Final Exam Study Guide

Includes
1. **Integrals & Antiderivative Rules**
2. **Definite Integrals (Integrals with bounds)**
3. **Area Between Two Curves - Region Bounded by Two Curves**
4. **Consumer and Producer Surplus**
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1. **Integrals & Antiderivatives**
   - **A. Rewrites**

     Remember to always stop and look for rewrites before you start trying to take the antiderivative. These are the three rewrites you need to look for.

     a. Radicals

Example	$\sqrt[root]{x^{power}}$	$\sqrt[r]{x^p}$	$\sqrt[3]{x^5}$
Rewrite	$x^{\frac{power}{root}}$	$x^{\frac{p}{r}}$	$x^{\frac{5}{3}}$
		Square root	$\sqrt{x}$
			$x^{\frac{1}{2}}$

b. x on the bottom

Example	$\frac{a\ number}{x^{power}}$	$\frac{a}{x^p}$	$\frac{5}{x^8}$
Rewrite	$a\ number \cdot x^{-power}$	$a \cdot x^{-p}$	$5 \cdot x^{-8}$

c. Both

Example	$\frac{a\ number}{\sqrt[root]{x^{power}}}$	$\frac{a}{\sqrt[r]{x^p}}$	$\frac{5}{\sqrt[3]{x^5}}$
1st Radical Rewrite	$\frac{a\ number}{x^{\frac{power}{root}}}$	$\frac{a}{x^{\frac{p}{r}}}$	$\frac{5}{x^{\frac{5}{3}}}$
2nd x on the bottom	$a\ number \cdot x^{-\frac{power}{root}}$	$a \cdot x^{-\frac{p}{r}}$	$5 \cdot x^{-\frac{5}{3}}$

B. Power Rule

This is the main rule that you will use when finding an antiderivative.
Rule:

Step 1: Add 1 to the power.
Step 2: Divide by the new power.

Example 1		$\int x^5 dx$	
Step 1	**Add 1 to the power.**	x^{5+1}	x^6
Step 2	**Divide by the new power.**	$\frac{x^{5+1}}{5+1}$	$\frac{x^6}{6}$
Final Result			$\frac{x^6}{6}+C$
Example 2	First apply Rewrites Second apply Power Rule	$\int \frac{5}{\sqrt[3]{x^5}} dx$	
Step 1: Radical Rewrite		$\int \frac{5}{x^{\frac{5}{3}}} dx$	
Step 2: x on bottom Rewrite		$\int 5 \cdot x^{-\frac{5}{3}} dx$	
Step 3: Power Rule	Add 1 to the power.	$5 \cdot x^{-\frac{5}{3}+1}$	$5 \cdot x^{-\frac{2}{3}}$
Step 4: Divide by the new power.	Divide by the new power.	$\frac{5 \cdot x^{-\frac{5}{3}+1}}{-\frac{5}{3}+1}$	$\frac{5 \cdot x^{-\frac{2}{3}}}{-\frac{2}{3}}$
Final Result			$\frac{5 \cdot x^{-\frac{2}{3}}}{-\frac{2}{3}}+C$

C. Special Cases

a. Natural Log = ln(x)

Example	$\int \frac{a\ number}{x} dx$	$\int \frac{a}{x} dx$	$\int \frac{5}{x} dx$
Result	$a\ number \cdot \ln(x) + C$	$a \cdot \ln(x) + C$	$5 \cdot \ln(x) + C$

b. e to a power

Example	$\int e^{(a\ number \cdot x)} dx$	$\int e^{(a \cdot x)} dx$	$\int e^{(5 \cdot x)} dx$
Result	$\frac{1}{a\ number} \bullet e^{(a\ number \cdot x)} + C$	$\frac{1}{a} \bullet e^{(a \cdot x)} + C$	$\frac{1}{5} \bullet e^{(5 \cdot x)} + C$

2. Definite Integrals (Integrals with bounds)

Example			$\int_3^5 \sqrt[3]{x} + 5x^3 + 7x + 5\,dx$
Step 1	Hit Y= on your calculator and input the equation into Y1	The cube root button is under the Math button.	$Y1 = \sqrt[3]{x} + 5x^3 + 7x + 5$
Step 2	Hit GRAPH Button	Make sure your x-window is large enough to take in your bounds. 3 & 5 in this case.	
Step 3	Hit: 2nd—>CALC —>#7 $\int f(x)\,dx$	-It will then ask you to enter a lower bound. Type 3 and hit Enter. -It will then ask you to enter an upper bound. Type 5 and hit Enter.	$\int_{LowerBound}^{UpperBound}$
Result		Calculator will output your answer at the bottom of the screen.	$\int f(x)dx = 749.16735$

3. Area Between Two Curves - Region Bounded by Two Curves

Equation	You will need to setup this equation to find the answer.	$Area = \int_a^b Top - (Bottom)\,dx$	
Example		Find the area of the region bounded by the graphs of the functions $y = x$ and $y = x^4$ in the first quadrant (where x ≥ 0 and y ≥ 0).	
Step 1	Hit Y= on your calculator and input the equations into Y1 & Y2		$Y1 = x$ $Y2 = x^4$
Step 2	Hit GRAPH Button	Adjust your window to be able to see the region bounded by the two curves.	Identify which graph is your **Top** equation (above), and which one is your **Bottom** equation (below).
Step 3	Find the upper and lower bounds for your situation.	Hit: 2nd—>CALC—>#5 Intersect You will need to do this process twice to find the two intersects.	-It will ask you to select the First Curve. Just hit Enter. -It will ask you to select the Second Curve. Just hit Enter. -It will ask you for a guess. Scroll closer to one of the interests and hit Enter
Step 4	Bring together your results from Step 2 and Step 3 to create your equation.		$\int_0^1 x - (x^4)\,dx$
Step 5	Enter your new equation into Y1		$Y1 = x - x^4$
Step 6	Hit GRAPH Button	Hit: 2nd—>CALC—>#7 $\int f(x)\,dx$	-It will ask you for a Lower Bound, enter 0. -It will ask you for an Upper Bound, enter 1.
Result	The answer will be at the bottom of the screen.		$\int f(x)\,dx = .3$

4. Consumer and Producer Surplus

Equations	You will need to setup these equations to find the answers.	Consumer Surplus	$\int_0^Q D(x)dx-(Q \bullet P)$
		Producer Surplus	$(Q \bullet P)-\int_0^Q S(x)dx$
		Equilibrium Point **(Q,P)**	$S(x)=D(x)$
Example		Find the Consumer and Producer Surplus given $D(x)=81-x^2$ $S(x)=x^2+4x+11$	
Step 1	Hit Y= on your calculator and input the supply and demand equations into Y1 & Y2		$Y1=81-x^2$ $Y2=x^2+4x+11$
Step 2	Hit GRAPH Button	Adjust your window to be able to see the intersections of the two graphs.	Note: You will see two intersections one in the positive x and one in the negative x. Always choose the positive.
Step 3	Find the intersection of the two graphs.	Hit: 2nd—>CALC—>#5 Intersect -It will ask you to select the First Curve. Just hit Enter. -It will ask you to select the Second Curve. Just hit Enter. -It will ask you for a guess. Scroll closer to one of the interests and hit Enter	You should find the intersect **(5,56)=(Q,P)** this is your equilibrium point.
Step 4	Setup your Consumer Surplus Equation	$\int_0^Q D(x)dx-(Q \bullet P)$	$\int_0^5 81-x^2\,dx-(5 \bullet 56)$
Step 5	Plug the Demand Equation into Y1.	$Y1=81-x^2$ Hit: 2nd—>CALC—>#7 $\int f(x)\,dx$ Lower bound enter: 0 Upper bound enter: 5	**Result = 363.33**
Step 6	Take Result from Step 5 and subtract off $(5 \bullet 56)=(280)$	**363.33-280 = 83.33**	**Consumer Surplus = 83.33**

Step 7	Setup your Producer Surplus Equation	$(Q \bullet P)-\int_0^Q S(x)dx$	$(5 \bullet 56)-\int_0^5 x^2+4x+11dx$
Step 8	Plug the Supply Equation into Y1.	$Y1=x^2+4x+11$ Hit: 2nd—>CALC—>#7 $\int f(x)\,dx$ Lower bound enter: 0 Upper bound enter: 5	**Result = 146.67**
Step 9	Take Result from Step 8 and subtract it from $(5 \bullet 56)=(280)$	**280-146.67 = 133.33**	**Producer Surplus = 133.33**

5. U-Substitution

Example		$\int (2x^2-3x+7)^3(4x-3)\,dx$	
Step 1	Select your **u.** Priority List for selecting **u** 1) **Inside Piece (like in a chain rule problem)** 2) **ln(x)** 3) **Bottom of a Fraction** 4) **Highest Power of x**		$u=2x^2-3x+7$
Step 2	Take the derivative of your **u.** Make sure to use this specific derivative notation.		$\frac{du}{dx}=4x-3$
Step 3	Solve for **dx.** (i.e. get **dx** by itself)	Multiply both sides by **dx**	$dx\bullet\frac{du}{dx}=(4x-3)\bullet dx$
		Result	$du=(4x-3)\bullet dx$
		Divide both sides by $(4x-3)$	$\frac{du}{(4x-3)}=\frac{(4x-3)\bullet dx}{(4x-3)}$
		Result	$\frac{du}{(4x-3)}=dx$
Step 4	Plug the **u** from **Step 1** & the **dx** from **Step 3** back into original equation.		$\int (u)^3(4x-3)\frac{du}{(4x-3)}$
Step 5	Simplify the Equation Cancel out the $(4x-3)$. All your **x's** should cancel out every time so that all you are left with is an equation with **u's.**		$\int (u)^3\,du$
Step 6	Apply the Power Rule		$\frac{(u)^4}{4}+C$
Step 7	Plug the **u** from **Step 1** back in.		$\frac{(2x^2-3x+7)^4}{4}+C$