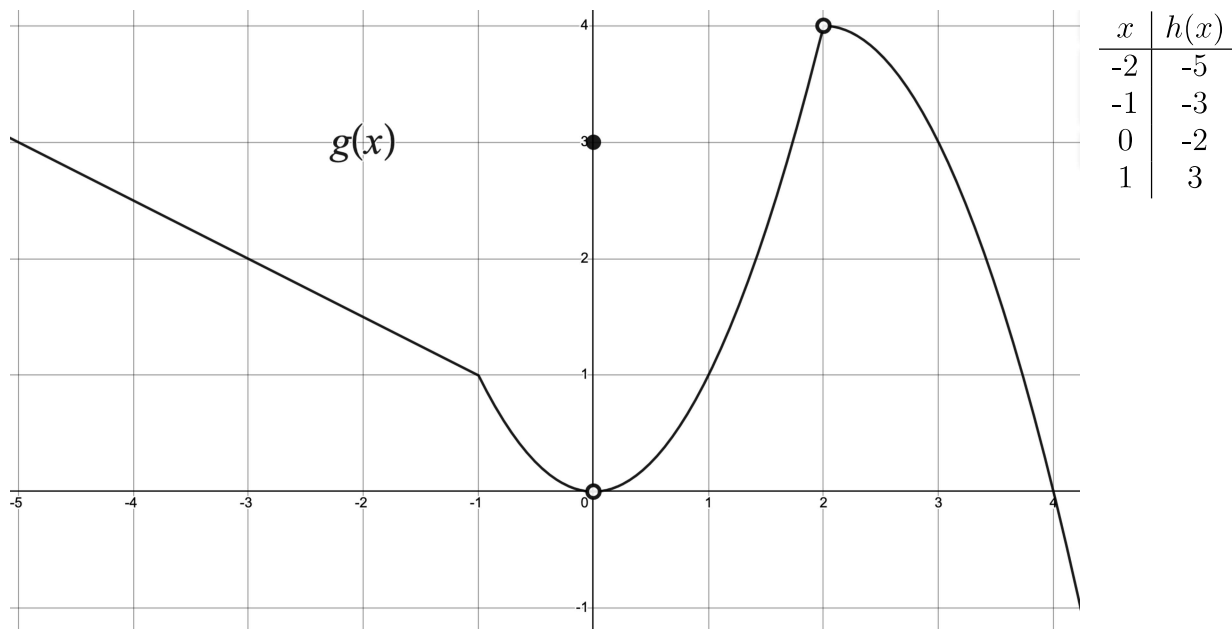


1. MIDTERM 1 SECTION

- (1) (25 points) Use the graph of the function $g(x)$ and the table of the invertible function $h(x)$ to answer each question below. Write “DNE” if a value does not exist.



(a) $g(0) =$

(d) $(h + g)(-1) =$

(b) $g(2) =$

(e) $h(g(1)) =$

(c) $h^{-1}(-2) =$

(f) $h^{-1}(h(-1)) =$

(g) Compute $AROC_{[-2,0]}$ for the function $h(x)$.

(h) Compute $AROC_{[-1,1]}$ for the function $g(x)$. Illustrate the meaning of this value on the graph of $g(x)$, and label it (h).

- (i) Rank the following values from smallest to largest. Enter only the numerals i, ii, and iii in the blanks provided. You need not make any computation to do so.

i. $\frac{g(1) - g(-4)}{1 - (-4)}$

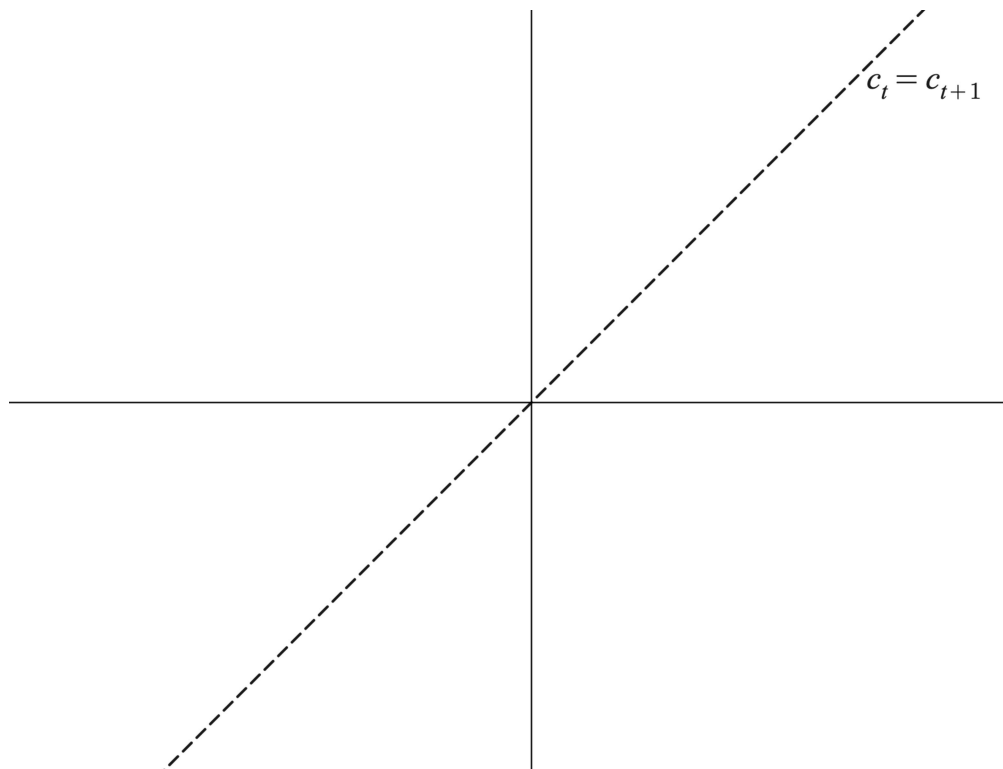
ii. $AROC_{[1,2.5]}$ for $g(x)$

iii. $\frac{g(3) - g(1.5)}{3 - 1.5}$

_____ < _____ < _____

(2) (25 points) Consider the updating function $c_{t+1} = 2c_t - 3$.

(a) Sketch the graph of the updating function, correctly labeling all axes.



(b) If $c_0 = 4$, complete the following table to determine the value of c_3 .

t	c_t	c_{t+1}
0		
1		
2		

$c_3 =$ _____

(c) Find the equilibrium point of this system algebraically. You must show the equation you are using to find the equilibrium point, as well as solve the equation.

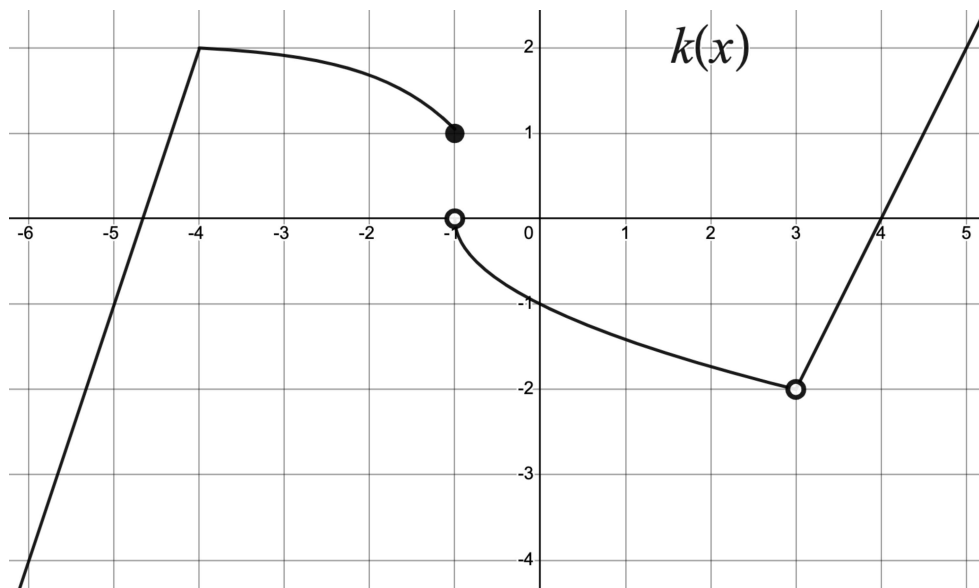
(d) Use *cobwebbing* on your graph from part (a) to classify the equilibrium point as *stable* or *unstable*. You must identify the equilibrium point from the previous point on your graph, and clearly illustrate cobwebbing to receive credit.

Stable

Unstable

2. MIDTERM 2 SECTION

- (3) (25 points) Use the graph of the function $k(x)$ to answer each question below. Write “DNE” if a value does not exist.



(a) $\lim_{x \rightarrow 0} k(x) =$

(f) $k'(-5) =$

(b) $\lim_{x \rightarrow 3} k(x) =$

(g) $k''(4) =$

(c) $\lim_{x \rightarrow -1^+} k(x) =$

(h) Let $j(x) = k(x^2)$.

$j'(x) =$ _____

(d) $\lim_{x \rightarrow -1} k(x) =$

$j'(2) =$ _____

(e) $\lim_{h \rightarrow 0} \frac{k(-4+h) - k(-4)}{h} =$

(i) $k''(0)$ is

Positive

Negative

Zero

DNE

- (j) You are given that $k'(0) = -0.5$. Write the linear approximation, $L(x)$, for $k(x)$ centered at $a = 0$.

$L(x) =$ _____

- (k) Sketch and label $L(x)$ on the graph above. Is $L(0.5)$ an overestimate or underestimate of $k(0.5)$?

Overestimate

Underestimate

(4) (25 points)

- (a) The updating function $p_{t+1} = 3p_t - (p_t)^2$ has equilibrium points at $p^* = 0$ and $p^* = 2$.
(i) Write the updating function rule as “ $f(x) =$ ”, then compute the derivative of the updating function rule.

(ii) According to the Stability Theorem, $p^* = 0$ is

Stable

Unstable

Inconclusive

You must show all work to receive credit.

(iii) According to the Stability Theorem, $p^* = 2$ is

Stable

Unstable

Inconclusive

You must show all work to receive credit.

- (b) Consider the function $\ell(x) = \frac{1}{3}x^3 - 3x^2 + 8x - 4$.
- (i) Explain why $x = 2$ and $x = 4$ are *critical numbers* of $\ell(x)$.
 - (ii) Use a **first derivative sign chart** to classify $x = 2$ and $x = 4$ as local maxima, local minima, or neither of $\ell(x)$.
 - (iii) At what x -values do the absolute maximum and absolute minimum of $\ell(x)$ occur on the interval $[0, 3]$? You must use calculus to justify your answer.

3. NEW MATERIAL SECTION

(5) (25 points)

- (a) For each limit below, write a new limit using leading behaviors. You do not need to compute the limit.

(i) $\lim_{x \rightarrow \infty} \frac{\sqrt{x} + \ln(x)}{x^{100} + e^x + x^{-2}} = \lim_{x \rightarrow \infty}$

(ii) $\lim_{x \rightarrow -\infty} \frac{x^{100} + e^x + x^{-2}}{0.5^x - x} = \lim_{x \rightarrow -\infty}$

- (b) Explain why L'Hospital's rule can be used to compute the following limit, then compute the limit.

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin(x)}$$

- (c) Consider the differential equation $k'(x) = 5 - 4x + 12x^2$ with initial condition $k(0) = -1$.
- (i) Use Euler's method with $\Delta x = 1$ to approximate $k(2)$.

- (ii) Find the specific solution, $k(x)$, using antiderivatives and the initial value $k(0) = -1$.

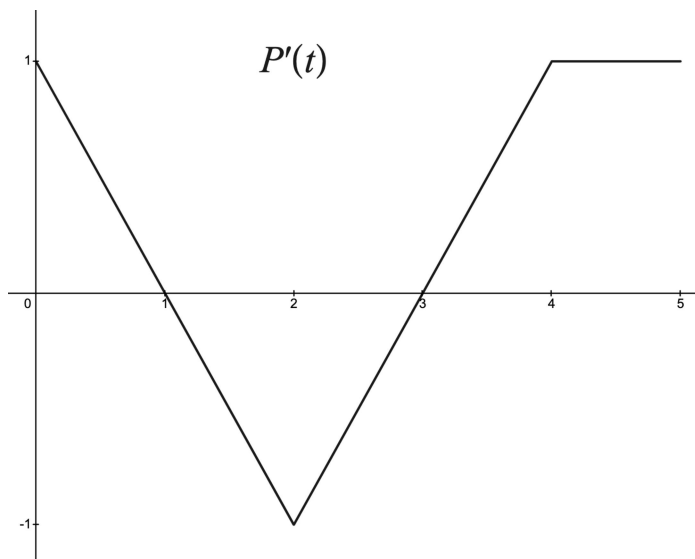
(6) (25 points)

(a) Evaluate each integral below using substitution.

(i) $\int \frac{(\ln(x))^3}{x} dx$

(ii) $\int_0^{\frac{\pi}{2}} \cos(x) e^{\sin(x)} dx$

(b) A population changes according to the graph $P'(t)$ (hundreds/year) below.



- (i) Sketch the rectangles for the Riemann sum L_3 (left with 3 rectangles) that would be used to estimate $\int_0^3 P'(t) dt$.
- (ii) Given that $P(0) = 20$, use the Fundamental Theorem of Calculus and the graph to compute $P(1)$.
- (iii) Use the graph to compute $\int_1^4 P'(t) dt$. Include units in your answer.
- (iv) Use the graph to compute the average value of $P'(t)$ on the domain $[0, 5]$.