Math 155 **Exam 1** **Spring 2018**

NAME: Key

SECTION: ____________________ TIME: ____________________

INSTRUCTOR: ____

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You can ask for scratch paper. You will have one hour and 50 minutes to work on the exam.

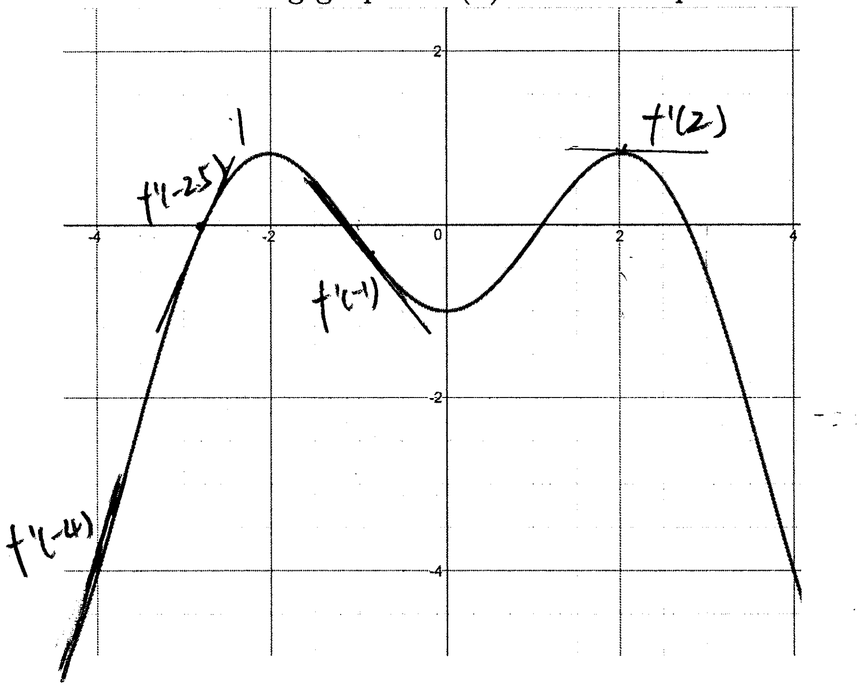
Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	10	
7	8	
8	10	
9	7	
10	8	
11	10	
12	10	
13	12	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

____
(Signature)

The following graph of f(x) is used for problems 1 and 2.



1. The notation $f'(-4)$ means the instantaneous rate of change of the function f at the point $x = -4$. Consider the instantaneous rate of change of $f(x)$ at the points $x = -4, x = -2.5, x = -1$ and $x = 2$. Order the instantaneous rates of change of f(x) at the given points from smallest to largest.

   A) $f'(-4) < f'(-2.5) < f'(-1) < f'(2)$

   B) $f'(2) < f'(-1) < f'(-2.5) < f'(-4)$

   C) $f'(-4) < f'(2) < f(-1) < f(-2.5)$

   D) $f'(-1) < f'(2) < f'(-2.5) < f'(-4)$

   E) None of the above.

When we go back to graph, we can draw the tangent line of each point, and we found $f'(-2.5) > 0$. $f'(-4) > 0$. $f'(2) < 0$, $f'(-1) <$

And Compared to $f'(-2.5)$ and $f'(-4)$, $f'(-4)$ is steeper which $f'(-2.5) < f'(4)$.

And Compared to $f'(-1)$ and $f'(2)$, $f'(2)$ is flatter, we have $f'(-1) < f'(2)$.

Conculsion: $f'(-1) < f'(2) < f'(-2.5) < f'(-4)$

2. Let $x_0 = 3$ and $\Delta x = 0.5$. The function $f(x)$ is shown on graph above. Estimate the slope of the secant line using:

$$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

   A) $\frac{1.5}{.5}$ B) $\frac{-1.5}{.5}$ C) $\frac{3.5-3}{.5}$ D) $\frac{-.5}{1}$

   E) None of the above.

First, we put the $x_0 = 3$ and $\Delta x = 0.5$ into secant line which has

$$\frac{f(3+0.5) - f(3)}{0.5} = \frac{f(3.5) - f(3)}{0.5}$$

From the graph, we can estimate

$f(3.5) \approx -2.0$ $f(3) \approx -0.5$

$f(3.5) - f(3) \approx -1.5$

3. Peanut the cat is sick. Peanut needs $\frac{2}{3}$ mL of medicine per 1 kg of body weight. The scale says that Peanut weighs 11.2 pounds. Using the following conversion factor find out how much medicine Peanut needs.

1 kg = 2.2 pounds

$$11.2\,\cancel{lb} \times \frac{1\,\cancel{kg}}{2.2\,\cancel{lb}} \times \frac{\frac{2}{3}\,mL}{1\,\cancel{kg}} = \frac{(11.2)(2)}{(2.2)(3)}$$

A) $\frac{(11.2)(3)}{(2.2)(3)}$ mL

B) $\frac{(11.2)(2.2)(2)}{3}$ mL

C) $\frac{(11.2)(3)}{(2.2)(3)}$ mL

D) $\frac{(2.2)(3)}{(11.2)(3)}$ mL

E) $\frac{(11.2)(2)}{(2.2)(3)}$ mL

4. On an alien planet they use goops and snarks to measure volume. A goop is $\frac{9}{7}$ times as large as a snark. Peanut the cat has a volume of 75 snarks. What is his volume in goops?

the volume of 1 goop > vol of 1 snark

vol goop = $\frac{9}{7}$ (vol snark)

7 goops = 9 snarks

# of goops should be smaller than the # of snarks

A) $\frac{(75)(9)}{(7)}$ goops

B) $\frac{(75)(7)}{(9)}$ goops

$$75\,\cancel{snarks} \times \frac{7\,goops}{9\,\cancel{snarks}} = \frac{(75)(7)}{9}\,goops$$

C) $\frac{(75)(9^3)}{(7^3)}$ goops

D) $\frac{(75)(7^3)}{(9^3)}$ goops

E) Not possible to determine with information given.

5. The model that describes the number of bacteria in a culture after t days has just been revised from $P(t) = 7e^{\ln(2)t}$ to $P(t) = 7e^{\ln(3)t}$.
What implications can you draw from this information?

A) The final number of bacteria is 3 times as much of the initial value instead of 2 times as much.

B) The initial number of bacteria is 3 instead of 2.

C) The number of bacteria triples every day instead of doubling every day.

D) The growth rate of the bacteria in the culture is ln(2) % per day instead of ln(3) % per day.

E) None of the above.

Model for Population growth:

$$P(t) = P_o e^{kt}$$ * k is now ln(3)

* Plug in k

$$P(t) = P_o e^{\ln(3)\,t}$$ we know that $e^{\ln(3)} = 3$

$$P(t) = P_o \left(e^{\ln(3)}\right)^t \implies P(t) = P_o (3)^t$$

So...

$$P(1) = P_o (3)^{(1)}$$

$$P(1) = 3P_o$$

* In one time interval t, the initial population is multiplied by 3, so it is tripled.

$\boxed{C \text{ - is correct}}$

Aside An updating function could be written as

$$P_{t+1} = 3P_t$$

BEGINNING OF FREE RESPONSE: SHOW WORK!

6. (10 points) The population ~~P~~ $P(t)$ of dogs at time t is modeled by the equation

$$P(t) = P_0 e^{0.23t}.$$

P_0 is the initial population size and t is measured in years.

(a) Find the time that it takes for the population to double in size.

$P = P_0 e^{(.23)t}$ * It doubled, $P = 2P_0$ or the pop. is now 2x initial population..... So...

$2\cancel{P_0} = \cancel{P_0} e^{(.23t)}$

* P_0 divide out

$2 = e^{(.23)t}$

$\ln(2) = \ln(e^{.23t})$

$\ln(2) = (.23)t \cancel{\ln(e)}$

Note: $\ln(e) = 1$

$\ln(2) = .23t$

$\boxed{t = \frac{\ln(2)}{.23} \approx 3.014 \text{ years}}$

(b) At time $t = 4$ the population of dogs is 5,000. What is the initial population P_0?

$P(4) = 5{,}000$, $t = 4$

Input $t=4$

Input $P = 5{,}000$

$5{,}000 = P_0 e^{.23(4)}$

$5{,}000 = P_0 e^{(.92)}$

$\frac{5{,}000}{e^{(.92)}} = P_0$

$P_0 = \frac{5000}{2.509}$

$\boxed{P_0 = 1992.6 \text{ dogs at } t = 0}$

7. (8 points) Consider the discrete-time dynamical system:

$$p_{t+1} = 1.13p_t.$$

p_t is the population of fish at time t. The population is measured in thousands of fish. Find the solution function to the discrete-time dynamical system, given the initial condition $p_0 = 3$ thousand. The solution function should give the number of fish measured in thousands as a function of time.

t	P_t
0	3
1	1.13×3
2	$1.13 \times 1.13 \times 3$
3	$1.13 \times 1.13 \times 1.13 \times 3$

Notice the pattern in the table. To calculate p at time t, we multiply the initial value 3 by 1.13 t times. Thus

$$\boxed{p(t) = 3 \times 1.13^t}$$

8. (10 points) One seal splashes in her saltwater pool at regularly spaced intervals of time. Each splash spills 40 L of saltwater.

The pool starts withs 5000 L of saltwater. The initial concentration of salt in the pool is s_0 mols per liter.

After each splash the zookeeper replaces the 40 L of lost saltwater with 40 L of new saltwater that has a concentration of 2 mol/L.

Fill in the blank boxes below to model the situation above.

Step	Volume (L)	Total Salt (mol)	Salt Concentration (mol/L)
H_20 in pool before seal jump in	5000	$5000 s_0$	s_0
Water lost	40	$40 s_0$	s_0
H_20 in pool after seal jump in	4960	$4960 s_0$	s_0
Water replaced	40	$40 \times 2 = 80$	2
H_20 in pool after replacing water	5000	$4960 s_0 + 80$	$\frac{4960 s_0 + 80}{5000}$

9. (7 points) 10 L of water with a salt concentration of 4 moles/L is mixed with 33 L of water with a salt concentration of 7 moles/L .

Find the salt concentration of the resulting mixture.

The concentration of the mixture is the amount of salt in the mixture divided by the amount of water in the mixture.

$$\text{Salt in mixture} = \left(10L \cdot \frac{4\,mol}{L}\right) + \left(33L \cdot \frac{7\,mol}{4}\right) = 271\ mol$$

$$\text{Water in mixture} = 10L + 33L = 43\ L$$

$$\text{Concentration of mixture} = \frac{271\ mol}{43\ L} = 6.302\ mol/L$$

10. (8 points) (a) Find all equilibria of the discrete-time dynamical system

$$v_{t+1} = \frac{cv_t}{10v_t + 4}$$

where c is a parameter.

$$v^* = \frac{cv^*}{10v^* + 4}$$

$$v^*(10v^*+4) = cv^*$$

$$v^*(10v^*+4) - cv^* = 0$$

factor out v^*

$$v^*(10v^*+4-c) = 0$$

This tells us that $v^* = 0$ or $10v^* + 4 - c = 0$. In simplified form, $\boxed{v^* = 0 \text{ or } v^* = \frac{c-4}{10}}$

(b) For what value or values of c are both equilibria the same value?

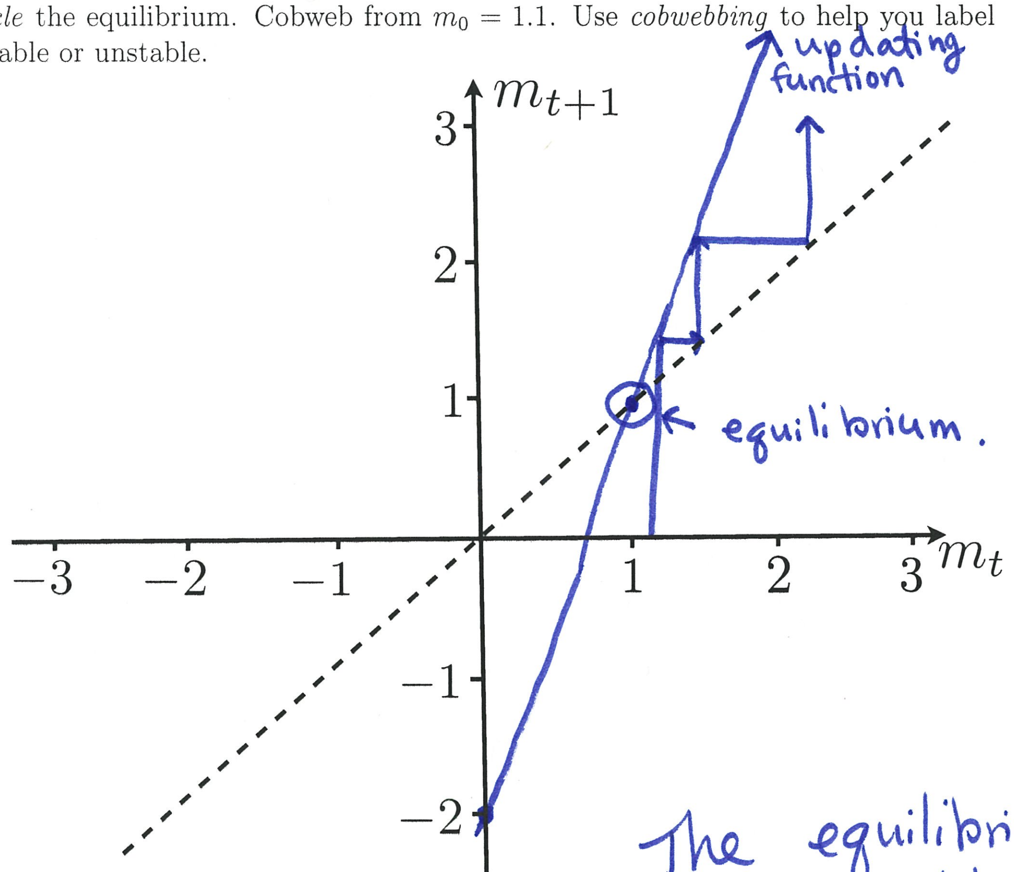
If $c = 4$, then $v^* = \frac{4-4}{10} = \frac{0}{10} = 0$. This makes both solutions from part a) equal to 0.

11. (10 points) Consider the updating function:

$$m_{t+1} = 3m_t - 2$$

a) *Graph* the updating function on the axes below (the diagonal $m_{t+1} = m_t$ is already graphed).

b) *Circle* the equilibrium. Cobweb from $m_0 = 1.1$. Use *cobwebbing* to help you label it as stable or unstable.



The equilibrium is unstable.

12. (10 points) Let V_t represent the voltage at the AV node in the heart model after t pulses from SA node.

$$V_{t+1} = \begin{cases} .78V_t + 1.2, & \text{if } .78V_t \leq 2 \\ .78V_t, & \text{if } .78V_t > 2 \end{cases}$$

Find V_1 and V_2. Show work.

Not on Fall, 2018 Test.

13. (12 points) Consider the function, $g(t) = t^2 + 3$.

(a) Find the average rate of change in $g(t)$ between time $t = 2$ and time $t = 2.5$. In other words, find the slope of the secant line that approximates $g(t)$ on the given interval.

$$\frac{g(t+\Delta t) - g(t)}{\Delta t} \qquad \frac{g(2+.5) - g(2)}{.5}$$

$$\frac{g(2.5) - g(2)}{.5} = \frac{2.5^2 + 3 - (2^2 + 3)}{.5} = \frac{2.5^2 - 4}{.5} = 4$$

(b) Find the average rate of change in $g(t)$ between time $t = 2$ and time $t = 2 + \Delta t$. In other words, find the slope of the secant line that approximates $g(t)$ on the given interval.

$$\frac{g(2+\Delta t) - g(2)}{\Delta t} = \frac{(2+\Delta t)^2 + 3 - ((2)^2 + 3)}{\Delta t} =$$

$$\frac{(2+\Delta t)^2 - 2^2}{\Delta t} = \frac{4 + 2\Delta t + 2\Delta t + (\Delta t)^2 - 4}{\Delta t} = \frac{4\Delta t + (\Delta t)^2}{\Delta t} = \boxed{4 + \Delta t}$$

(c) Find the instantaneous rate of change of $g(t)$ at $t = 2$ by representing the average rate of change and then letting the change in time approach zero. In other words, use the following formula:

$$\lim_{\Delta t \to 0} \boxed{\frac{g(t+\Delta t) - g(t)}{\Delta t}}$$

we already simplified this - we don't have to do algebra again.

$$\lim_{\Delta t \to 0} 4 + \Delta t = 4$$