Math 141 Final Exam Fall 2019

Make sure you fill in your answers clearly on the scantron answer sheet provided. Fill in your name, student ID number, section number, and exam version on the answer sheet.You may use this exam to work out the problems. You must hand in this exam as well as the scantron answer sheet. When you hand in your exam, be prepared to show your picture ID. No notes or formula sheets are allowed for this exam. Approved calculators such as TI-83 or TI-84 calculators are permitted for this exam, however TI-89 calculators and any CAS calculators are prohibited. No cell phones are permitted outside your bag at any time during the exam.

Name, and section: _____

ID#: _____

Exam version: A

1. Given $f'(x) = 6x^2 + 6x$ and $f(-1) = 5$, find $f(x)$.

   A. $f(x) = 2x^3 + 3x^2 + C$
   B. $f(x) = 2x^3 + 3x^2 + 4$
   C. $f(x) = 2x^3 + 3x^2 + 5$
   D. $f(x) = 2x^3 + 3x^2$

2. Compute $\lim_{x \to 3} \dfrac{x^2 + 3x - 18}{x^2 - 9}$.

   A. $-\dfrac{3}{2}$ B. $\dfrac{3}{2}$ C. 0 D. does not exist

3. Compute $\displaystyle\int \frac{4x}{2x^2 + 2}\, dx$.

   A. $\dfrac{2x^2}{\frac{2}{3}x^3 + 2x} + C$
   B. $\dfrac{8 - 8x^2}{(2x^2 + 2)^2} + C$
   C. $\ln(4x) + C$
   D. $\ln(2x^2 + 2) + C$

4. Find the derivative of $y = \dfrac{4x}{1 + x^2}$

   A. $2\ln(1 + x^2)$
   B. $\dfrac{2}{x}$
   C. $\dfrac{4 + 12x^2}{(1 + x^2)^2}$
   D. $\dfrac{4 - 4x^2}{(1 + x^2)^2}$

5. Find the area below the graph of the equation $y = e^x$ and above the x-axis over the interval $[0, 4]$.

A. $4e$
B. $4e - 1$
C. e^4
D. $e^4 - 1$

6. Find the derivative of $f(x) = 10x^2 + \dfrac{1}{x} + 2\sqrt{x}$.

A. $\dfrac{10}{3}x^3 + \ln(x) + \dfrac{2}{3}x^{3/2}$
B. $\dfrac{10}{3}x^3 + \ln|x| + \dfrac{2}{3}x^{3/2}$
C. $20x - x^{-2} + x^{-1/2}$
D. $20x + x^{-2} - x^{-1/2}$

7. Which of the following is **false**?

A. $\displaystyle\int (f(x) \cdot g(x))\, dx = \int f(x)\, dx \cdot \int g(x)\, dx.$
B. $\displaystyle\int (f(x) - g(x))\, dx = \int f(x)\, dx - \int g(x)\, dx.$
C. $\displaystyle\int (2f(x) + 3g(x))\, dx = 2\int f(x)\, dx + 3\int g(x)\, dx.$
D. If $a < c < b$, then $\displaystyle\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx.$

Use the following information for problems 8-10.

For a certain product, $D(x) = (x-7)^2$ is the price per unit, in dollars, that consumers are willing to pay for x units and $S(x) = x^2 + 2x + 1$ is the price per unit, in dollars, that the suppliers are willing to accept to sell x units.

8. Find the equilibrium point.

   A. $(3, 16)$.
   B. $(25, 324)$
   C. $(25, 676)$
   D. $(-25, 1024)$

9. Find the consumer surplus at the equilibrium point.

   A. \$27
   B. \$45
   C. \$2241.67
   D. \$6041.70

10. Find the producer surplus at the equilibrium point.

    A. \$27
    B. \$45
    C. \$2241.67
    D. \$6041.70

11. Find the derivative of $y = e^{2x^3 - x^2 + 4}$

    A. $e^{2x^3 - x^2 + 4}$
    B. $(6x^2 - 2x)e^{2x^3 - x^2 + 4}$
    C. $(2x^3 - x^2 + 4)e^{2x^3 - x^2 + 3}$
    D. $(6x^2 - 2x)e^{2x^3 - x^2 + 3}$

12. Use implicit differentiation to find $\frac{dy}{dx}$ if $4x^2 + 8xy + 9y^2 + 15y - 4 = 0$.

A. $\frac{-8x - 8y}{8x + 18y + 15}$

B. $\frac{8x + 8y}{8x + 18y + 15}$

C. $\frac{8x + 18y + 15}{-8x - 8y}$

D. $\frac{8x + 18y + 15}{8x + 8y}$

13. Find the derivative of $y = \ln\left(x^2(e^x + 1)\right)$.

A. $\frac{2}{x} + \frac{e^x}{e^x + 1}$

B. $\frac{e^x}{x} + \frac{2}{e^x + 1}$

C. $\frac{2e^x}{x(e^x + 1)}$

D. $\frac{1}{x^2(e^x + 1)}$

14. Find the absolute maximum and minimum values of $f(x) = x^3 - 6x^2$ on the interval $[0, 8]$

A. The absolute minimum value is 0 and the absolute maximum value is 128.

B. The absolute minimum value is -32 and the absolute maximum value is 0.

C. The absolute minimum value is -32 and the absolute maximum value is 128.

D. The absolute minimum value is 0 and the absolute maximum value is 4.

15. Find the general form of the function that satisfies $\frac{dA}{dt} = -7A$.

A. $A(t) = -7Ce^t$

B. $A(t) = Ce^{-7t}$

C. $A(t) = -7Ce^{-t}$

D. $A(t) = Ce^{7t}$

Use the following information for problems 16 and 17.

A clothing company determines that in order to sell x suits, the price per suit should be given by $p = 110 - 0.5x$, and the cost of producing x suits is given by $C(x) = 2000 + 0.75x^2$.

16. What the maximum profit?

    A. \$44
    B. \$110
    C. \$420
    D. \$2000

17. What price per suit should be charged in order to maximize the profit?

    A. \$44
    B. \$88
    C. \$110
    D. \$420

18. Which of the following is **true**?

    A. If $(c, f(c))$ is an inflection point, then $f''(c) = 0$
    B. If $(c, f(c))$ is a relative minimum point, then c is a critical value for f.
    C. If $(c, f(c))$ is a relative minimum point, then $f'(c) = 0$
    D. None of the above

19. Find all inflection points of $f(x) = x^4 - 8x^3 + 18x^2 - 5$.

    A. $x = 0$ only
    B. $x = 0$ and $x = 3$
    C. $x = 1$ and $x = 3$
    D. $x = 0$, $x = 1$, and $x = 3$

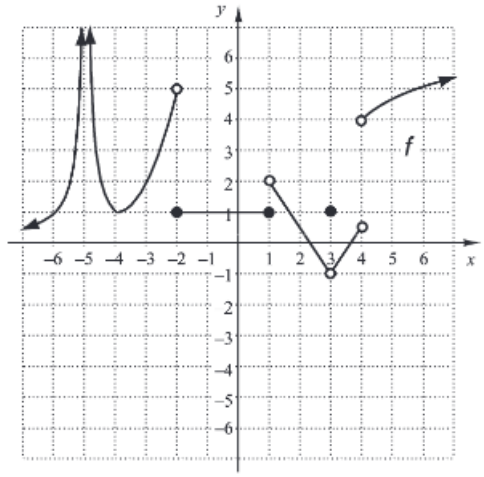

Figure 1: **Use this graph to answer questions 20-22.**

20. What is $\lim_{x \to -2^-} f(x)$?

    A. 1 B. 5 C. undefined D. none of the above

21. What is $\lim_{x \to -2^+} f(x)$?

    A. 1 B. 5 C. undefined D. none of the above

22. What is $\lim_{x \to -2} f(x)$?

    A. 1 B. 5 C. does not exist D. None of the above

23. Compute $\int \left(e^{2x} + \frac{1}{x^2} \right) dx$.

    A. $\frac{1}{2}e^{2x} - \frac{1}{x} + C$
    B. $2e^{2x} - \frac{2}{x^3} + C$
    C. $\frac{1}{2}e^{2x} + \ln(x^2) + C$
    D. $2e^{2x} - \ln(x^2) + C$

24. Which of the following is equal to the derivative $f'(x)$ of a function $f(x)$?

A. $\dfrac{f(x+h)-f(x)}{h}$

B. $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

C. $\dfrac{f(x)+h-f(x)}{h}$

D. $\lim\limits_{h\to 0}\dfrac{f(x)+h-f(x)}{h}$

25. Find an equation of the tangent line to the graph of the function $f(x)=x^2+\sqrt{x}$ at the point $(4,18)$.

A. $y=\dfrac{33}{4}x-15$

B. $y=\dfrac{33}{4}x+14$

C. $y=\dfrac{17}{2}x+18$

D. $y=\dfrac{17}{2}x-16$

26. If $y=-2x+1$ is the equation of the tangent line to the graph of $y=f(x)$ at $x=3$, then

A. $f(3)=1$ and $f'(3)=-2$.

B. $f(3)=-5$ and $f'(3)=-2$.

C. $f(3)=-5$ and $f'(3)=-1$.

D. $f(3)=-5$ but there is not enough information to determine $f'(3)$.

27. Compute $\displaystyle\int \frac{t^4}{\sqrt[3]{2+t^5}}\,dt$.

A. $\dfrac{4t^3}{\frac{5}{3}t^4(2+t^5)^{-2/3}}+C$

B. $\dfrac{4t^3(2+t^5)^{1/3}-\frac{5}{3}t^8(2+t^5)^{-2/3}}{(2+t^5)^{2/3}}+C$

C. $\dfrac{\frac{1}{5}t^5}{\frac{3}{4}(2+t^5)^{4/3}}+C$

D. $\dfrac{3}{10}(2+t^5)^{2/3}+C$

28. Find the derivative of $y = \ln(x^{99})$.

A. $\dfrac{99}{x}$

B. $99\ln(x^{98})$

C. $\dfrac{99\ln(x^{98})}{x}$

D. $\dfrac{\ln(x^{100})}{100}$

29. A 51 foot long ladder leans against a vertical wall. If the lower end of the ladder is being moved away from the wall at a rate of 3 feet per second, how fast is the height of the top changing (this will be a negative rate) when the lower end is 24 feet from the wall?

A. $-\dfrac{45}{8}$ feet per second

B. $-\dfrac{8}{45}$ feet per second

C. $-\dfrac{8}{5}$ feet per second

D. $-\dfrac{5}{8}$ feet per second

30. Find the area of the region bounded by the graphs of the equations $y = 6x - x^2$ and $y = x$.

A. $\dfrac{125}{6}$

B. $-\dfrac{125}{6}$

C. $\dfrac{275}{6}$

D. $-\dfrac{275}{6}$

31. Find the derivative of $f(x) = (2x-3)^{12}(3x+4)^{20}$.

A. $1440(2x-3)^{11}(3x+4)^{19}$

B. $240(2x-3)^{11}(3x+4)^{19}$

C. $24(2x-3)^{11}(3x+4)^{20} + 60(2x-3)^{12}(3x+4)^{19}$

D. $12(2x-3)^{11}(3x+4)^{20} + 20(2x-3)^{12}(3x+4)^{19}$