| 1st Derivative Test <br> Uses: Finding Local Max and Min Finding intervals where a function is increasing \& decreasing. |  | 2nd Derivative Test <br> Uses: Finding intervals where a function is concave up (like a cup) or concave down (like a frown). |  |
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| Step 1 | Find the first derivative. $\boldsymbol{f}^{\prime}(\boldsymbol{x})$ | Step 1 | Find the second derivative. $\boldsymbol{f}^{\boldsymbol{\prime}}(\boldsymbol{x})$ |
| Step 2 | Find the critical values. <br> Places where $f^{\prime}(x)=0$ or $f^{\prime}(x)$ is undefined <br> - Set $f^{\prime}(x)=0$ and solve for $\boldsymbol{x}$. <br> - If you have a fraction, set the denominator equal to zero and solve for $x$ to determine if there are any places where $f^{\prime}(x)$ is undefined. | Step 2 | Find the inflection points. <br> Places where $f^{\prime \prime}(x)=0$ or $f^{\prime \prime}(x)$ is undefined <br> - Set $\boldsymbol{f}^{\prime \prime}(\boldsymbol{x})=\mathbf{0}$ and solve for $\boldsymbol{x}$. <br> - If you have a fraction, set the denominator equal to zero and solve for $\boldsymbol{x}$ to determine if there are any places where $f^{\prime \prime}(x)$ is undefined. |
| Step 3 | Do the first derivative test. (\# Line Game) <br> - Draw a \# line, label the critical values, and choose test values on the left and right of all the critical values. <br> - Plug test values into the first derivative $f^{\prime}(x)$. <br> - If the value is positive put a plus and draw an increasing arrow above that test value to indicate that region is increasing. <br> - If the value is negative put a minus and draw a decreasing arrow above the test value to indicate that region is decreasing. | Step 3 | Do the second derivative test. (\# Line Game) <br> - Draw a \# line, label the inflection points, and choose test values on the left and right of all the inflection points. <br> - Plug test values into the second derivative $f$ " $(x)$. <br> - If the value is positive put a plus and draw $\mathbf{U}$ for concave up (like a cup) above that test value to indicate that region is concave up. <br> - If the value is negative put a minus and draw an upside down $\mathbf{U}$ for concave down (like a frown) above that test value to indicate that region is concave down. |
| Step 4 | Draw conclusions. <br> You have all the data you need to answer any questions involving local max's, local mins, and the intervals where the original $f(x)$ is increasing or deceasing. <br> NOTE: You have only found x -values at this point, if they want to know the actual max or min values you will need to plug those $x$-values back into the original $f(x)$ to determine the $y$-values that go with them. | Step 4 | Draw conclusions. <br> You have all the data you need to answer any questions involving the intervals where the original $f(x)$ is concave up or down. <br> NOTE: You have only found $x$-values at this point, if they want to know the actual inflection point you will need to plug those $x$-values back into the original $f(x)$ to determine the $y$-values that go with them. |
| Determine Absolute Max and Absolute Mins <br> Note: Absolute Max/Min problems generally have an interval [a,b] that you are provided with in addition to the equation. |  | The Stability Test <br> Use for determining if an equilibrium point is stable or unstable. |  |
| Step 1 | Find the first derivative. $\boldsymbol{f}^{\prime}(\boldsymbol{x})$ | Step 1 | Find the equilibrium points. |
| Step 2 | Find the critical values. <br> Places where $f^{\prime}(x)=0$ or $f^{\prime}(x)$ is undefined <br> - Set $\boldsymbol{f}^{\prime}(\mathbf{x})=\mathbf{0}$ and solve for $\boldsymbol{x}$. <br> - If you have a fraction, set the denominator equal to zero and solve for $\boldsymbol{x}$ to determine if there are any places where $f^{\prime}(x)$ is undefined. <br> - Check that all the critical values you found are in the interval $[a, b]$ that you are given and discard any that are not inside the interval. | Step 2 | Create and $f(x)$ equation based on the DTDS you were given, and use it to find the first derivative, $f^{\prime}(x)$. |
|  |  | Step 3 | Plug your equilibrium values from Step 1 into your derivative $f^{\prime}(x)$ and to get your comparison value (the $y$-value). |
| Step 3 | Plug all critical values, that you found in Step 2, and end points (the $\mathbf{a}$ and $\mathbf{b}$ from the interval you were given) into the original equation $f(x)$. <br> - The largest $y$-value from these wins the Absolute Max award. <br> - The smallest $y$-value from these wins the Absolute Min award. <br> Note: You will always have both. | Step 4 | Take the absolute value of each of your comparison values from Step 3. <br> - If the value is less than $1,<\mathbf{1}$, then the point is stable. <br> - If the value is greater than $1,>\mathbf{1}$, then the point is unstable. <br> - If the value is equal to $1,=\mathbf{1}$, then the test failed (and you wasted your time. |

