Math 155 **Final Exam** **Fall 2017**

NAME: Key

SECTION: ____________ TIME: ____________

INSTRUCTOR: ____________

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. You can ask for scratch paper from a proctor. Turn in any scratch paper that you use during the exam. You will have two hours to work on the exam.

Problem	Points	Score
1	12	
2	7	
3	15	
4	10	
5	8	
6	5	
7	10	
8	8	
9	5	
10	6	
11	4	
12	5	
13	5	
Bonus	5	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us. You can pick up final exams after grades are posted in the front office of Weber, the math building.

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(Signature)

1. (12 points) Suppose that the population w_t of yellow warblers satisfies the discrete-time dynamical system

$$w_{t+1} = \frac{3}{5} w_t (k - w_t),$$

where $k > 0$ is a positive parameter.

(a) Find all equilibria.

$$\frac{W^*}{W^*} = \frac{\frac{3}{5} W^* (K - W^*)}{W^*}$$

$$1 = \frac{3}{5}(K - W^*)$$

$$\frac{5}{3} = K - W^* \rightarrow \boxed{W^* = K - \frac{5}{3}}$$

* check if $W^* = 0$ is solution

$$0 = \frac{3}{5} \times 0 (K - 0) \checkmark \quad \boxed{W^* = 0}$$

(b) For what values of k is there more than one equilibrium greater than or equal to zero?

$$K - 5/3 \geq 0$$

$$K \geq 5/3$$

(c) For the equilibrium $w^* = 0$, use the Stability Theorem/Criterion to determine the values of k for which that equilibrium is stable. Use correct notation in your steps that show how you used the Stability Theorem/Criterion.

Stable if $|W'(0)| < 1$

$$W' = \left(\frac{3}{5} W\right)(-1) + (K - W)\left(\frac{3}{5}\right)$$

$$= -\frac{3}{5} W + \frac{3}{5} K - \frac{3}{5} W$$

$$W' = -\frac{6}{5} W + \frac{3}{5} K$$

$$W'(0) = \frac{3}{5} K$$

$$\left|\frac{3}{5} K\right| < 1$$

$$-1 < \frac{3}{5} K < 1$$

$$\boxed{-\frac{5}{3} < K < \frac{5}{3}}$$

2. (7 points)

Consider the function $f(x) = xe^{-3x^2}$.

Find *all* values of x where the rate of change of $f(x)$ is zero or undefined. These x values are called critical points of $f(x)$.
Hint: The value of $y = e^{-3x^2}$ *is always positive.*

critical points: when $f'(x) = 0$ or is undefined

$f'(x) = (x)(-6xe^{-3x^2}) + (e^{-3x^2})(1)$

$f'(x) = -6x^2e^{-3x^2} + e^{-3x^2}$

$f'(x) = e^{-3x^2}(-6x^2+1)$

never $= 0$
or
undefined

$-6x^2+1=0$

$1 = 6x^2$ → never undefined

$\frac{1}{6} = x^2$

$\boxed{x = \pm\sqrt{\frac{1}{6}}}$

3. (15 points) Evaluate the following definite and indefinite integrals. Show all of your work.

(a) $\int_1^3 x^2 dx$

$$\int_1^3 x^2 dx = \frac{x^3}{3}\Big|_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = \frac{27}{3} - \frac{1}{3} = \boxed{\frac{26}{3}}$$

(b) $\int_1^3 \frac{-x^2+x^7}{x^3} + e^2\, dx$

$$= \int_1^3 \left(\frac{-1}{x} + x^4 + e^2\right) dx$$

$$= -\ln|x| + \frac{x^5}{5} + xe^2 \Big|_1^3$$

$$= \left[-\ln|3| + \frac{3^5}{5} + 3e^2\right] - \left[-\cancel{\ln|1|}^{\,0} + \frac{1^5}{5} + e^2\right]$$

$$= \boxed{47.301}$$

(c) Use integration by parts to evaluate $\int -xe^{7x}\, dx$. $= uv - \int v\,du$

$u = -x \quad dv = e^{7x} dx$

$du = -1 dx \quad v = \frac{1}{7}e^{7x}$

$$= (-x)\left(\frac{1}{7}e^{7x}\right) - \int \left(\frac{1}{7}e^{7x}\right)(-1) dx$$

$$= \frac{-x}{7}e^{7x} + \int \frac{1}{7}e^{7x} dx$$

$$= \boxed{\frac{-x}{7}e^{7x} + \frac{1}{49}e^{7x} + C}$$

4. (10 points) A runner is running a marathon. At time $t = 0$ the runner is -0.1 miles from the starting line because she did not arrive in time. When the runner has a negative distance it means she is before the starting line. She runs at a rate of

$$10e^{-0.1t} \text{miles/hour}$$

(a) Let $P(t)$ represent the distance the runner is from the starting line at time t (hours). Write a pure-time differential equation and an initial condition for the situation.

$\boxed{P(0) = -0.1}$

$$\frac{dP}{dt} = 10e^{-0.1t}$$

(b) Estimate distance the runner is from the starting line at $t = 1.5$ assuming the same initial condition as part a. Pretend the rate of change is constant on intervals of size $\Delta t = 0.5$. Show work.

Note: You can use Euler's method or Riemann sums with left-hand estimates.

Reimann Sum method

$$\frac{dP}{dt}(0) = 10e^{-0.1(0)} = 10 \text{ m/s}$$

$$\frac{dP}{dt}(0.5) = 10e^{-0.1(0.5)} = 9.512 \text{ m/s}$$

$$\frac{dP}{dt}(1) = 10e^{-0.1(1)} = 9.048 \text{ m/s}$$

$$\Delta P = (10 \text{ m/s})(0.5 \text{ s}) + (9.512 \text{ m/s})(0.5 \text{ s}) + (9.048 \text{ m/s})(0.5 \text{ s})$$

$\boxed{\Delta P = 14.28 \text{ m}}$

5. (8 points) $P(t)$ is the position (in meters) of at car at time t (in seconds). The car's velocity is given by

$$\frac{dP}{dt} = 3t.$$

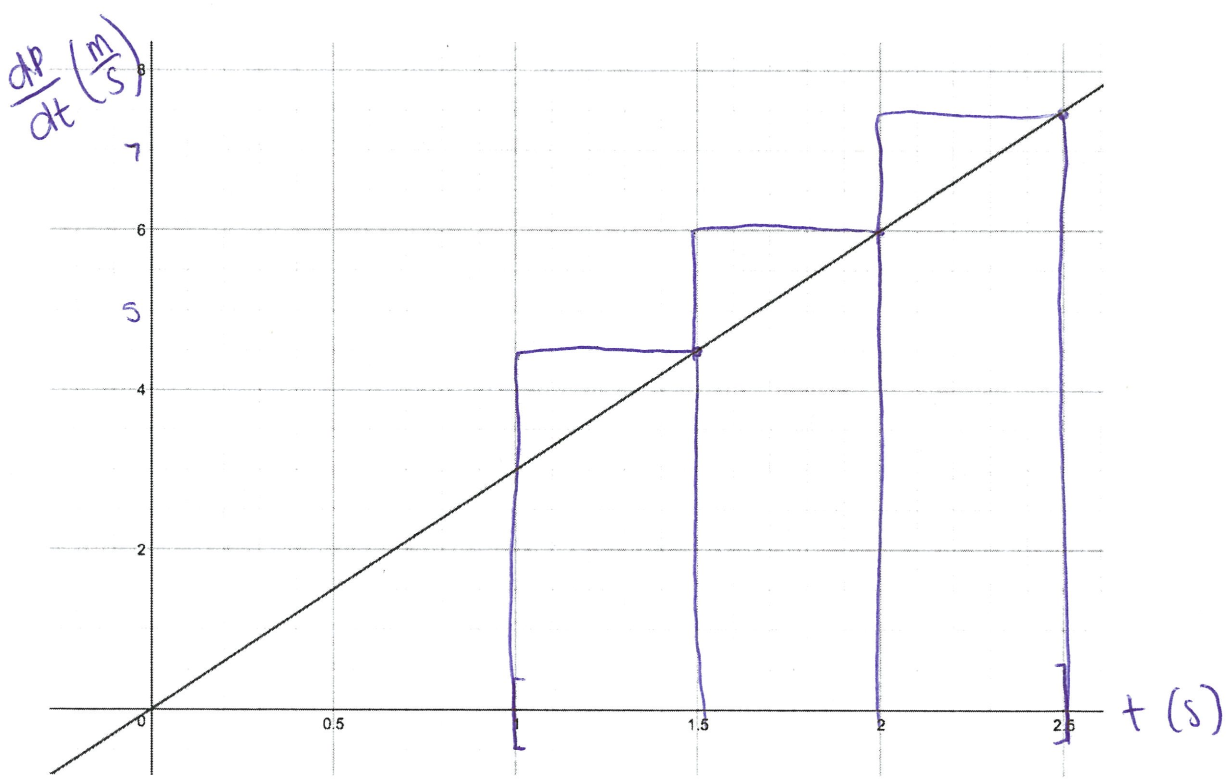
a) Label both axes on graph.

b) Estimate the total change in $P(t)$ between times $t = 1$ and $t = 2.5$ using a right-hand Riemann Sum with $\Delta t = 0.5$. Draw your rectangles or step functions on the graph below.

$\Delta P(t) = (4.5\ m/s)(.5s) + (6\ m/s)(.5s) + (7.5\ m/s)(.5s)$

$\Delta P(t) = 2.25m + 3m + 3.75m$

$\boxed{\Delta P(t) = 9m}$



6. (5 points) A horse starts with a concentration of medicine in her bloodstream equal to 4 milligrams per liter (mg/L). Each morning the horse has used up 12% of the medicine in her bloodstream. Each afternoon the horse gets enough medication to increase the concentration of medicine in her bloodstream by 5 mg/L. Let M_t = concentration of medicine on day t.

Write down a discrete-time dynamical system, together with an initial condition, that describes this situation.

$M_0 = 4\text{mg/L}$

$M_{t+1} = .78 M_t + 5$

7. (10 points) Let $V(t)$ equal the volume (in milliliters) of blood at time t (in seconds) in the kidney of a bird. Suppose that

$$\frac{dV}{dt} = 0.6\sin(2t+\pi).$$

(a) Use a definite integral to determine the total change in the volume of blood in the bird's kidney between times $t = \frac{\pi}{4}$ and $t = 3\pi$. Show work. You will only get partial credit for finding the final answer with your calculator if there is not work.

$$\int_{\pi/4}^{3\pi} 0.6\sin(2t+\pi) = \frac{-0.6}{2}\cos(2t+\pi)\Big]_{\pi/4}^{3\pi}$$

$$= -0.3\underbrace{\cos(7\pi)}_{=-1} + 0.3\,\cancel{\cos\left(\tfrac{3\pi}{2}\right)}^{\to 0}$$

$$= \boxed{0.3}$$

(b) Determine $V(t)$ if $V(0) = 4$. (That is, find a solution to the differential equation $\frac{dV}{dt} = 0.6\sin(2t+\pi)$ with initial condition $V(0) = 4$.)

$V(t) = \frac{-0.6}{2}\cos(2t+\pi) + C$

$V(t) = -0.3\cos(2t+\pi) + C$

$V(0) = 4 = -0.3\cos(2(0)7+\pi) + C$

$4 = -0.3\cos(\pi) + C$

$4 = 0.3 + C \rightarrow C = 3.7$

$\boxed{V(t) = -0.3\cos(2t+\pi) + 3.7}$

8. (8 points) A population p_t of mice obeys the discrete-time dynamical system

$$p_{t+1} = 1.3p_t.$$

(a) Write down the solution to this discrete-time dynamical system if $p_0 = 747$. (In other words, find a function that gives population for any time t.)

$p(t) = 747 \times 1.3^t$

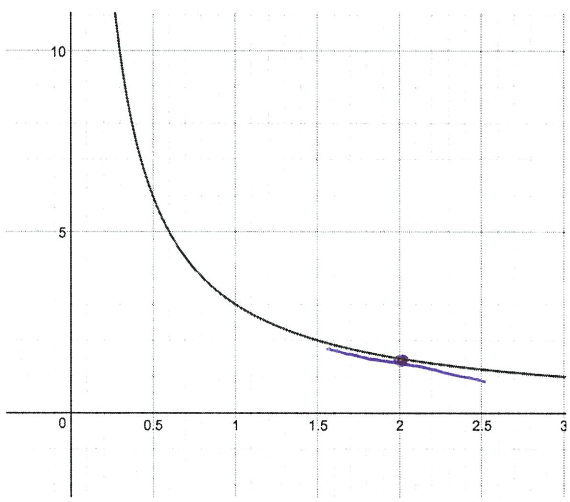
(b) If $p_0 = 747$, at what time will the population reach size 3000?

$3000 = 747\,(1.3)^t$

$4.016 = 1.3^t \rightarrow \ln(4.016) = t\ln(1.3)$

$t = \dfrac{\ln(4.016)}{\ln(1.3)}$ $\boxed{t = 5.299 \text{ sec/hr/units of time}}$

9. (5 points) The graph of the **mass** of an object $y = M(x)$ is given below.

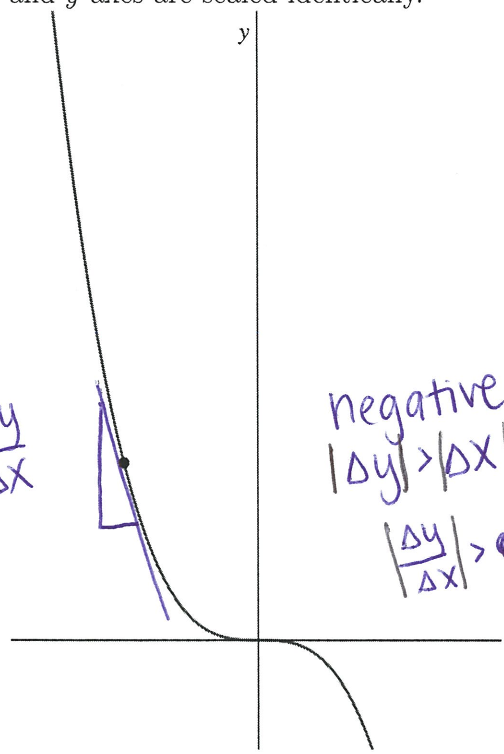


The value of the **rate of change** of mass at time $x = 2$ is:

A) Positive
B) Negative
C) Undefined
D) Zero
E) Not enough information.

10. (6 points) Estimate the slope of the function at the point shown. See below. The x and y axes are scaled identically.

$\frac{\Delta y}{\Delta x}$



negative slope

$|\Delta y| > |\Delta x|$

$\left|\frac{\Delta y}{\Delta x}\right| > 1$

$\frac{\Delta y}{\Delta x} < -1$

A) The slope is approximately 2

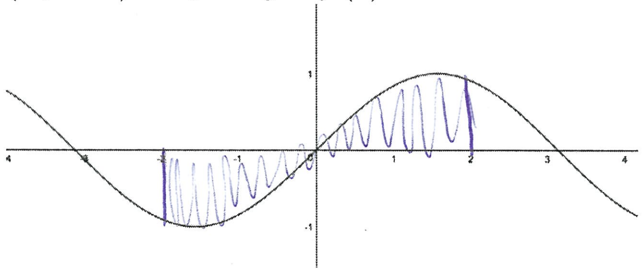
B) The slope is approximately -4

C) The slope is approximately -20

D) The slope is approximately 4

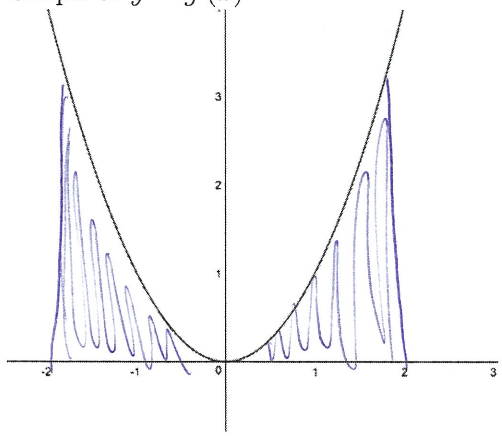
E) The slope is approximately $\frac{-1}{4}$

11. (4 points) Graph of $y = f'(x)$



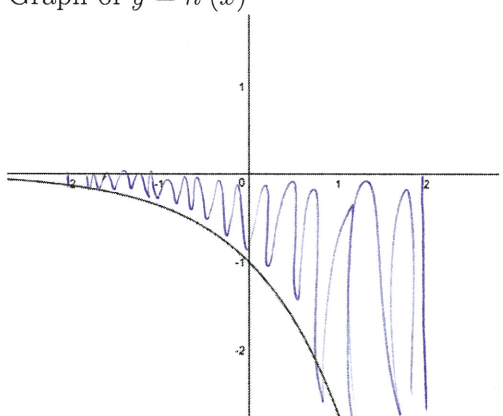
$\int_{-2}^{2} f'(x)dx \approx 0$

Graph of $y = g'(x)$



$\int_{-2}^{2} g'(x)dx > 0$

Graph of $y = h'(x)$



$\int_{-2}^{2} h'(x)dx < 0$

Which of the following is true?

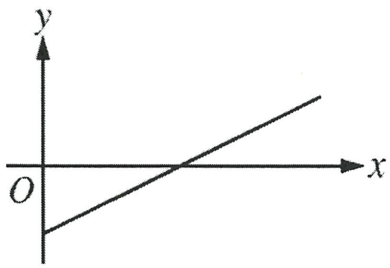
A) $\int_{-2}^{2} f'(x)dx < \int_{-2}^{2} g'(x)dx < \int_{-2}^{2} h'(x)dx$

B) $\int_{-2}^{2} g'(x)dx < \int_{-2}^{2} h'(x) < B) \int_{-2}^{2} f'(x)dx$

C) $\int_{-2}^{2} h'(x)dx < \int_{-2}^{2} f'(x)dx < \int_{-2}^{2} g'(x)dx$

D) $\int_{-2}^{2} f'(x)dx < \int_{-2}^{2} h'(x)dx < \int_{-2}^{2} g'(x)dx$

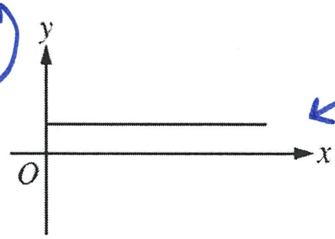
E) $\int_{-2}^{2} h'(x)dx < \int_{-2}^{2} g'(x)dx < \int_{-2}^{2} f'(x)dx$



Slope is positive, constant

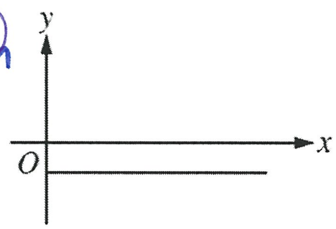
12. (5 points)The figure above shows the graph of $f(x)$. Which of the following could be the graph of its derivative, $f'(x)$?

(A)

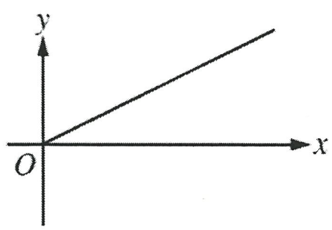


positive values for y.

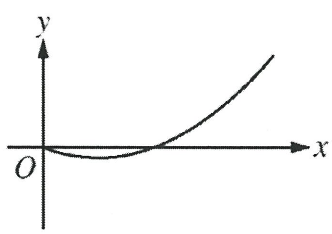
(B)



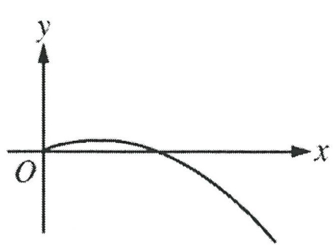
(C)



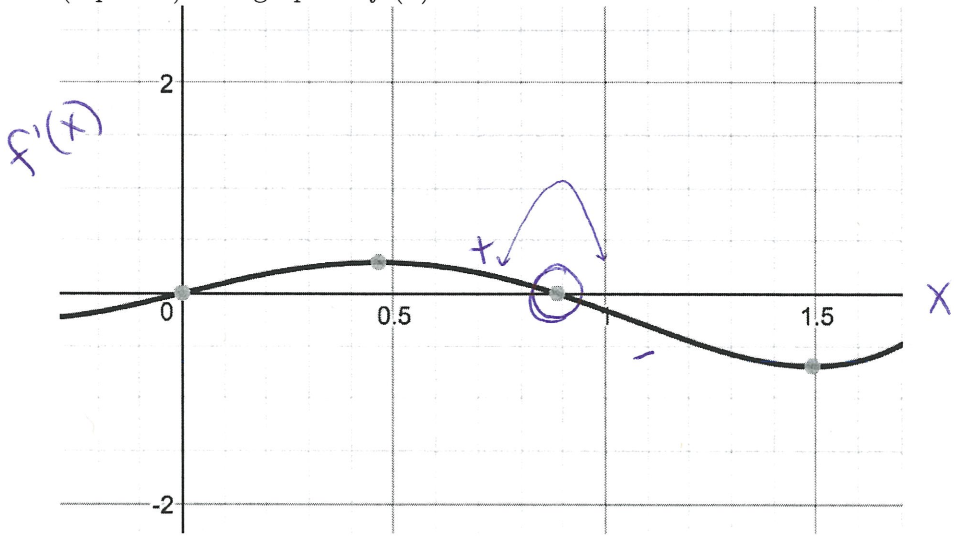
(D)



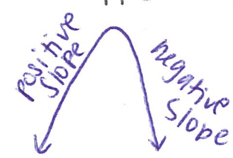
(E)



13. (5 points) The graph of $f'(x)$:



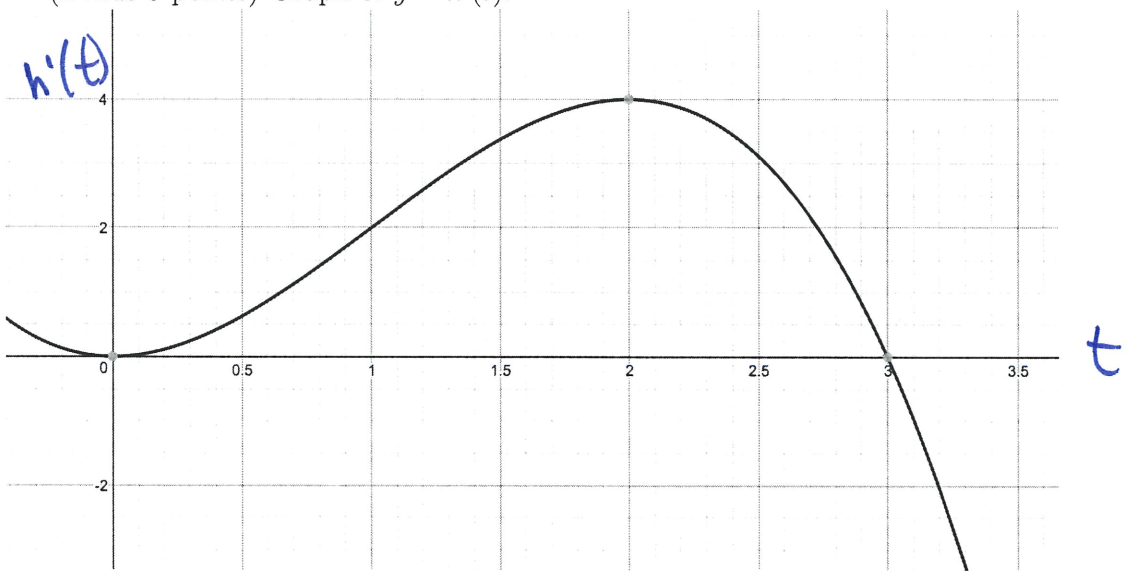
graph of f(x) near x=.88



The graph of f', the derivative of f, is shown in the figure above. The **function** f has a local maximum at $x =$

A) 0
B) 0.46
C) 0.886
D) 1.49
E) The function has more than one local maximum on interval shown.

(Bonus-5 points) Graph of $y = h'(t)$.



The graph of a function $h'(t)$ is shown above. Let

$$h(x) = \int_0^x h'(t)dt$$

Which of the following is true?

A) $h(3) < h'(3) < h''(3)$

B) $h(3) < h''(3) < h'(3)$

C) $h''(3) < h'(3) < h(3)$

D) $h''(3) < h(3) < h'(3)$

E) $h'(3) < h(3) < h''(3)$

$h(3) = \int_0^3 h'(t)dt$ = positive
(area under curve)

$h'(3) = 0$ (from graph)

$h''(3) =$ negative (~~concavity~~)

$h''(3)$ is slope of $h'(x)$ at $x=3$

$h''(3)$ is negative.