Math 155 **Exam 2** **Fall 2016**

NAME: ______________________________

SECTION: _______________ TIME: _______________

INSTRUCTOR: ______________________________

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to *clearly* show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have one hour and 50 minutes to work on the exam.

Problem	Points	Score
1	15	
2	12	
3	18	
4	15	
5	14	
6	14	
7	12	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

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(Signature)

1. (15 points) Compute the derivatives of the following functions. You do NOT need to simplify your answer. Use parentheses to indicate multiplication where appropriate, and make sure that your notation is correct.

   (a) $g(x) = -(3\pi)^4 + 6x^4 - 4x^{-\frac{5}{4}}$

   (b) $f(x) = (x^3 + 5x)^6 \ln(2x^2 + 1)$

   (c) $c(t) = \cos\left(t^2 e^{t^2}\right)$

   (d) $q(x) = \frac{x^3+1}{x-2}$

   (e) $L(x) = \tan\left(k(x^2 - 2)\right)$, where k is a constant.

2. (12 points) Suppose that the number W_t of whiskers on Mark's magical dog on day t obeys the discrete-time dynamical system

$$W_{t+1} = rW_t(3 - W_t) - W_t,$$

where r is a positive parameter.

(a) Find all equilibria of the discrete-time dynamical system. For what range of values of the parameter r is there a positive equilibrium?

(b) For the nonzero equilibrium in (a) use the Stability Theorem to find the range of r for which that equilibrium is stable.

3. (18 points)

(i) For the following statements answer True or False. No partial credit will be given.

(a) The derivative of the updating function of a discrete time dynamical system is given by $f'(x) = \frac{2x-3}{x^2+5x+1}$. The equilibrium at $x^* = 1$ is stable.

(b) If $x = c$ is a critical point of a function $f(x)$, then $f''(c)$ can always be used to determine if $f(x)$ has a local maximum or minimum at $x = c$.

(c) The derivative of the function $g(x) = x^3 + \ln(\pi)$ is $g'(x) = 3x^2 + \frac{1}{\pi}$.

(d) $W^* = -\ln(\frac{1+h}{2})$ is an equilibrium of the discrete time dynamical system with updating function $W_{t+1} = 2W_t e^{-W_t} - hW_t$, where h is a positive parameter.

(e) The derivative of a function is given by $h'(x) = (x-5)(x+1)$. There is a local minimum at $x = -1$.

(ii) Evaluate the following limit. Show all of your work. If you use leading behavior, justify your answer by explaining all of your steps. If you use L'Hopital's Rule, justify why it can be applied each time you use it.

(a) $\displaystyle\lim_{x\to 0} \frac{\sin(x^2)}{x^2}$

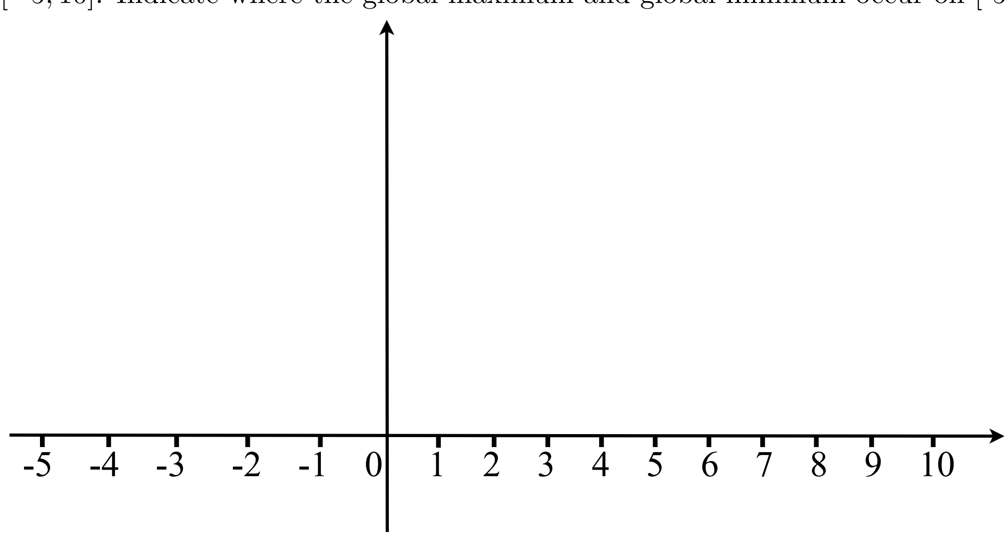
4. (15 points) Consider the function $f(x) = \frac{1}{3}x^3 - 4x + 10$ on the interval $[-5, 10]$.

(a) Calculate $f'(x)$, and use this to find all the critical points of $f(x)$.

(b) Calculate $f''(x)$, and use this to find regions where $f(x)$ is concave up or concave down.

(c) For each critical point, determine if $f(x)$ has a local maximum or a local minimum there. Justify your answer using the first or second derivative test.

(d) Use the information found above to sketch a graph of the function $f(x)$ on the interval $[-5, 10]$. Indicate where the global maximum and global minimum occur on [-5,10].



5. (14 points) Suppose that the rate at which a bee collects nectar is given by

$$R(t) = \frac{t}{5(t^2+1)}.$$

i) Find the critical point of $R(t)$ on the interval $[0, 2]$ and use either the first or second derivative test to determine if there is a local maximum or local minimum at that point. For full credit, make sure that your work and answers are clear and well organized.

ii) Determine the global maxima and minima of $R(t)$ on the interval $[0, 2]$, and express your answer in coordinate form (as in "There is a global maximum at $(t, R(t))$").

6. (14 points) Consider the discrete-time dynamical system

$$T_{t+1} = \frac{1}{3}(4 - T_t)T_t - hT_t$$

describing a population of trout being harvested at rate h, where $0 \le h \le 1$.

(a) Find the nonzero equilibrium population T^* as a function of h.

(b) The equilibrium harvest is given by $P(h) = hT^*$, where T^* is the equilibrium you found in part (a). Find the value of h that maximizes $P(h)$ on the interval $0 \le h \le 1$. Use ***Calculus*** to justify your answer.

7. (12 points) For the following functions, $g(x)$, find $g_\infty(x)$, the leading behavior of $g(x)$ as $x \to \infty$, and $g_0(x)$, the leading behavior of $g(x)$ as $x \to 0$.

(a) $g(x) = 12x^{-\pi} + 5x^{12} + \mathrm{e}^{-x} + 0.0001\mathrm{e}^{0.001x} + 10\mathrm{e}^{-5x}$

(b) $g(x) = \dfrac{5 + 3x^3}{x^{-2} + 3x^4 + 4x^2}$

(c) For the function in part (b), use the method of matched leading behaviors to sketch the graph of $g(x)$ for $x \geq 0$. Graph and indicate where you have graphed $g_\infty(x)$, $g_0(x)$, and $g(x)$.