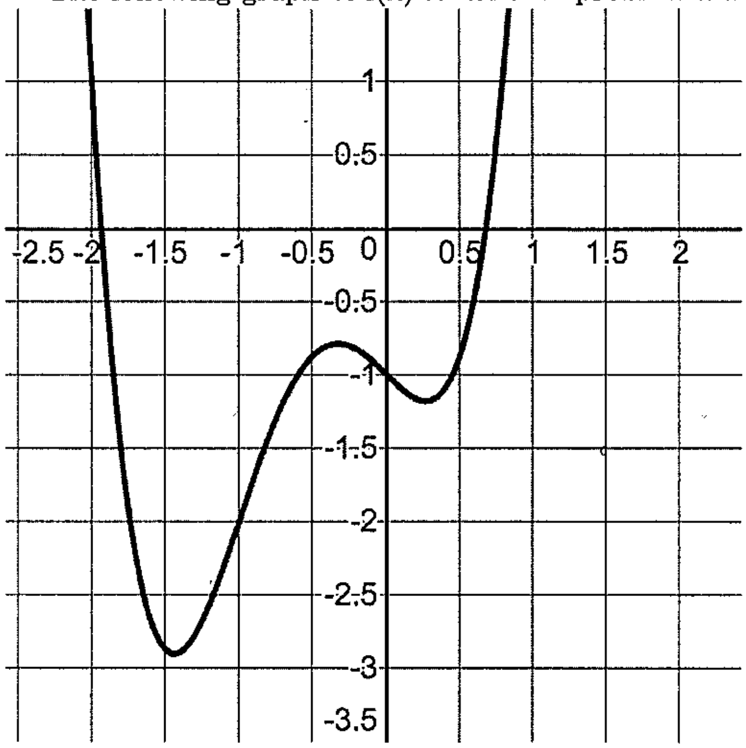
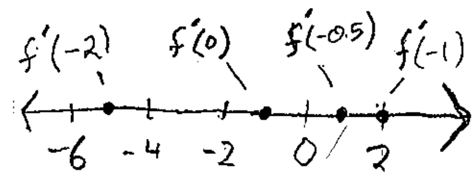
Fall 2018 Midterm 1

The following graph of f(x) is used for problem 1 and 2





1. The notation $f'(-1)$ means the instantaneous rate of change of the function f at the point $x = -1$. Consider the instantaneous rate of change of $f(x)$ at the points $x = -2, x = -1, x = -.05$ and $x = 0$. Order the instantaneous rates of change of f(x) at the given points from smallest to largest.

A) $f'(-0.5) < f'(0) < f'(-1) < f'(-2)$

B) $f'(-2) < f'(0) < f'(-0.5) < f'(-1)$

C) $f'(-.5) < f'(-1) < f(-2) < f(0)$

D) $f'(-2) < f'(-1) < f'(-0.5) < f'(0)$

$$\left.\begin{aligned} f'(-2) &\approx -5 \\ f'(-1) &\approx 2 \\ f'(-0.5) &\approx 1 \\ f'(0) &\approx -1 \end{aligned}\right\}$$

2. Let $x_0 = -2$ and $\Delta x = 1$. The function $f(x)$ is shown on graph above. Which is the best estimate of the slope of the secant line using:

$$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

A) $\frac{-3}{-3}$ B) $\frac{-1}{1}$ C) $\frac{-3}{1}$ D) $\frac{-2}{1}$ E) 3

$$\frac{f(-2+1) - f(-2)}{1} = \frac{f(-1) - f(-2)}{1} \approx \frac{-2-1}{1} = \frac{-3}{1}$$

use the graph to find f(-1) and f(-2).

3. On an alien planet they use gibs and bits to measure volume. A gib is $\frac{7}{5}$ times as large as a bit. Peanut the cat has a volume of 32 gibs. What is his volume in bits?

A) $\frac{(32)(7)}{(5)}$ bits

B) $\frac{(32)(5)}{(7)}$ bits

C) $\frac{(32)(7^3)}{(5^3)}$ bits

D) $\frac{(32)(5^3)}{(7^3)}$ bits

E) Not possible to determine with information given.

$1 \text{ gib} = \frac{7}{5} \text{ bit}$

$\Rightarrow 1 = \frac{\frac{7}{5} \text{bit}}{1 \text{gib}} \Rightarrow 1 = \frac{7}{5} \frac{\text{bit}}{\text{gib}}$

$32 \cancel{\text{gibs}} \cdot \frac{7}{5} \frac{\text{bit}}{\cancel{\text{gib}}} = \frac{32 \cdot 7}{5} \text{ bits}$

4. 45 L of water with a salt concentration of 3 moles/L is mixed with 31 L of water with a salt concentration of 10 moles/L .

Find the salt concentration of the resulting mixture.

A. 137 moles per liter
B. 9.88 moles per liters
C. $\frac{13}{2}$ moles per liter
D. 5.86 moles per liter
E. 2.93 moles per liter

$45 L + 31 L = 76 L$

$\frac{45}{76} \cdot 3 \frac{\text{mole}}{L} + \frac{31}{76} \cdot 10 \frac{\text{Mole}}{L} \approx 5.855 \frac{\text{mole}}{L}$

5. Consider the updating function (DTDS):

$$p_{t+1} = 1.72 p_t.$$

p_t is the population of insects at time t. The population is measured in thousands of insects. Find the solution function to the discrete-time dynamical system, given the initial condition $p_0 = 4$ thousand. The solution function should give the number of insects measured in thousands as a function of time.

A) $P(t) = 1.72 + 4t$

B) $P(t) = 4 + 1.72t$

C) $P(t) = 1.72(4^t)$

D) $P(t) = 4(1.72^t)$

E) None of the above.

$P_0 = 4$

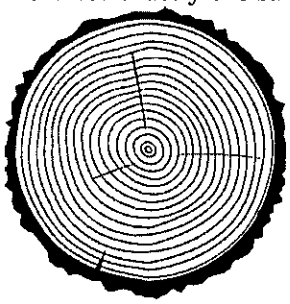
$P_1 = 1.72(4)$

$P_2 = 1.72(1.72(4)) = 1.72^2 \cdot 4$

$P_3 = 1.72(1.72(1.72(4))) = 1.72^3 \cdot 4$

$P(t) = P_t = 1.72^t \cdot 4$

[Context for Problems 5 and 6]
The diameter of a tree increases by roughly the same amount each year. This growth is visible in tree rings. Create a model of tree growth with the assumption the tree's diameter increases exactly the same amount each year.



In 1979 a tree's diameter was 11 inches. In 1988 the tree's diameter was 12.35 inches. Assume $t = 0$ in 1979 and that t gives the number of years passed since 1979.

6. Write an equation that models the tree's diameter as a function of the number of years past 1979. Note that Dr. Byerley looked up actual information about tree growth rates so your answer should be logical.

   A) $d(t) = 1.35t + 11$
   B) $d(t) = 1.35t + 12.35$
   C) $d(t) = .15t + 11$
   D) $d(t) = .15t + 12.35$
   E) $d(t) = .15t$

$m = \text{slope} = \dfrac{12.35 - 11}{9 - 0} = 0.15$

slope intercept form : $d(t) = (m)\, t + c$

: $d(t) = 0.15\, t + c$

Plug in $(0, 11)$ to solve for c

$11 = 0.15(0) + c$

$\boxed{c = 11}$

$d(t) = (0.15)t + 11$

7. Write an updating function (DTDS) that shows the relationship between the diameter in inches of a tree at time t d_t and 1 year later, d_{t+1}.

   A) $d_{t+1} = d_t + 0.15$
   B) $d_{t+1} = d_t + 11$
   C) $d_{t+1} = (0.15)d_t + 11$
   D) $d_{t+1} = (1.35)d_t$
   E) C) $d_{t+1} = (1.35)d_t + 11$

slope is a measure of how much the output changes for unit change in input

in question 6 slope = 0.15

so $d_{t+1} = d_t + 0.15$

Problem 8: Choice B. See website for updated image for # 8.

BEGINNING OF FREE RESPONSE: SHOW WORK!

9. (8 points) The length $L(T)$ of an insect is a function of the outside temperature, T, during development according to $L(T) = 11 + \frac{T}{9}$.

   The volume of an insect is a function of the length according to $V(L) = 0.4L^3$.
   Length is measured in millimeters(mm), volume is measured in cubic mm, and temperature is measured in $°C$. You do not have to simplify to answer this problem.

   a) Write a function expressing the volume of an insect in cubic mm as a function of the outside temperature in $°C$.

   Want: $V(L(T))$ $\quad V(L) = 0.4L^3, \; L(T) = 11 + \frac{T}{9}$

   $$V(T) = V(L(T)) = 0.4\left(11 + \frac{T}{9}\right)^3$$

   b) The average density of an insect is 0.26 milligrams per cubic millimeter. Write a function that gives the mass of an insect in mg as a function of the outside temperature in $°C$.

   $$\underbrace{\text{density}}_{0.26} \cdot \underbrace{\text{Volume}}_{V(T)} = \underbrace{\text{mass}}_{M(T)}$$

   $$M(T) = 0.26\,V(T)$$
   $$= 0.26\left[0.4\left(11 + \frac{T}{9}\right)^3\right]$$

10. (10 points) The population P of birds at time t is modeled by the equation

$$P(t) = P_0 e^{0.3t}.$$

P_0 is the initial population size and t is measured in years.

(a) Find the time that it takes for the population to double in size.

We want : $P(t) = 2P_0$

$$P_0 e^{0.3t} = 2P_0$$

$$e^{0.3t} = 2$$

$$\ln(e^{0.3t}) = \ln(2)$$

$$0.3t = \ln(2)$$

$$\boxed{t = \frac{\ln(2)}{0.3}}$$

(b) At time $t = 7$ the population of birds is 4,000. What is the initial population P_0? Round to the nearest whole bird.

$$P(7) = P_0 e^{0.3(7)} = 4{,}000.$$

$$P_0 e^{2.1} = 4{,}000$$

$$P_0 = \frac{4{,}000}{e^{2.1}} \approx \boxed{490 \text{ birds}}$$

11. (9 points) A person breaths in and out .55L of air in each breath. Together their lungs have 3 L of air when full. Let s_t denote the concentration of chemical in the lungs at the beginning of each breath(when the lungs are full). On each breath in they inhale .55 L of air with concentration of chemical of 2 mol/L.

a) Fill in the blank boxes below to model the situation above.

Step	Volume (L)	Total Chemical (mol)	Chemical Conc. (mol/L)
Air in lung when full.	3	$3\cancel{L} \cdot S_t \frac{mol}{\cancel{L}}$	s_t
Air lost from breath out	.55	$0.55 \cdot S_t$	s_t
Air remaining in lung	$3-0.55 = 2.45$	$2.45 S_t$	s_t
Air replaced by inhale	.55	$2(0.55)=1.10$	2 mol/L
Air in lung after inhale	3	$2.45 S_t + 2(0.55)$	$\frac{2.45 S_t + 1.1}{3}$

b) Write an updating function(DTDS) that relates s_t and s_{t+1}.

$$S_{t+1} = \frac{2.45 S_t + 1.1}{3}$$

c) What is the equilibrium concentration of chemical in the lung? Find the equilibrium algebraically. (If you find it with a graph you can still get partial credit.)

$$s^* = \frac{2.45 s^* + 1.1}{3}$$

$$3s^* = 2.45 s^* + 1.1$$
$$-2.45^* \quad -2.45^*$$

$$0.55 s^* = 1.1$$

$$s^* = \frac{1.1}{0.55} = 2$$

12. (6 points) (a) Find all equilibria of the discrete-time dynamical system

$$v_{t+1} = \frac{Kv_t + 7}{3v_t}$$

where K is a parameter.

$$V^* = \frac{KV^* + 7}{3V^*}$$

$$3(V^*)^2 = KV^* + 7$$

$$3(V^*)^2 - KV^* - 7 = 0$$

$$V^* = \frac{K \pm \sqrt{K^2 - 4(3)(-7)}}{2(3)}$$

$$= \frac{K \pm \sqrt{K^2 + 84}}{6}$$

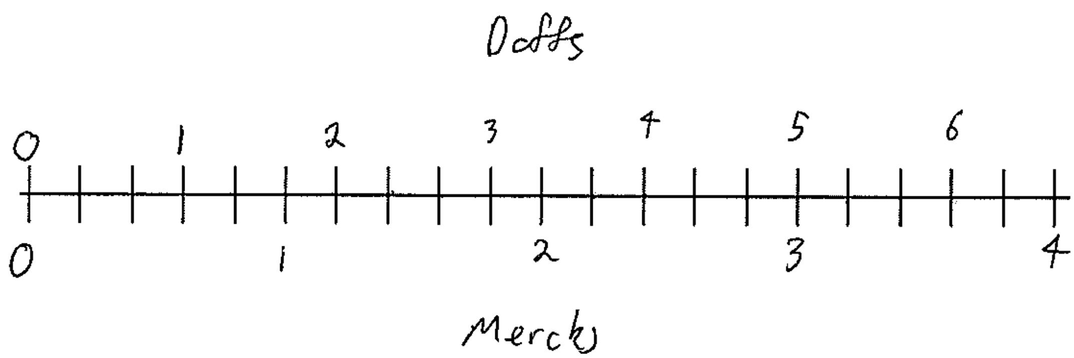
Either is acceptable. So is anything else that is equivalent.

13. (8 points) On an alien planet they use doffs and mercks to measure length. The size of a merck is $\frac{5}{3}$ times as large as the size of a doff.

1 merck = $\frac{5}{3}$ doff, $\frac{1}{5}$ merck = $\frac{1}{3}$ doff

a) Draw a ruler that has both doffs and mercks and is at least 6 doffs long. Use the line provided to make the ruler as accurate as possible.

3 merck = 5 doff



b) The length of a stick is $\frac{8}{3}$ doffs. What is the length of the stick measured in merks?

$$\frac{8}{3}\,\cancel{\text{doffs}} \cdot \frac{1 \text{ merck}}{5/3\ \cancel{\text{doffs}}} = \frac{8}{3}\,\frac{3}{5} \text{ merck} = \frac{8}{5} \text{ mercks}$$

14. (12 points) Consider the function, $g(t) = 3t^2 + 1$.

(a) Find the average rate of change in $g(t)$ between time $t = 1$ and time $t = 1.5$. In other words, find the slope of the secant line that approximates $g(t)$ on the given interval.

Average Rate of Change $\dfrac{g(t+\Delta t) - g(t)}{\Delta t}$ $1.5 - 1 = .5 = \Delta t$

$$\frac{g(1+.5) - g(1)}{.5} = \frac{g(1.5) - g(1)}{.5} = \frac{(3(1.5)^2+1) - (3(1)^2+1)}{.5}$$

$$= \frac{3(1.5)^2 - 3}{.5} = 7.5$$

(b) Find the average rate of change in $g(t)$ between time $t = 1$ and time $t = 1 + \Delta t$. In other words, find the slope of the secant line that approximates $g(t)$ on the given interval.

$$\frac{g(1+\Delta t) - g(1)}{\Delta t} = \frac{3 + 6\Delta t + 3(\Delta t)^2 + 1 - 4}{\Delta t}$$

$$= \frac{(3(1+\Delta t)^2+1) - (3(1)^2+1)}{\Delta t} = \frac{6\Delta t + 3(\Delta t)^2}{\Delta t}$$

$$= \frac{3(1 + 2\Delta t + (\Delta t)^2) + 1 - 4}{\Delta t} \quad \boxed{= 6 + 3\Delta t}$$

(c) Find the instantaneous rate of change of $g(t)$ at $t = 1$ by representing the average rate of change and then letting the change in time approach zero. In other words, use the following formula:

$$\lim_{\Delta t \to 0} \frac{g(t + \Delta t) - g(t)}{\Delta t}$$

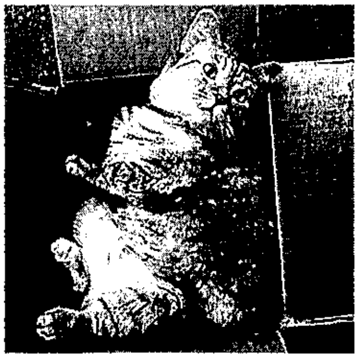
We already simplified $\dfrac{g(t+\Delta t) - g(t)}{\Delta t}$ in part b

$$= \lim_{\Delta t \to 0} 6 + 3\Delta t = 6 + 3(0) = 6.$$

15. (10 points) Pebbles the cat is in box. The box is 32 cm long, 23 cm wide, and 12.5 cm tall. Assume the volume of the box and the volume of Pebbles are the same.

Assume the volume of a cell is $20^3 \mu m^3$.
$1 \mu m = 10^{-4} cm$.

Estimate the number of cells in Pebbles.



$V_{box} = (32\,cm)(23\,cm)(12.5\,cm) = 9{,}200\ cm^3$

$V_{Pebbles} = 9{,}200\,cm^3$

$V_{cell} = 20^3 \mu m^3$

$$\frac{1\ \text{cell}}{20^3 \mu m^3} \cdot \frac{1 \mu m^3}{10^{-12} cm^3} \cdot 9{,}200\,cm^3 = \left(\frac{1}{20^3} \cdot \frac{1}{10^{-12}} \cdot 9{,}200\right) \text{cells}$$