Math 155, Fall 2019

Midterm 1

Instructions

- Answer all the following questions to the best of your ability. In the case you do not know how to make progress on a problem, we encourage you to describe related material you do know.
- Your solutions to open response questions must demonstrate understanding of the material. Correct answers to these questions without any explanation or work shown will receive **zero** credit.
- You may not consult with anyone other than the proctors or consult any materials during the exam without prior approval. Note sheets and formula sheets are not allowed without prior approval. Only that scratch paper which is attached here or provided by proctors is allowed.
- The **only** tools allowed without prior approval are writing utensils and an approved calculator (non-CAS graphing calculators such as TI-83/84 versions or lower).
- If you are caught cheating, your exam will be taken and you will receive a zero on the midterm.
- If you need to use the bathroom, you must leave your exam, calculator, and scratch paper with a proctor and retrieve them upon returning.
- When you finish, bring your exam and any scratch paper to the proctors. You may not take your scratch paper away from the exam room.

Affirm the Honor Pledge below, fill in your information below, and wait for the signal to begin.

I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

Name (sign): _____

Name (print): _____

Section: _____

Date: _____

Scores:

Problem 1	:	_____ / 36
Problem 2	:	_____ / 12
Problem 3	:	_____ / 16
Problem 4	:	_____ / 12
Problem 5	:	_____ / 20
Problem 6	:	_____ / 16
Problem 7	:	_____ / 16
Problem 8	:	_____ / 16
Problem 9	:	_____ / 20
Problem 10	:	_____ / 16
Problem 11	:	_____ / 20
Total	:	_____ / 200

1. Answer the following multiple choice questions to the best of your ability. Circle your selection clearly. Every question has precisely one correct answer.

   (a) Quantities in a mathematical model such as the volume of your filled lungs V, an ambient chemical concentration γ, a population growth rate r, and so on, are known as

   A) Parameters.
   B) Units.
   C) Coefficients.
   D) Variables.
   E) These have no special name.

   (b) Which of the following expressions exactly represents the trigonometric function graphed below?

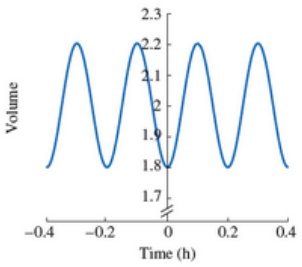
   A) $1.8 + 0.2\cos\left(\frac{2\pi}{0.2}(t - 0.1)\right)$

   B) $2 + 0.2\cos\left(\frac{2\pi}{0.2}(t - 0.1)\right)$

   C) $2 + \cos\left(\frac{2\pi}{0.1}(t - 0.1)\right)$

   D) $1.8 + 0.4\cos\left(\frac{2\pi}{0.2}(t - 0.1)\right)$

   E) $2 + 0.2\cos\left(\frac{2\pi}{0.2}(t - 0.2)\right)$



   (c) What is the derivative of $f(x) = 150$?

   A) 0 B) 15 C) $150x$ D) 149 E) $150x^{-1}$

   (d) What is the derivative of $q(t) = 12t - 5$?

   A) 12 B) $12t - 5$ C) -5 D) 0 E) $6t^2 - 5t$

   (e) Which of the following concepts is **precisely** the same as the instantaneous rate of change of a function $f(t)$ at the point $t = 3$?

   A) $\frac{df}{dt}$
   B) The slope of a secant line to the function at $t = 3$.
   C) The slope of the tangent line to the function at $t = 3$.
   D) The average rate of change at a point $t = 3$.
   E) There are multiple concepts listed here that are precisely the same.

   (f) Given the following discrete-time dynamical system, what is the solution function?

$$x_{t+1} = 1.6x_t, \qquad x_0 = 3$$

   A) $x(t) = 1.6 \cdot 3^t$
   B) $x^* = 5$
   C) $x(t) = 3 + 1.6t$
   D) $x(t) = 3 \cdot 1.6^t$
   E) $x_1 = 4.8$

2. One argument in favor of improved bicycle lanes is their superior traffic density. Suppose a road is a long rectangle which is 10 feet wide and one city block long. By footprint, suppose 1 bicyclist = 50ft^2. Dr. Aminian later goes on Wikipedia and determines that 1 city block = 900ft. Determine the number of bicyclists that can fit on this road.

3. The function $q(x) = x^{3/4}$ takes an animal's mass x in kilograms as input and returns its metabolic rate as output. The function $m(t) = 80 - \frac{76}{1+0.1t}$ takes a person's age in years as an input and returns the mass of a human in kilograms as output.

   For each of the following, calculate the composition function. Then, if the composition is invalid because of the biological context, explain why. You do not need to simplify your expressions.

   (a) $(q \circ m)(t)$

   (b) $(m \circ q)(x)$

4. Determine whether or not each of the following functions is linear. Explain your reasoning.

   (a) $y = \sqrt{2} \cdot x - 10$

   (b)

r	s
-1	2
0	4
1	8
2	16

   (c)

x	y
0	4
2	5
4	6
5	6.5

5. A person breathes in and out 0.5L of air in each breath. The volume of air in the lungs is 2.5L when full. Let c_t denote the concentration of chemical in the lungs at the beginning of each breath (when the lungs are full). On each breath in they inhale 0.5L of the ambient air which has a chemical concentration of 2 mol/L.

   (a) Complete the table below. One value has been filled in for you.

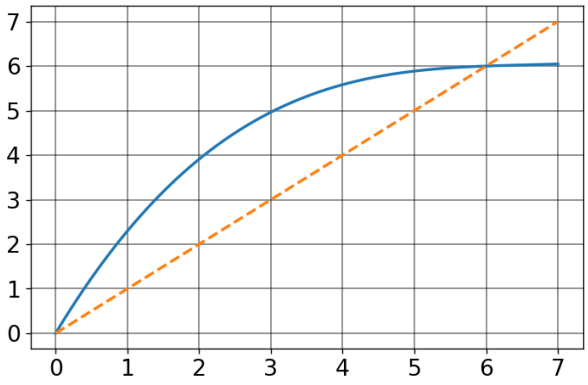
Step	Volume (L)	Total Chemical (mol)	Chemical Conc. (mol/L)
Air in lungs when full.			c_t
Air lost from breath out			
Air remaining in lungs			
Air replaced by inhale			
Air in lungs after inhale			

   (b) Given the results of the table, write the updating function of a discrete-time dynamical system that relates the concentration in the lungs between one breath and the next; c_t and c_{t+1}.

   (c) What is the equilibrium concentration of chemical in the lung? Find the equilibrium algebraically.

6. For the diagram corresponding to a discrete-time dynamical system graphed below,

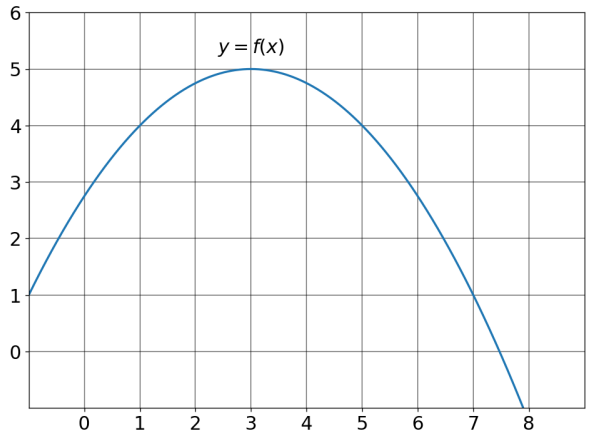
   (a) Identify all equilibria **graphically**. Then apply cobwebbing beginning with $x_0 = 1$ to fill out the corresponding table. You may estimate each iteration to one digit past the decimal point (the tenths place).



x_0	x_1	x_2	x_3	x_4
1.0				

   (b) Which equilibria are stable and which are unstable?

7. In the given figure, draw a secant line between the points $x = 1$ and $x = 5$. Use the secant line to estimate the average rate of change of the function on the interval $[1, 5]$.



8. The populations of two separate species of rabbits are well-modeled by the following equations:

$$r(t) = 60 \cdot 2^{t/8}, \qquad j(t) = 20 \cdot 2^{t/4}.$$

where t is the number of years since the first observation of both populations.

(a) What are the doubling times for each species? Show your work, or explain your reasoning.

(b) When will the population $j(t)$ reach 100 rabbits? You do not need to simplify your answer.

(c) When will the two populations be equal? You do not need to simplify your answer.

9. (a) Find the updating function $k_{t+1} = f(k_t)$ for the population of kokanee salmon in a lake in Colorado based on the following description:

Each year, the kokanee population increases by 10% due to natural growth. Then, 1000 kokanee are removed due to fishing.

(b) Find the population of kokanee up to year two (k_2) given that $k_0 = 5000$.

(c) Algebraically find the point of equilibrium k^* in your model.

10. Find the derivatives of the following functions using differentiation rules. **You do not need to simplify your answers.**

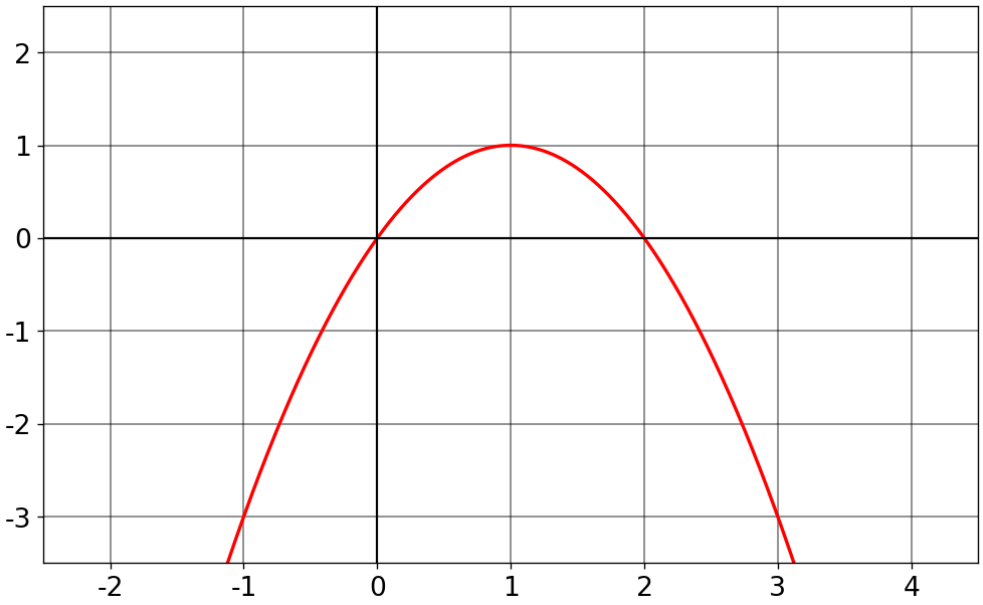
(a) $q(r) = -r^{150} + \pi r^2 - e$

(b) $p(t) = (t^3 - t)(t^2 + 4t + 1)$

(c) $M(x) = \dfrac{x^3 - 1}{1 + x^2}$

11. This problem is about the function $f(x) = 2x - x^2$.

(a) On the graph of $f(x)$ below, draw the **tangent line** to the function at $x = 2$.



(b) Find the slope of the tangent line at $x = 2$ **by using the limit definition of the derivative.**

(c) Use **derivative rules** to find $f'(x)$ and evaluate it at $x = 2$.

$f'(x) =$ _____ $f'(2) =$ _____

(d) Write the equation of the **tangent line** at $x = 2$.