Math 155 **Exam 2** **Fall 2017**

NAME: ______________________________________

SECTION: ___________________ TIME: ___________________

INSTRUCTOR: ______________________________________

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to *clearly* show your work on each problem for full credit. Work that is crossed out or erased will not be graded. If you need scratch paper ask for some from the proctor. Turn in any scratch paper that you use during the exam. You will have one hour and 50 minutes to work on the exam.

Problem	Points	Score
1	16	
2	11	
3	10	
4	9	
5	12	
6	10	
7	12	
8	5	
9	5	
10	5	
11	5	
12	5	
13	5	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

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(Signature)

1. (16 points) Compute the derivatives of the following functions. You do NOT need to simplify your answer. Use parentheses to indicate multiplication where appropriate, and make sure that your notation is correct.

   (a) $f(x) = x^3 + e^{3x}$

   (b) $f(t) = t^3 \ln(t)$

   (c) $k(x) = (\sin(x)\mathrm{e}^{x})^3$

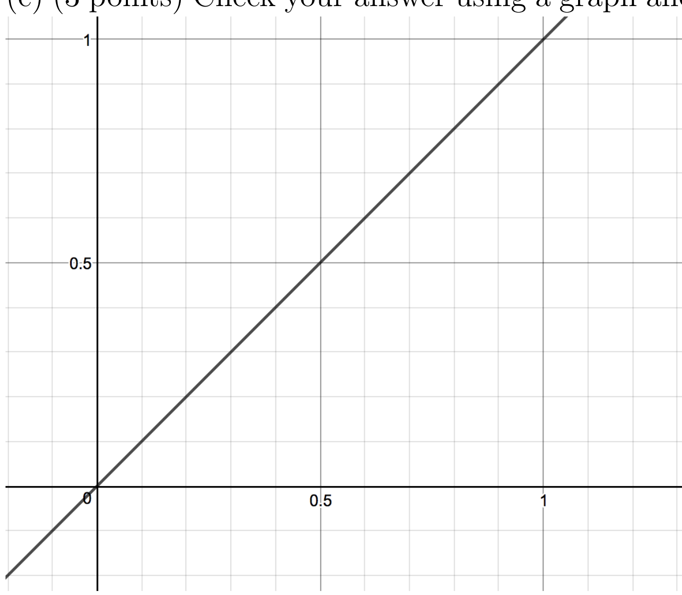
   (d) $r(t) = \cos\left(c(t^5 - 3c)\right)$, where c is a constant.

2. (11 points) Consider the discrete time dynamical system $x_{t+1} = 2.5x_t(1 - x_t)$.

(a) (4 points) Find the equilibrium for the DTDS.

(b) (4 points) Is the non-zero equilibrium stable or unstable? Justify your answer using the Stability Theorem.

(c) (3 points) Check your answer using a graph and cobwebbing. Label your axes.



3. (10 points) Consider the logistic growth model.

$$x_{t+1} = 3.2rx_t(1 - x_t)$$

(a) What condition on r guarantees that the equilibrium $x^* = 0$ is stable? Justify your answer with the Stability Theorem. Use correct notation in your work.

(b) Find the non-zero equilibrium in terms of parameter r.

4. (9 points)

A ball is thrown vertically upward from the roof of a 12 foot building with a velocity of 85 feet per second. The ball's height above the ground after t seconds is given by the function

$$s(t) = 12 + 85t - 16t^2$$

(a) What is the maximum height the ball reaches? Find and justify your answer using properties of the derivative of the function. Check your answer with a graphing calculator.

(b) Suppose:

$$s'(t_1) > 0$$

According to the model is the ball going up or down after t_1 seconds? Justify your choice using the meaning of a derivative.

(c) What is the velocity of the ball when it hits the ground? The ground has height zero.

5. (12 points) Suppose the function $M(t)$ gives the mass of an insect(in grams) and the function V(t) gives the volume of an insect in cubic centimeters. Assume $t > 0$.
Let $M(t) = 3t$ and $V(t) = 2 + t^2$.

(a) Density is computed by comparing the mass of an object to its volume.

$$Density = \frac{Mass}{Volume}$$

If the mass of an insect increases while the volume of the insect stays constant what happens to the density of the insect?

(b) Write a formula giving the density ($D(t)$) of the insect at time t with mass $M(t)$ and volume $V(t)$.

(c) Find the positive value of t where the rate of change of the insect's density is zero. This value of t is also called a critical point.

(d) Consider the value of time t you found in part (c). Does this point correspond to a local (relative) maximum or local (relative) minimum value of density or neither? Justify using graph or derivative properties.

6. (10 points) Consider the discrete-time dynamical system

$$T_{t+1} = 2(1 - T_t)T_t - hT_t$$

describing a population of fish being harvested at rate h, where $0 \leq h \leq 1$.

(a) Find the nonzero equilibrium population T^* as a function of h.

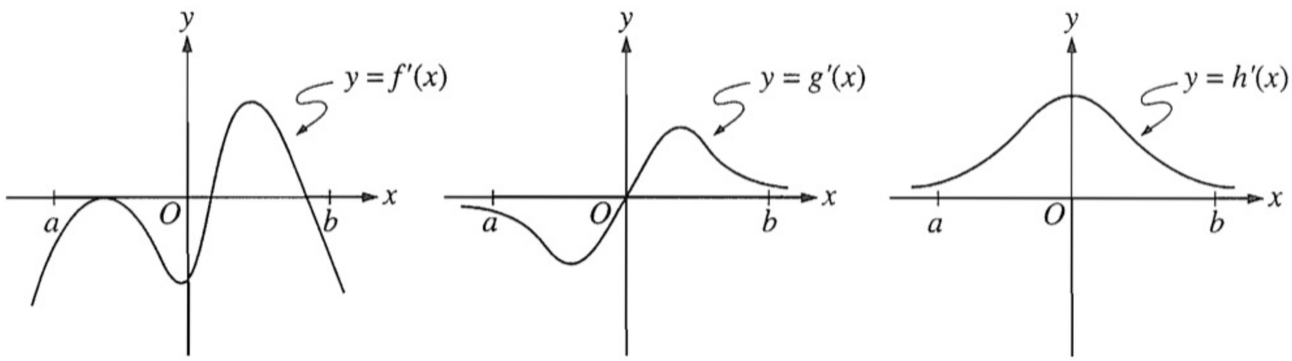
(b) The equilibrium harvest is given by $P(h) = hT^*$, where T^* is the equilibrium you found in part (a). Find the value of h that maximizes $P(h)$ on the interval $0 \leq h \leq 1$. Use ***derivative properties*** to justify your answer.

7. (12 points) Evaluate the following limits. Show all of your work. If you use leading behavior, justify your answer by explaining all of your steps. If you use L'Hopital's Rule, justify why it can be applied each time you use it.

(a) $\lim_{x\to\infty} \dfrac{x^3+3}{\ln(x)+2x}$

(b) $\lim_{x\to 0} \dfrac{\sin(x)}{\mathrm{e}^{3x}+10x^{-2}}$

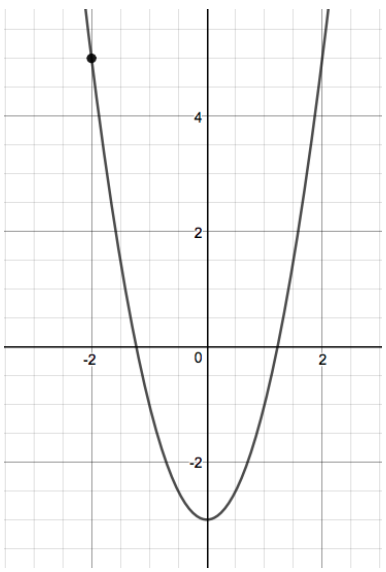
(c) $\lim_{x\to\infty} \dfrac{e^x+5x^3+x^{-1}}{\ln(x)+2x^3}$



8. (5 points) The graphs of the **derivatives** of the function f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum(local maximum) on the open interval $a < x < b$?

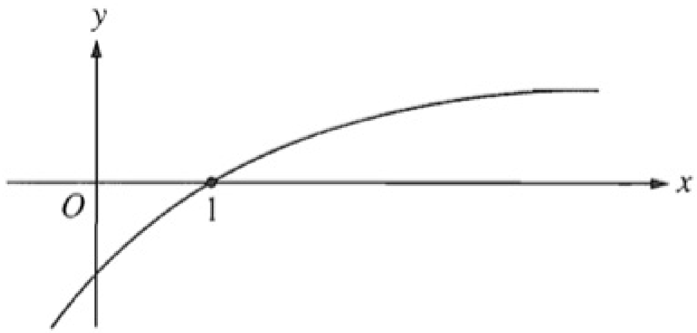
A) f only
B) g only
C) h only
D) f and g only
E) f, g, and h

9. (5 points) The graph of $y = f(x)$ is given below.



The value of $f'(-2)$ is:

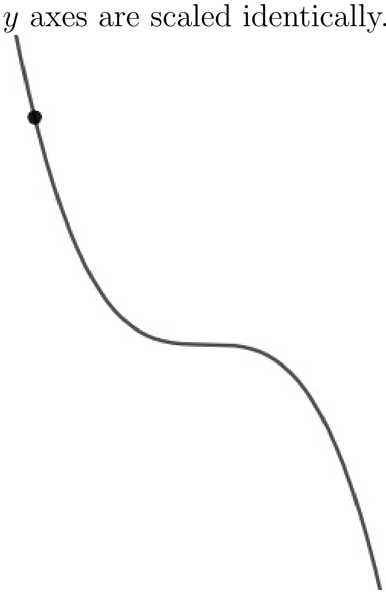
A) Positive
B) Negative
C) Undefined
D) Zero
E) Not enough information.



10. (5 points) The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

A) $f(1) < f'(1) < f''(1)$
B) $f(1) < f''(1) < f'(1)$
C) $f'(1) < f(1) < f''(1)$
D) $f''(1) < f(1) < f'(1)$
E) $f''(1) < f'(1) < f(1)$

11. (5 points) Estimate the slope of the function at the point shown. See below. The x and y axes are scaled identically.



A) The slope is approximately 2
B) The slope is approximately -4
C) The slope is approximately -20
D) The slope is approximately 4
E) The slope is approximately $\frac{-1}{4}$