 1b above.*	
Step 3	Compare the results of Step 1 and Step 2. **LHL = 1** **RHL = 1** **Since the LHL = RHL the Overall Limit is the same 1.**	
Answer	$\lim_{x \to 3} f(x) = 1$	**Read:** Limit of **f(x)** as **x** approaches **3** equals **1**.

b. Example 2

Question 2a		
Left Hand Limit	$\lim_{x \to -1^-} f(x) =$	**Read:** Limit of **f(x)** as **x** approaches **-1** from the **left** equals **?**.
Solution 2a		
Step 1	Follow along the graph moving from **Left** to Right and head toward the **x-value** of **-1**.	
Step 2	The **y-value** that it looks like you are headed towards is the answer. *NOTE: The Limit Does Not Care if there is a Dot or an Open Circle. It only cares what it looks like it is headed towards.*	
Answer	$\lim_{x \to -1^-} f(x) = -2$	**Read:** Limit of **f(x)** as **x** approaches **-1** from the **left** equals **-2**.

Question 2b		
Right Hand Limit	$\lim_{x \to -1^{+}} f(x) =$	**Read:** Limit of **f(x)** as **x** approaches **-1** from the **right** equals **?**.
Solution 2b		
Step 1	Follow along the graph moving from **Right** to Left and head toward the **x-value** of **-1**.	
Step 2	The **y-value** that it looks like you are headed towards is the answer. *NOTE: The Limit Does Not Care if there is a Dot or an Open Circle. It only cares what it looks like it is headed towards.*	
Answer	$\lim_{x \to -1^{+}} f(x) = -1$	**Read:** Limit of **f(x)** as **x** approaches **-1** from the **right** equals **-1**.

<table>
<tr><td>Question 2c</td><td colspan="2"></td></tr>
<tr><td>Overall Limit</td><td>$\lim_{x \to -1} f(x) =$</td><td>Read: Limit of f(x) as x approaches -1 equals ?.</td></tr>
<tr><td>Solution 2c</td><td colspan="2"></td></tr>
<tr><td>Note</td><td colspan="2">When finding an Overall Limit, we have to answer the question, Does the LHL=RHL? If they do equal each other, then the overall limit is the same. If they do not equal each other, then the overall limit Does Note Exist (DNE).</td></tr>
<tr><td>Step 1</td><td colspan="2">Find the Left Hand Limit (LHL) as x approaches -1.
$\lim_{x \to -1^-} f(x) = -2$
Note: We found this result in Question 2a above.</td></tr>
<tr><td>Step 2</td><td colspan="2">Find the Right Hand Limit (RHL) as x approaches -1.
$\lim_{x \to -1^+} f(x) = -1$
Note: We found this result in Question 2b above.</td></tr>
<tr><td>Step 3</td><td colspan="2">Compare the results of Step 1 and Step 2.
LHL = -2
RHL = -1
Since the LHL $\neq$ RHL the Overall Limit Does Note Exist (DNE).</td></tr>
<tr><td>Answer</td><td>$\lim_{x \to -1} f(x) = DNE$</td><td>Read: Limit of f(x) as x approaches -1 Does Note Exist.</td></tr>
</table>

B. Using Equations

a. Example 1

Question	$\lim_{x \to -3^-} x^2 - 3x + 7 =$	**Read:** Limit of $x^2 - 3x + 7$ as **x** approaches **-3** from the **Left**.
Solution	**Step 1**	Pick up your TI-83/84 calculator, or whatever graphing calculator you own.
	Step 2	Hit the **Y=** button located on the top left of your calculator.
	Step 3	Enter the equation $x^2 - 3x + 7$ into **Y1** on your calculator.
	Step 4	Hit the **Graph** button on your calculator, located in the top right.
	Step 5	Hit **2nd—>CALC—>#1Value** Then enter the **x-value** we are looking for, which is **-3**, and hit **Enter**
	Step 6	Read off the **y-value** of **25** from your calculator, this is your answer.
Answer	$\lim_{x \to -3^-} x^2 - 3x + 7 = 25$	**Read:** Limit of $x^2 - 3x + 7$ as **x** approaches **-3** from the **Left** is **25**.
Note	*Notice that when using this method, the fact that it was a **left hand limit** **did not** matter when calculating the solution.*	

b. Example 2

Question	$\lim_{x \to -3^+} x^2 - 3x + 7 =$	**Read:** Limit of $x^2 - 3x + 7$ as **x** approaches **-3** from the **Right**.
Solution	**Step 1**	Pick up your TI-83/84 calculator, or whatever graphing calculator you own.
	Step 2	Hit the **Y=** button located on the top left of your calculator.
	Step 3	Enter the equation $x^2 - 3x + 7$ into **Y1** on your calculator.
	Step 4	Hit the **Graph** button on your calculator, located in the top right.
	Step 5	Hit **2nd—>CALC—>#1Value** Then enter the **x-value** we are looking for, which is **-3**, and hit **Enter**
	Step 6	Read off the **y-value** of **25** from your calculator, this is your answer.
Answer	$\lim_{x \to -3^+} x^2 - 3x + 7 = 25$	**Read:** Limit of $x^2 - 3x + 7$ as **x** approaches **-3** from the **Right** is **25**.
Note	*Notice that when using this method, the fact that it was a **right hand limit did not** matter when calculating the solution.*	

c. **Example 3**

Question	$\lim_{x \to -3} x^2 - 3x + 7$	**Read:** Limit of $x^2 - 3x + 7$ as **x** approaches **-3**.
Solution	**Step 1**	Pick up your TI-83/84 calculator, or whatever graphing calculator you own.
	Step 2	Hit the **Y=** button located on the top left of your calculator.
	Step 3	Enter the equation $x^2 - 3x + 7$ into **Y1** on your calculator.
	Step 4	Hit the **Graph** button on your calculator, located in the top right.
	Step 5	Hit **2nd—>CALC—>#1Value** Then enter the **x-value** we are looking for, which is **-3**, and hit **Enter**
	Step 6	Read off the **y-value** of **25** from your calculator, this is your answer.
Answer	$\lim_{x \to -3} x^2 - 3x + 7 = 25$	**Read:** Limit of $x^2 - 3x + 7$ as **x** approaches **-3** is **25**.
Note	*Notice that when using this method, the fact that it was a* ***overall limit did not*** *matter when calculating the solution.*	

d. Example 4

Question	$\lim_{x \to -2^-} \frac{x^2+5x+6}{x+2} =$	**Read:** Limit of $\frac{x^2+5x+6}{x+2}$ as **x** approaches **-2** from the **Left**.
Solution	**Step 1**	Pick up your TI-83/84 calculator, or whatever graphing calculator you own.
	Step 2	Hit the **Y=** button located on the top left of your calculator.
	Step 3	Enter the equation $\frac{x^2+5x+6}{x+2}$ into **Y1** on your calculator. *Note: When entering a fraction into your calculator you need to make sure that you put the whole top in parenthesis, and the whole bottom in parenthesis.* *So your equation should look like this* $\frac{(x^2+5x+6)}{(x+2)}$ *in your **Y1**.* *If you have one of the new TI-84's there is actually a very cool button that will save you from having to worry about parenthesis at all. Just hit **Alpha—> Y= —> n/d** and your calculator will create a fraction for you.*
	Step 4	Hit the **Graph** button on your calculator, located in the top right.
	Step 5	Hit **2nd—>CALC—>#1Value** Then enter the **x-value** we are looking for, which is **-2**, and hit **Enter**
	Step 6	You will notice that the **y-value** comes up as blank. This does not tell us anything about the answer, just that this first method was not enough. Since we are doing a **left hand limit** we will just want to repeat **Step 5** with a value **very very very** close to **-2** from the **left**. Try **Step 5** again using the **x-value** of **-2.0000000001**.
	Step 7	Read off the **y-value** of **1** from your calculator, this is your answer.
Answer	$\lim_{x \to -2^-} \frac{(x^2+5x+6)}{(x+2)} = 1$	**Read:** Limit of $\frac{(x^2+5x+6)}{(x+2)}$ as **x** approaches **-2** from the **Left** is **1**.
Note	*Notice that when using this method, the fact that it was a **left hand limit** **did** matter when calculating the solution.*	

e. **Example 5**

Question	$\lim\limits_{x \to -2^+} \frac{(x^2+5x+6)}{(x+2)} =$	**Read:** Limit of $\frac{x^2+5x+6}{x+2}$ as **x** approaches **-2** from the **Right**.
Solution	**Step 1**	Pick up your TI-83/84 calculator, or whatever graphing calculator you own.
	Step 2	Hit the **Y=** button located on the top left of your calculator.
	Step 3	Enter the equation $\frac{x^2+5x+6}{x+2}$ into **Y1** on your calculator. *Note: When entering a fraction into your calculator you need to make sure that you put the whole top in parenthesis, and the whole bottom in parenthesis.* *So your equation should look like this* $\frac{(x^2+5x+6)}{(x+2)}$ *in your **Y1**.* *If you have one of the new TI-84's there is actually a very cool button that will save you from having to worry about parenthesis at all. Just hit **Alpha—> Y= —> n/d** and your calculator will create a fraction for you.*
	Step 4	Hit the **Graph** button on your calculator, located in the top right.
	Step 5	Hit **2nd—>CALC—>#1Value** Then enter the **x-value** we are looking for, which is **-2**, and hit **Enter**
	Step 6	You will notice that the **y-value** comes up as blank. This does not tell us anything about the answer, just that this first method was not enough. Since we are doing a **right hand limit** we will just want to repeat **Step 5** with a value **very very very** close to **-2** from the **right**. Try **Step 5** again using the **x-value** of **-1.9999999999**.
	Step 7	Read off the **y-value** of **1** from your calculator, this is your answer.
Answer	$\lim\limits_{x \to -2^-} \frac{(x^2+5x+6)}{(x+2)} = 1$	**Read:** Limit of $\frac{(x^2+5x+6)}{(x+2)}$ as **x** approaches **-2** from the **Right** is **1**.
Note	*Notice that when using this method, the fact that it was a **right hand limit did** matter when calculating the solution.*	

f. **Example 6**

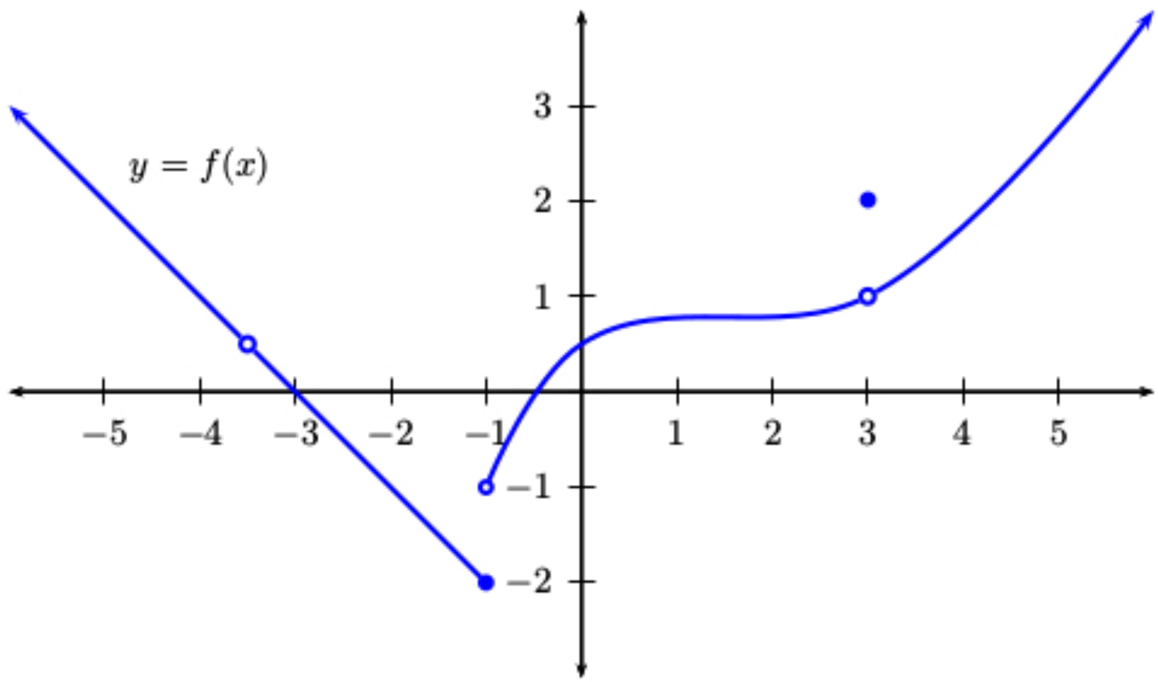
Question	$\lim_{x \to -2} \frac{(x^2+5x+6)}{(x+2)} =$	**Read:** Limit of $\frac{x^2+5x+6}{x+2}$ as **x** approaches **-2**.
Solution	**Step 1**	Pick up your TI-83/84 calculator, or whatever graphing calculator you own.
	Step 2	Hit the **Y=** button located on the top left of your calculator.
	Step 3	Enter the equation $\frac{x^2+5x+6}{x+2}$ into **Y1** on your calculator. *Note: When entering a fraction into your calculator you need to make sure that you put the whole top in parenthesis, and the whole bottom in parenthesis.* *So your equation should look like this* $\frac{(x^2+5x+6)}{(x+2)}$ *in your* ***Y1****.* *If you have one of the new TI-84's there is actually a very cool button that will save you from having to worry about parenthesis at all. Just hit* ***Alpha—> Y= —> n/d*** *and your calculator will create a fraction for you.*
	Step 4	Hit the **Graph** button on your calculator, located in the top right.
	Step 5	Hit **2nd—>CALC—>#1Value** Then enter the **x-value** we are looking for, which is **-2**, and hit **Enter**
	Step 6	You will notice that the **y-value** comes up as blank. This does not tell us anything about the answer, just that this first method was not enough. Since we are doing an **Overall Limit** we will need to find the values of the **LHL** and **RHL**'sj and compare their results. This requires us to follow the steps from Example 4 & Example 5.
	Step 7	Example 4 provides a **Left Hand Limit** of **1** Example 5 provides a **Right Hand Limit** of **1** Since the **LHL** = **RHL**, the **Overall Limit** is the same **1**
Answer	$\lim_{x \to -2} \frac{(x^2+5x+6)}{(x+2)} = 1$	**Read:** Limit of $\frac{(x^2+5x+6)}{(x+2)}$ as **x** approaches **-2** is **1**.
Note	*Notice that when using this method, the fact that it was a* ***overall limit did*** *matter when calculating the solution.*	

3. Determining Continuity

A. Common Language Definition

At a very basic level, a graph is continuous if you can draw it without lifting your pencil. Any time you have to lift your pencil to draw a part of a graph, at that point the function (graph) is not continuous.

EXAMPLE: When you look at the graph of **f(x)** below, if you tried to draw the graph you would need to pick up your pencil at three different places in order to complete the graph. At those 3 points the graph is **NOT** continuous. The three places would be at **x = -3.5**, **x = -1**, and **x = 3**.



B. Formal Definition

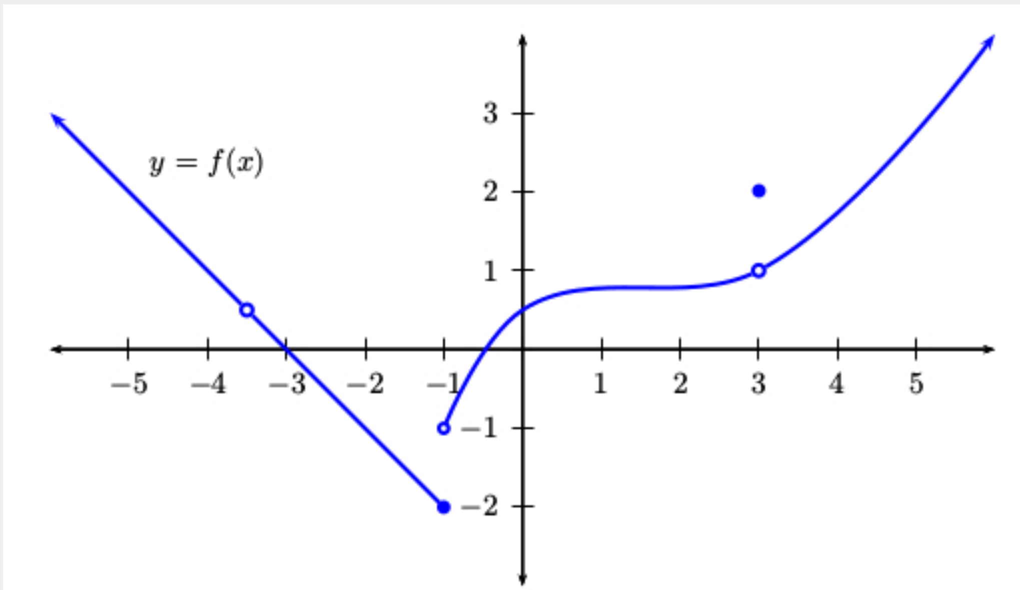
The formal definition of a function being continuous looks like this:

$$\lim_{x \to a} f(x) = f(a)$$

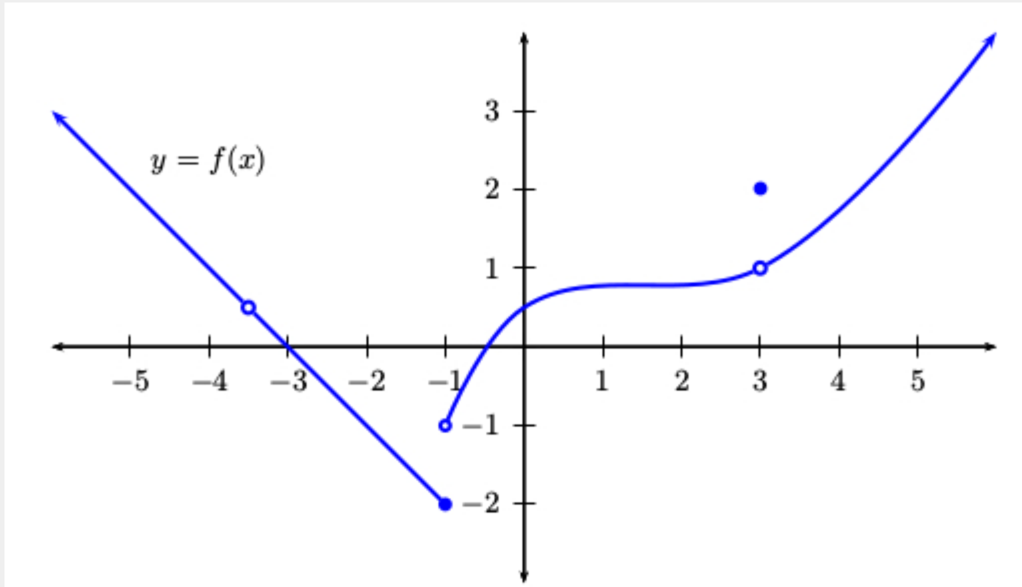
What this math sentence is really saying in common language is:

$$\textit{Does the Limit} = \textit{The Dot on the Graph}$$

EXAMPLE 1:

Question	Is the graph of f(x) below continuous at **x = 3**?	
		
Step 1	Find the limit as x approaches **3**.	$\lim_{x \to 3} f(x) = ?$
	Left Hand Limit	$\lim_{x \to 3^-} f(x) = 1$
	Right Hand Limit	$\lim_{x \to 3^+} f(x) = 1$
	Overall Limit	$\lim_{x \to 3} f(x) = 1$
Step 2	Find the function value at **x = 3**. In other words what is the **y-value** or the dot on the graph.	$f(3) = ?$
	Dot on the Graph	$f(3) = 2$
Step 3	Compare results from Step 1 (Limit) and Step 2 (Dot on the Graph).	$\lim_{x \to 3} f(x) = 1 \neq f(3) = 2$
Answer	The limit does not equal the dot on the graph so f(x) is not continuous at **x=3**.	**f(x) is NOT continuous at x = 3.**

EXAMPLE 2:

Question	Is the graph of f(x) below continuous at **x = -3**?	
		
Step 1	Find the limit as x approaches **-3**.	$\lim_{x \to -3} f(x) = ?$
	Left Hand Limit	$\lim_{x \to -3^-} f(x) = 0$
	Right Hand Limit	$\lim_{x \to -3^+} f(x) = 0$
	Overall Limit	$\lim_{x \to -3} f(x) = 0$
Step 2	Find the function value at **x = -3**. In other words what is the **y-value** or the dot on the graph.	$f(-3) = ?$
	Dot on the Graph	$f(-3) = 0$
Step 3	Compare results from Step 1 (Limit) and Step 2 (Dot on Graph).	$\lim_{x \to -3} f(x) = 0 = f(-3) = 0$
Answer	The limit **does equal** the dot on the graph so f(x) is **continuous** at **x=3**.	**f(x) is continuous at x= -3.**

4. Limit Definition of the Derivative

The limit definition of the derivative is really just a history lesson on the way that people used to do derivatives before the quicker derivative rules, which are located in Section 6 were developed. This is really just a big algebra problem. You will generally only have 1 or 2 of these problems on your exam because the algebra is so long, not hard, just long.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE: Below is an example of how to apply the limit definition to an equation **f(x)** to find the final answer, which is the derivative **f'(x)**.

Question	Use the Limit Definition of the Derivative to find **f'(x)** for the given **f(x)**.
Equation	$f(x)=3x^2-5x+9$
Step 1	Find $f(x+h)$
Replace all **x's** in **f(x)** with **x+h**	$f(x+h)=3(x+h)^2-5(x+h)+9$
Expand out the equation by rewriting $(x+h)^2$	$f(x+h)=3(x+h)(x+h)-5(x+h)+9$
FOIL or Multiply together the $(x+h)(x+h)$	$f(x+h)=3(x^2+xh+xh+h^2)-5(x+h)+9$
Distribute the **3** through the first set of parenthesis and the **-5** through the second set.	$f(x+h)=3x^2+3xh+3xh+3h^2-5x-5h+9$
Combine like terms and we are done with **Step 1**.	$f(x+h)=3x^2+6xh+3h^2-5x-5h+9$
Step 2	Plug $f(x+h)$ from **Step 1** and original $f(x)$ into Limit Definition of the Derivative.
Limit Definition of the Derivative	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
Plug in $f(x+h)$ and $f(x)$ *Make sure to put the entire* $f(x)$ *equation in parenthesis.*	$f'(x)=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-5x-5h+9-(3x^2-5x+9)}{h}$
Distribute the negative sign through the parenthesis.	$f'(x)=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-5x-5h+9-3x^2+5x-9}{h}$
Combine like terms. Every time you do this the entire back piece should cancel entirely.	$f'(x)=\lim_{h\to 0}\frac{\cancel{3x^2}+6xh+3h^2-\cancel{5x}-5h+\cancel{9}-\cancel{3x^2}+\cancel{5x}-\cancel{9}}{h}=\lim_{h\to 0}\frac{6xh+3h^2-5h}{h}$
Factor an **h** out of the top and cancel with the **h** on the bottom.	$f'(x)=\lim_{h\to 0}\frac{h(6x+3h-5)}{h}=\lim_{h\to 0}\frac{\cancel{h}(6x+3h-5)}{\cancel{h}}=\lim_{h\to 0}6x+3h-5$
Plug **0** in for **h** and simplify.	$f'(x)=6x+3(0)-5=6x-5$
Final Answer	$f'(x)=6x-5$

5. Algebraic Rewrite Rules

Before you start taking a derivative using your derivative rules, you **ALWAYS** need to look to rewrite the equation to make sure that it is setup properly for you. There are two rewrite rules from Algebra that you will need to look to apply. Below are those two rules plus the situation where you would need to use both rules at once.

A. Radicals or Roots (Square Roots)

Any time you have an **x** underneath a radical (root) you will need to rewrite the radical so that it is a power.

Example	$\sqrt[root]{x^{power}}$	$\sqrt[r]{x^{p}}$	$\sqrt[3]{x^{5}}$
Rewrite: Change to an exponent with the **power** written over the **root**.	$x^{\frac{power}{root}}$	$x^{\frac{p}{r}}$	$x^{\frac{5}{3}}$
One of the most common radicals to have to rewrite is a square root. However, the square root has its **power = 1** and its **root = 2** unwritten.	$\sqrt{x}$	$\sqrt[2]{x^{1}}$	$x^{\frac{1}{2}}$

B. X on the Bottom of a Fraction

Whenever there is a single **x** to a power on the bottom of a fraction (denominator) with a single number on the top of the fraction (numerator), you will want to rewrite the fraction to move the **x** from the bottom to the top.

Example	$\frac{number}{x^{power}}$	$\frac{n}{x^{p}}$	$\frac{5}{x^{8}}$
Rewrite: Bring the x from the bottom to the top and change the **power** to a negative.	$number \cdot x^{-power}$	$n \cdot x^{-p}$	$5 \cdot x^{-8}$

C. Both at Once

The final rewrite you need to look out for is just a combination of the first two rewrites. The situation is when you have a radical (root) on the bottom of a fraction.

Example	$\dfrac{number}{\sqrt[root]{x^{power}}}$	$\dfrac{n}{\sqrt[r]{x^{p}}}$	$\dfrac{5}{\sqrt[3]{x^{5}}}$
Step 1: Do the Radical Rewrite	$\dfrac{number}{x^{\frac{power}{root}}}$	$\dfrac{n}{x^{\frac{p}{r}}}$	$\dfrac{5}{x^{\frac{5}{3}}}$
Step 2: Do the X on the Bottom Rewrite	$number \cdot x^{-\frac{power}{root}}$	$n \cdot x^{-\frac{p}{r}}$	$5 \cdot x^{-\frac{5}{3}}$

6. Derivative Rules

There are 4 main derivative rules; the Power Rule, the Product Rule, the Quotient Rule and the Chain Rule. For the First Exam in Math 141 you will only need to know the first three.

When we find a derivative we are finding an equation that allows us to find the slope of the tangent line or *instantaneous* rate of change at a single point. In your past Algebra life you found the slope of a line (the **m** in y=**m**x+b), which is the *average* rate of change or the rate of change between two points.

Notation:

There are 3 main ways to notate a derivative. They all mean the same thing, they are just different ways of stating a derivative.

1) If you start with **f(x) =** , then the derivative would be notated **f'(x) =** , and read **f prime of x**.
2) If you start with **y =** , then the derivative would be notated as **y' =** , and read **y prime of x**.
3) Another option if you start with y = , would have the derivative notated $\frac{dy}{dx}$ and read **d-y-d-x.**

A. Power Rule

The Power Rule is the main rule that you will use when taking a derivative. All other rules are just methods for taking something big and crazy looking and breaking it back down into the Power Rule.

The POWER RULE Saying

Bring the power down, then subtract 1 from the power.

Example 1	**Original Equation**	**Derivative**	
Derivative of a Constant	$f(x)=5$	$f'(x)=0$	The derivative of a number by itself (constant) is always **0** because it has **no change**.
Example 2	**Original Equation**	**Derivative**	
Derivative of just an x	$f(x)=x$	$f'(x)=1$	You use the **Power Rule**.
More details	$f(x)=x^1$	$f'(x)=1x^{1-1}$	An **x** by itself has an unwritten **1** so when we apply the power rule we bring that power down and then subtract **1** from the power.
Simplify		$f'(x)=1x^0$	Doing the math in the exponent leaves us with **0**.
Final Result		$f'(x)=1$	Anything to the **0** power is always just **1**.

Example 3	Original Equation	Derivative	
Derivative of a single term equation.	$f(x)=3x^5$	$f'(x)=15x^4$	You use the **Power Rule**.
More details	$f(x)=3x^5$	$f'(x)=5 \bullet 3x^{5-1}$	Bring the **Power** down and subtract **1** from the **Power.**
Simplify		$f'(x)=15x^4$	

Example 4	Original Equation	Derivative	
Derivative of a multiple term equation	$f(x)=3x^5+7x^4-2x-9$	$f'(x)=15x^4+28x^3-2$	You use the **Power Rule** on each individual term.
Term 1	$f(x)=3x^5$	$f'(x)=5 \bullet 3x^{5-1}=15x^4$	Bring the **Power** down and subtract **1** from the **Power.**
Term 2	$f(x)=7x^4$	$f'(x)=4 \bullet 7x^{4-1}=28x^3$	Bring the **Power** down and subtract **1** from the **Power.**
Term 3	$f(x)=-2x$	$f'(x)=-2$	Derivative of just an **x**, the **x** drops off.
Term 4	$f(x)=-9$	$f'(x)=0$	Derivative of a **Constant** is always **zero**.

Example 5	Original Equation	Derivative	
Derivative of an equation with rewrites.	$f(x)=\sqrt{x}+\sqrt[5]{x^7}-\frac{3}{x^8}-\frac{7}{\sqrt[3]{x^4}}$	$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}+\frac{7}{5}x^{\frac{2}{5}}+24x^{-9}+\frac{28}{3}x^{-\frac{7}{3}}$	You use the **Power Rule** on each individual term.
Term 1 **Square Root Rewrite**	$f(x)=\sqrt{x}=x^{\frac{1}{2}}$	$f'(x)=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}$	Bring the **Power** down and subtract **1** from the **Power.**
Term 2 **Radical Rewrite**	$f(x)=\sqrt[5]{x^7}=x^{\frac{7}{5}}$	$f'(x)=\frac{7}{5}x^{\frac{7}{5}-1}=\frac{7}{5}x^{\frac{2}{5}}$	Bring the **Power** down and subtract **1** from the **Power.**
Term 3 **X on the bottom Rewrite**	$f(x)=-\frac{3}{x^8}=-3x^{-8}$	$f'(x)=-8 \bullet -3x^{-8-1}=24x^{-9}$	Derivative of just an **x**, the **x** drops off.
Term 4 **Both Rewrites**	$f(x)=-\frac{7}{\sqrt[3]{x^4}}=-\frac{7}{x^{\frac{4}{3}}}=-7x^{-\frac{4}{3}}$	$f'(x)=-\frac{4}{3} \bullet -7x^{-\frac{4}{3}-1}=\frac{28}{3}x^{-\frac{7}{3}}$	Derivative of a **Constant** is always **zero**.

B. Product Rule

The Product Rule is used whenever you have two or more equations being **multiplied**. It is really just a method for taking a large crazy looking equation and breaking it down into small bite size pieces that just require the *Power Rule*.

The PRODUCT RULE Saying

F G-Prime Plus G F-Prime

$$f \bullet g' + g \bullet f'$$

Example		
The Problem	**Differentiate (i.e. Take the derivative)** $H(x)=(3x^5-9x^3-7x)(4x^2-2x^{-3}+5)$	
Break Equation into two smaller equations. Label one f and one g.	$f=(3x^5-9x^3-7x)$	$g=(4x^2-2x^{-3}+5)$
Take the derivative of each one individually, applying the Power Rule.	$f'=(15x^4-27x^2-7)$	$g'=(8x+6x^{-4})$
Bring the Pieces together using the Product Rule saying.	**F G-Prime Plus G F-Prime** $f \bullet g' + g \bullet f'$	
The Final Answer ***Do Not Simplify***	$H'(x)=(3x^5-9x^3-7x)(8x+6x^{-4})+(4x^2-2x^{-3}+5)(15x^4-27x^2-7)$	

C. Quotient Rule

The Quotient Rule is used whenever you have two equations being **divided**. It is really just a method for taking a large crazy looking equation involving division and breaking it down into small bite size pieces that just require the *Power Rule*.

The QUOTIENT RULE Saying

LOW D-Hi Minus Hi D-LOW All Over LOW Squared

$$\frac{(Low)(D-Hi)-(Hi)(D-Low)}{(Low)^2}$$

Example		
The Problem	**Differentiate (i.e. Take the derivative)** $H(x)=\frac{(3x^5-9x^3-7x)}{(4x^2-2x^{-3}+5)}$	
Break Equation into two smaller equations. Label the one from the top Hi and one from the bottom Low.	$Hi=(3x^5-9x^3-7x)$	$Low=(4x^2-2x^{-3}+5)$
Take the derivative of each one individually, applying the Power Rule.	$D-Hi=(15x^4-27x^2-7)$	$D-Low=(8x+6x^{-4})$
Bring the Pieces together using the Quotient Rule saying.	**LOW D-Hi Minus Hi D-LOW All Over LOW Squared** $\frac{(Low)(D-Hi)-(Hi)(D-Low)}{(Low)^2}$	
The Final Answer ***Do Not Simplify***	$H'(x)=\frac{(4x^2-2x^{-3}+5)(15x^4-27x^2-7)-(3x^5-9x^3-7x)(8x+6x^{-4})}{(4x^2-2x^{-3}+5)^2}$	

D. Chain Rule

The Chain Rule is used whenever you have one equations **inside** another equation. People ofter refer to these types of equations as composite equations because it is one equation composed (or made up) of multiple equations. It is really just a method for taking a large crazy looking equation involving division and breaking it down into small bite size pieces that just require the *Power Rule*.

<u>The CHAIN RULE Saying</u>

D-IN Times D-OUT or Derivative of the Inside Times Derivative of the Outside

$$(D-IN)\bullet(D-OUT)$$

Example		
The Problem	**Differentiate (i.e. Take the derivative)** $f(x)=(3x^5-9x^3-7x)^5$	
Break Equation into two smaller equations. Label the one **inside** as **In** and the one on the **outside** as **Out**.	$IN=3x^5-9x^3-7x$	$OUT=(\ )^5$
Take the derivative of each one individually, applying the **Power Rule**.	$D-IN=15x^4-27x^2-7$	$D-OUT=5(\text{—})^4$
<u>Bring the Pieces Together:</u> Use the **Chain Rule** saying.	**D-IN Times D-OUT** $(D-IN)\bullet(D-OUT)$ $f'(x)=(15x^4-27x^2-7)\bullet 5(\text{—})^4$	
<u>The Final Answer:</u> Put the **In** back inside the **D-Out** and you are done. ***Do Not Simplify***	$f'(x)=(15x^4-27x^2-7)\bullet 5(3x^5-9x^3-7x)^4$	

7. Concept Questions

Exam 1’s Concept questions revolve around understanding what **f(x)** represents graphically and what **f’(x)** represents graphically.

- **f(x)** is asking you about the y-values.
- **f’(x)** is asking you about the instantaneous rate of change, the slope of the tangent, or in common language is the graph increasing, decreasing, or neither.

<table>
<tr><th>Concept</th><th>f(x) = y-values</th></tr>
<tr><td>

- If the graph is above the x-axis, then the y-value is positive and f(x)>0.
- If the graph is below the x-axis, then the y-value is negative and f(x)<0.
-If the graph is on the x-axis, then the y-value is zero and f(x)=0.

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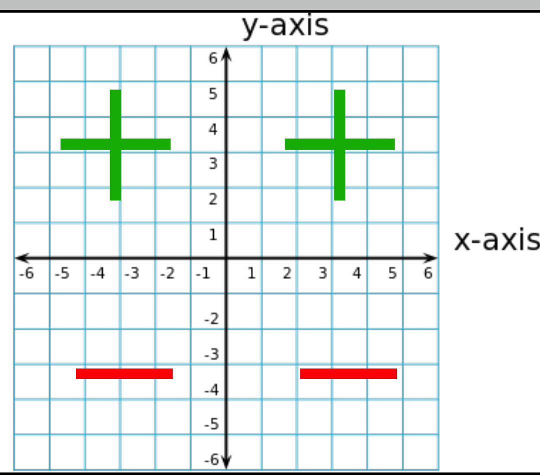

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<tr><th>Concept</th><th>f’(x) = Slopes of Tangent Lines</th></tr>
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- If the graph is increasing, then f’(x) is positive or f’(x)>0.
- If the graph is decreasing, then f’(x) is negative or f’(x)<0.
-If the graph is at a Max, Min, or goes horizontal, then f’(x) is zero or f’(x)=0.

Note: *ALWAYS READ the GRAPH Left to Right.*

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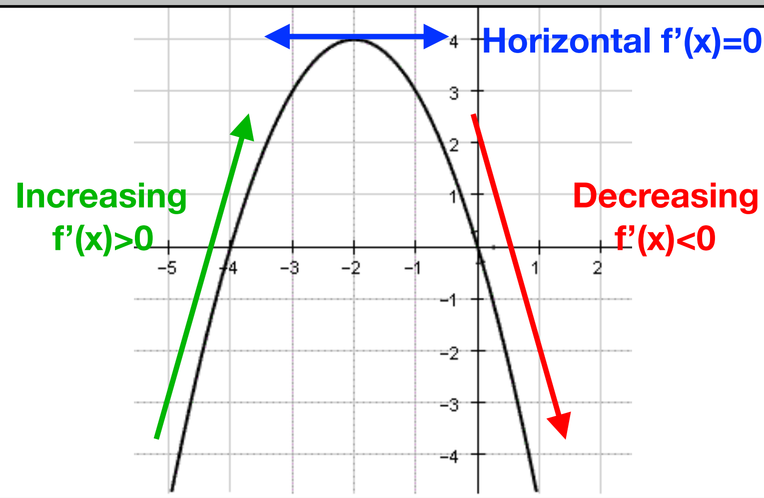

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