

NAME: \_\_\_\_\_

SECTION: \_\_\_\_\_ TIME: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

**Instructions:** The exam is closed book and closed notes. You may use an approved calculator, but be sure to *clearly* show your work on each problem for full credit. Work that is crossed out or erased will not be graded. If you need scratch paper ask for some from the proctor. Turn in any scratch paper that you use during the exam. You will have one hour and 50 minutes to work on the exam.

Problem	Points	Score
1	12	
2	9	
3	10	
4	9	
5	9	
6	9	
7	6	
8	5	
9	6	
10	5	
11	5	
12	5	
13	5	
14	5	
Bonus	3	
Total	100	

## CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us in class.

\_\_\_\_\_  
(Signature)

1. (12 points) Compute the derivatives of the following functions. You do NOT need to simplify your answer. Use parentheses to indicate multiplication where appropriate, and make sure that your notation is correct.

(a)  $f(x) = \sqrt{x} + e^{3x^2}$

(b)  $f(t) = \ln(\sin(t))$

(c)  $p(x) = (e^x + 1)(x^2 - 2x + 1)$

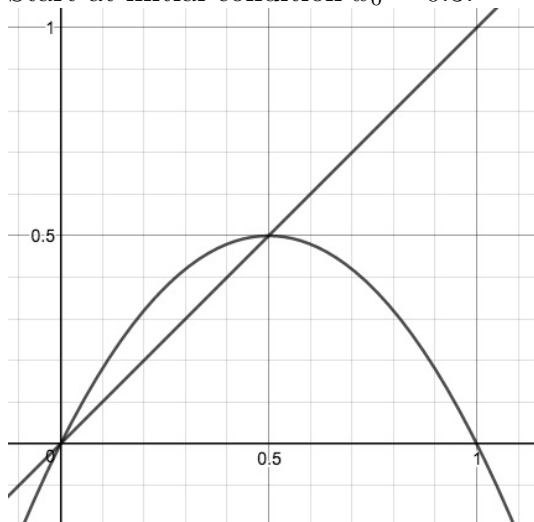
(d)  $r(t) = \cos\left(c\left(\frac{1}{t} - 3c\right)\right)$ , where  $c$  is a constant.

2. (9 points) Consider the discrete time dynamical system  $x_{t+1} = 2x_t(1 - x_t)$ .

(a) (3 points) Find the equilibrium for the DTDS.

(b) (3 points) Is the non-zero equilibrium stable or unstable? Justify your answer using the Stability Theorem.

(c) (3 points) Check your answer using a graph and cobwebbing. Label your axes. Start at initial condition  $x_0 = 0.3$ .



3. (10 points) Consider the logistic growth model.

$$x_{t+1} = \frac{x_t}{r + x_t^2}$$

- (a) Assume parameter  $r$  is greater than 0. What condition on  $r$  guarantees that the equilibrium  $x^* = 0$  is stable? Justify your answer with the Stability Theorem. Use correct notation in your work.

- (b) Find the positive, non-zero equilibrium in terms of parameter  $r$ .

4. (9 points)

A ball is thrown vertically upward from the roof of a 3 foot building with a velocity of 2 feet per second. The ball's height above the ground after  $t$  seconds is given by the function

$$s(t) = 3 + 2t - 16t^2$$

(a) What is the maximum height the ball reaches? Find and justify your answer using properties of the derivative of the function. Check your answer with a graphing calculator.

(b) For what values of  $t$  is the height of the ball increasing? Justify your answer with the derivative.

(c) What is the velocity of the ball when it hits the ground? The ground has height zero.

5. (9 points) Suppose the function  $M(t)$  gives the mass of an alien (in grams) and the function  $V(t)$  gives the volume of an alien in cubic centimeters. Assume  $t > 0$ . Let  $M(t) = .02e^t$  and  $V(t) = 1 + t^2$ .

(a) Density is computed by comparing the mass of an object to its volume.

$$Density = \frac{Mass}{Volume}$$

If the mass of an alien increases while the volume of the alien stays constant what happens to the density of the alien?

(b) Write a formula giving the density ( $D(t)$ ) of the alien at time  $t$  with mass  $M(t)$  and volume  $V(t)$ .

(c) Find the positive value of  $t$  where the rate of change of the alien's density is zero. This value of  $t$  is also called a critical point.

6. (9 points) Consider the discrete-time dynamical system

$$T_{t+1} = 3(1 - T_t)T_t - hT_t$$

describing a population of fish being harvested at rate  $h$ , where  $0 \leq h \leq 1$ .

- (a) Find the nonzero equilibrium population  $T^*$  as a function of  $h$ .

- (b) The equilibrium harvest is given by  $P(h) = hT^*$ , where  $T^*$  is the equilibrium you found in part (a). Find the value of  $h$  that maximizes  $P(h)$  on the interval  $0 \leq h \leq 1$ . Use any method to justify answer(including calculator).

7. (6 points) Evaluate the following limits. Show all of your work. If you use leading behavior, justify your answer by explaining all of your steps. If you use L'Hopital's Rule, justify why it can be applied each time you use it.

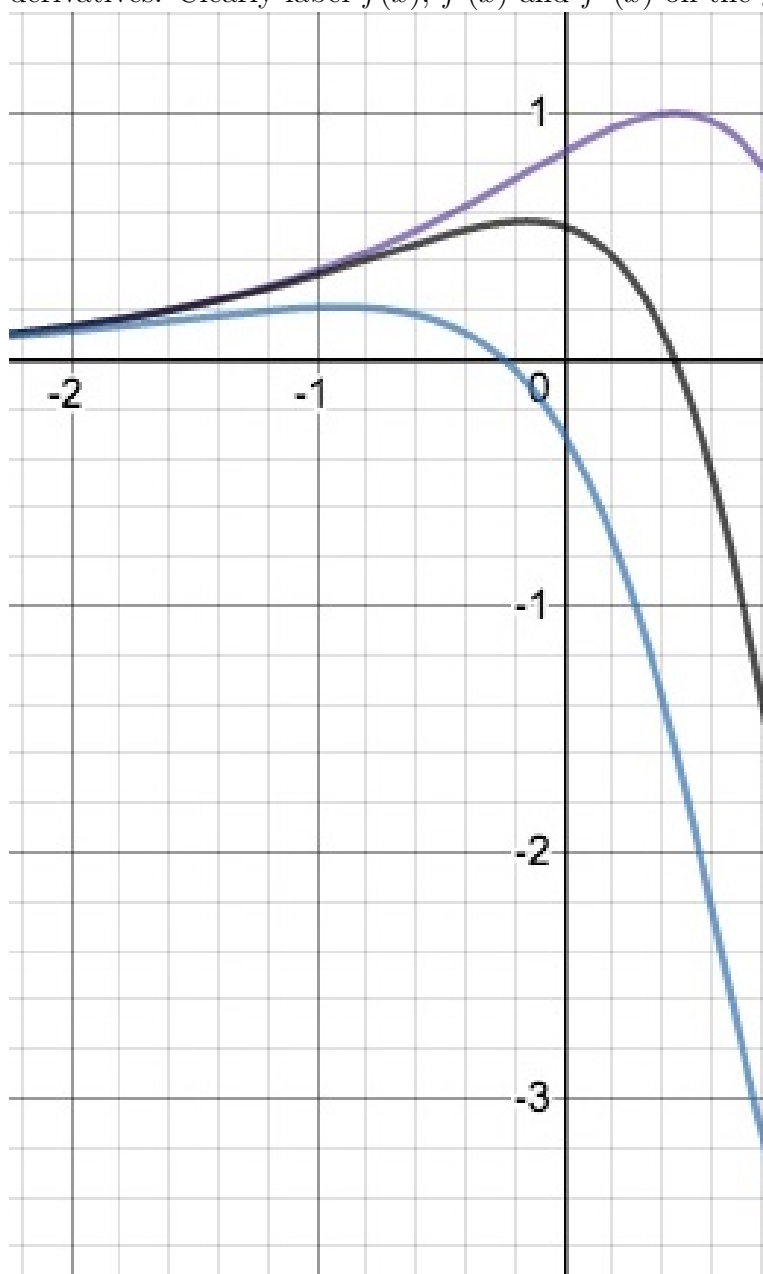
(a)  $\lim_{x \rightarrow \infty} \frac{5x^2 + \frac{1}{x}}{0.05e^x}$

(b)  $\lim_{x \rightarrow 3} \frac{(x-3)}{(x^2-9)}$

(c)  $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x^2}$



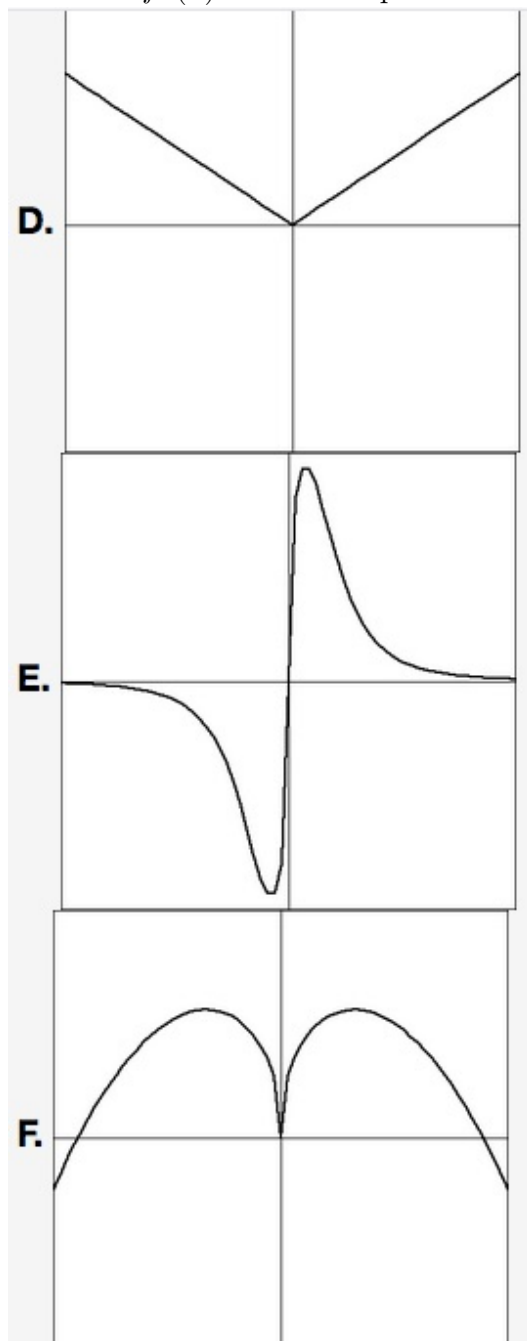
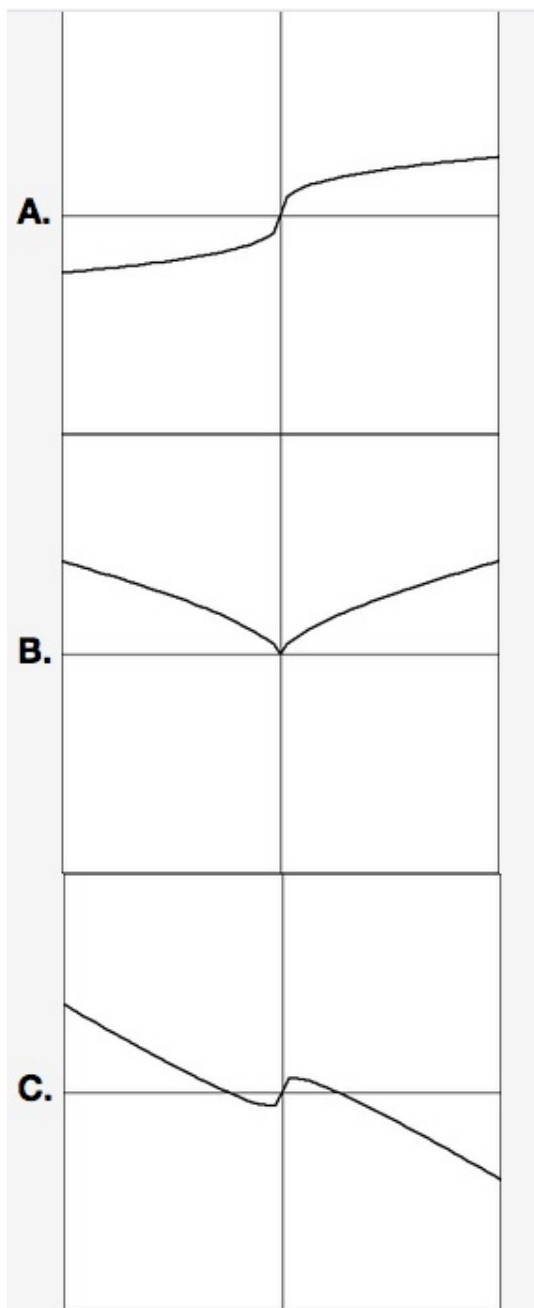
8. (5 points) The following graph shows a function  $f(x)$  and its first and second derivatives. Clearly label  $f(x)$ ,  $f'(x)$  and  $f''(x)$  on the graph.



9. (6 points) Label the given descriptions with the letter of the matching graph. Every graph shows a function  $f(x)$ .

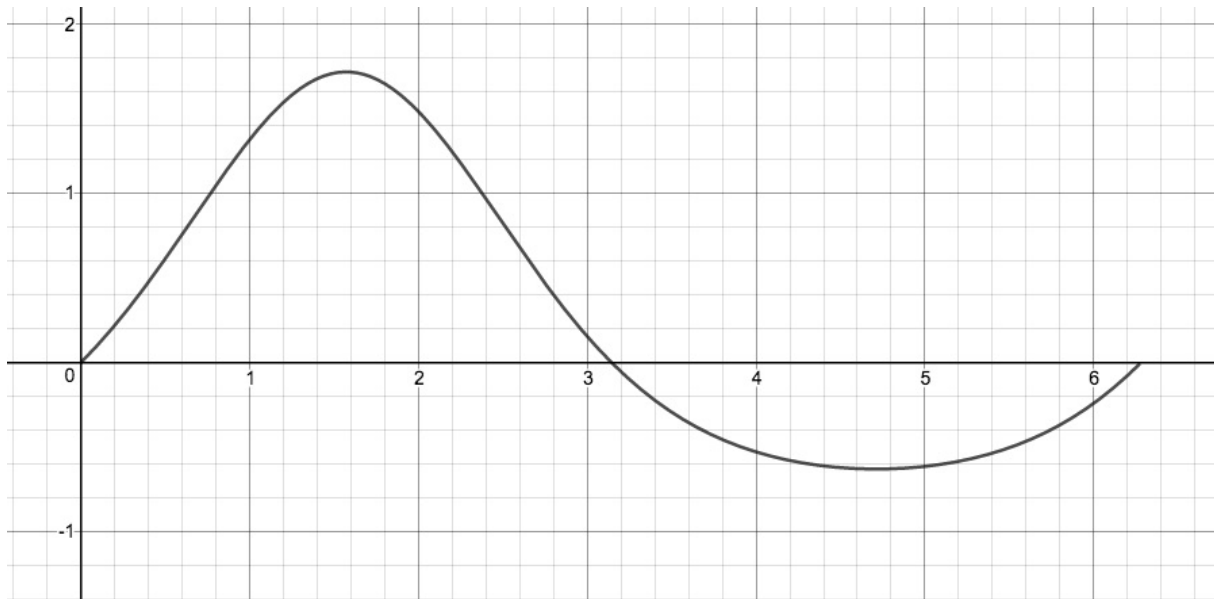
i)  $f'(x) = 0$  for two values of  $x$ .  $f(x)$  changes concavity three times. \_\_\_\_\_

ii)  $f'(x)$  is always greater than or equal to 0 and  $f''(x) = 0$  at one point. \_\_\_\_\_



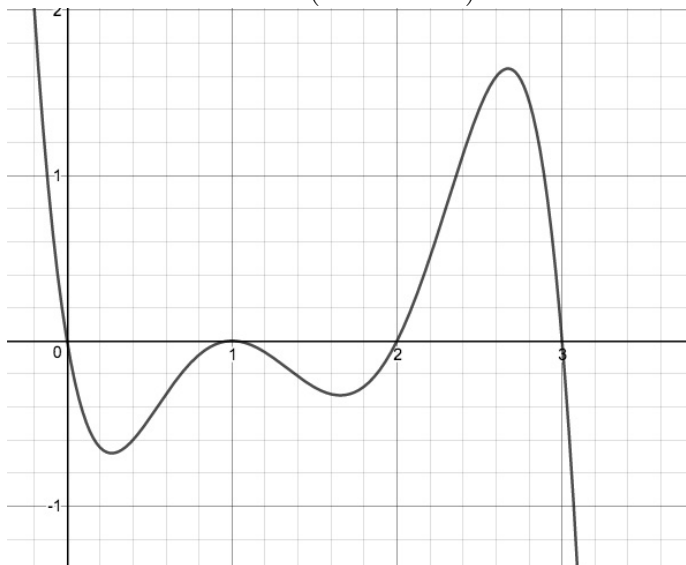
For problems 10-13 you can earn 1 point partial credit per problem for correctly labeling vertical axis as either  $f(x)$  or  $f'(x)$ .

10. (5 points) The following graph shows the *rate of change*  $f'(x)$  of a population of fish as a function of time on the closed interval  $[0, 6.3]$ . What is the largest interval where the population of fish is decreasing?

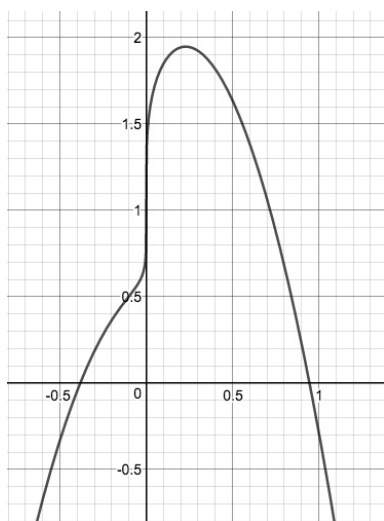


- a)  $3.2 < x < 4.6$
- b)  $1.6 < x < 4.6$
- c)  $0.6 < x < 2.6$
- d)  $3.2 < x < 6.3$
- e)  $2.6 < x < 6.3$

11. The graph of the **derivative** of the function  $f$  is shown below. Where does  $f(x)$  have a relative maximum? (Label axes).



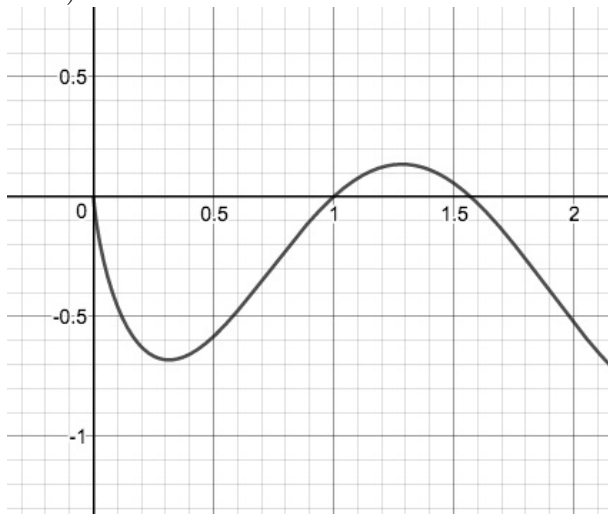
- A)  $x = 1$  and  $x = 2.7$   
 B)  $x = 3$   
 C)  $x = 2$   
 D)  $x = 0$  and  $x = 3$   
 E)  $x = 1$
12. (5 points) The graph of  $y = f(x)$  is given below. (Label axes).



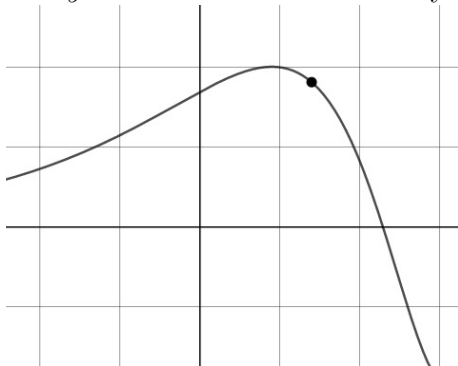
The value of the **derivative** of  $f(x)$  at  $x = .5$  is:

- A) Positive  
 B) Negative  
 C) Undefined  
 D) Zero  
 E) Not enough information.

13. (5 points) The graph of a function  $f$  is shown in the figure below. The function  $f$  has a first and a second derivative for all values of  $x$ . Which of the following is true? (Label axes).



- A)  $f(1) < f'(1) < f''(1)$   
 B)  $f(1) < f''(1) < f'(1)$   
 C)  $f'(1) < f(1) < f''(1)$   
 D)  $f''(1) < f(1) < f'(1)$   
 E)  $f''(1) < f'(1) < f(1)$
14. (5 points) Estimate the slope of the function at the point shown. See below. The  $x$  and  $y$  axes are scaled identically.



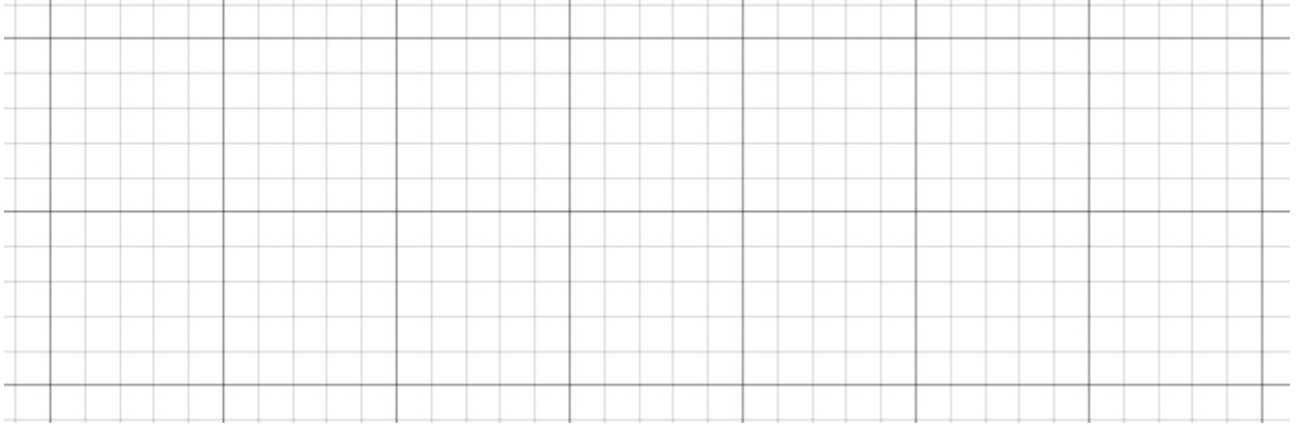
- A) The slope is approximately  $\frac{1}{4}$   
 B) The slope is approximately  $\frac{4}{5}$   
 C) The slope is approximately  $-20$   
 D) The slope is approximately  $-\frac{4}{5}$   
 E) The slope is approximately  $-\frac{1}{4}$

### Bonus-3 points

On an alien planet they use *mumps* and *togs* to measure length.

The length of one mump is  $\frac{7}{3}$  times as large as the length of one tog.

1. On the grid paper draw a ruler that has both mumps and togs. The ruler should show that a mump is  $\frac{7}{3}$  times as large as a tog. The ruler should be at least 1 mump long. (There is more than one acceptable way to draw this ruler).



2. Peanut the cat has a length of 33.4 mumps. What is his length in togs?

a)  $\frac{(33.4)(7)}{(3)}$  togs

b)  $\frac{(33.4)(3)}{(7)}$  togs

3. Garbanzo the cat is measured from nose to tail in both mumps and togs. Garbanzo is  $Q$  mumps long and  $R$  togs long. Is the measure of Garbanzo's length in mumps ( $Q$ ) larger or smaller than the measure of his length in togs ( $R$ )?

a) His measure in mumps is larger than his measure in togs.

b) His measure in togs is larger than his measure in mumps.