Math 155 **Final Exam: Version A** **Fall 2018**

NAME: ______________________________

SECTION: ______________ TIME of your Class: ______________

CIRCLE INSTRUCTOR: Cameron Byerley, Colin Roberts, Lara Kassab, Alex Williams, Brian Collery, Amie Bray, Tarun Mukthineni, Clayton Craig.

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. You can ask for scratch paper from a proctor. Turn in any scratch paper that you use during the exam. You will have two hours.

Problem	Points	Score
MC1	6	
MC2	6	
MC3	6	
MC4	6	
MC5	6	
MC6	6	
MC7	6	
MC8	Bonus	
MC9	Bonus	
10	14	
11	11	
12	15	
13	14	
14	16	
15	14	
16	12	
17	12	
Total	150	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us. You can pick up final exams after grades are posted in the front office of Weber, the math building.

____

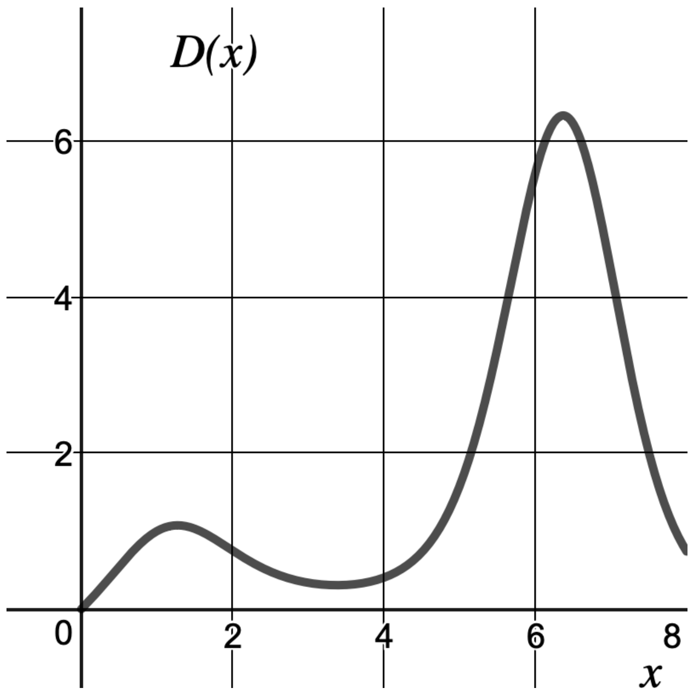
(Signature)

Blank Page.

START OF MULTIPLE CHOICE.

Please Bubble Version A on Scantron. Scantron must have your student ID to be counted for a grade.

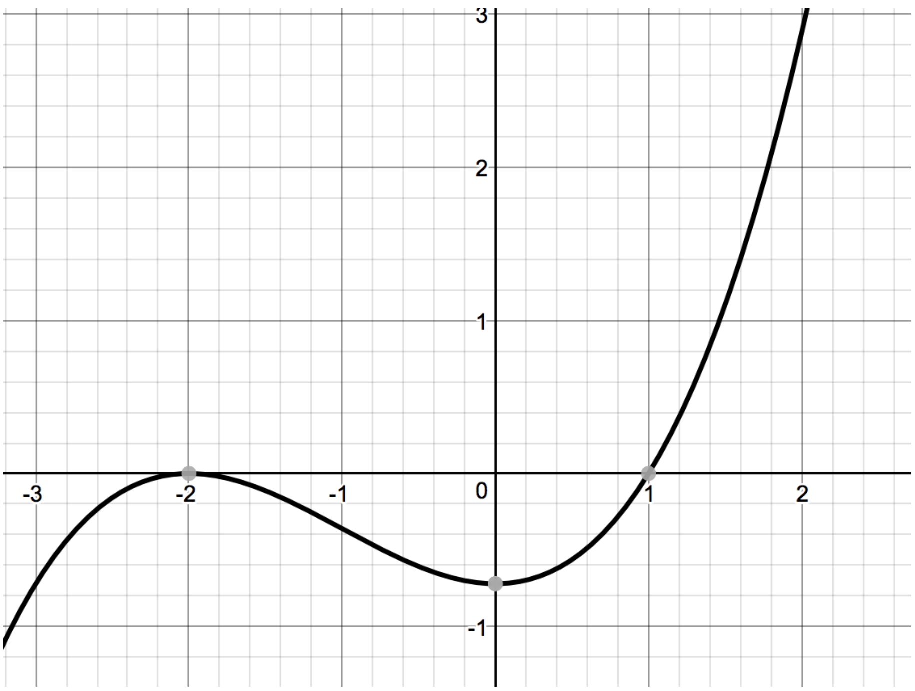
1. The graph of the density of an object $y = D(x)$ is given below.



The value of the **rate of change** of density at time $x = 2$ is:

A) Positive
B) Negative
C) Undefined
D) Zero
E) Not enough information.

2. The graph below shows the **rate of change** of a runner in meters per second. Positive rate of change means the runner is going towards his home and negative rate of change means the runner is going away from his home. When is the runner getting closer to his home on the interval $-3 < t < 2$?



A) I. $-3 < t < -2$
B) II. $0 < t < 1$
C) III. $1 < t < 2$
D) All three intervals: I, II, and III
E) Both I and II

3. Colin takes foods that have high *caloric densities* backpacking. Caloric density is a measure of calories per mass. One package of dried mashed potatoes has 440 calories and weighs 3.9 **ounces**. One package of Ramen Noodles has 380 calories and weighs 86 **grams**. Which food has higher caloric density?
Note: 1 ounce = 28.3 grams.

A) Ramen Noodles

B) Dried Potatoes

C) They have the same caloric density.

D) It is not possible to tell with the given information.

4. If temperature changes by 1 degree Fahrenheit, then it changes by $\frac{9}{5}$ degrees Celsius. A group of scientist estimates that the global average temperature will increase by 3.2 degrees Celsius over the next 100 years. How much does this group estimate the global average temperature will change in degrees Fahrenheit?

A. $\frac{9(3.2)}{5}$ $^\circ F$

B. $\frac{5(3.2)}{9}$ $^\circ F$

C. $\frac{9}{5(3.2)}$ $^\circ F$

D. $\frac{5}{9(3.2)}$ $^\circ F$

E. None of the above.

(Problems 5 and 6) The table below gives information about functions f and g. Let h be defined as $h(x) = f(g(x))$.

Value of x	1	2	3	4
f(x)	2	1	10	7
Rate of change of f(x)	7	3	-7	1
g(x)	3	-2	0.5	3
Rate of change of g(x)	4	3	-8	8

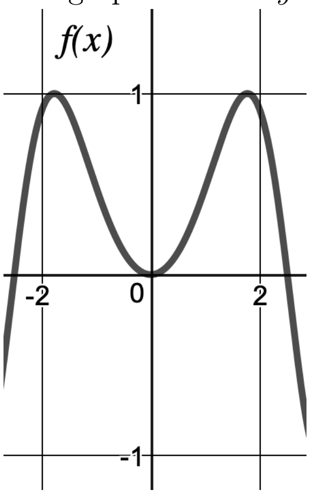
5. What is the **value** of h at $x = 1$?

a.) 6 b) 2 c) 7 d) 28 e) 10

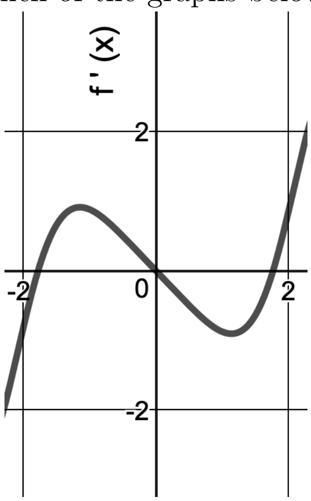
6. What is the **rate of change** of h at $x = 4$?

a.) 8 b) 1 c) -56 d) -7 e) 80

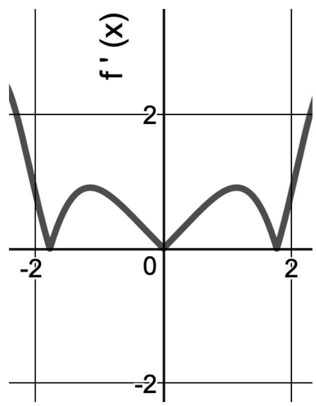
7. The graph below is $y = f(x)$



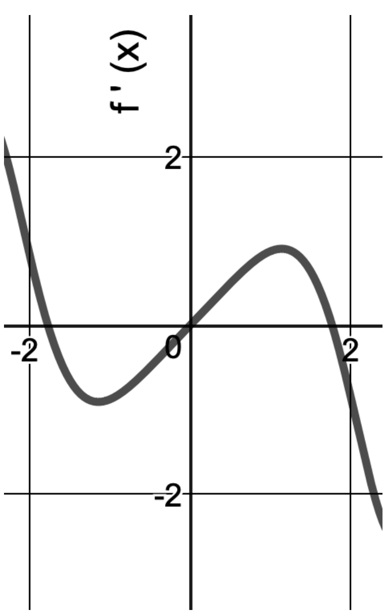
Which of the graphs below could be the graph of its derivative, $f'(x)$?



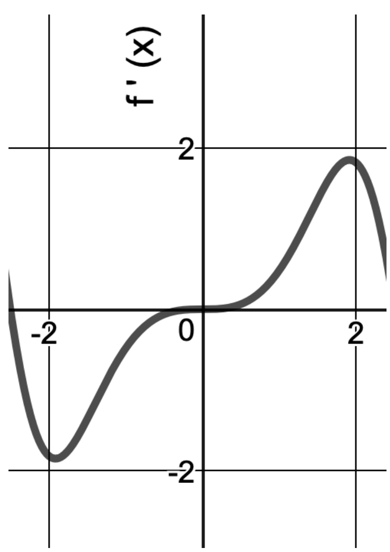
A)



B)



C)



D)

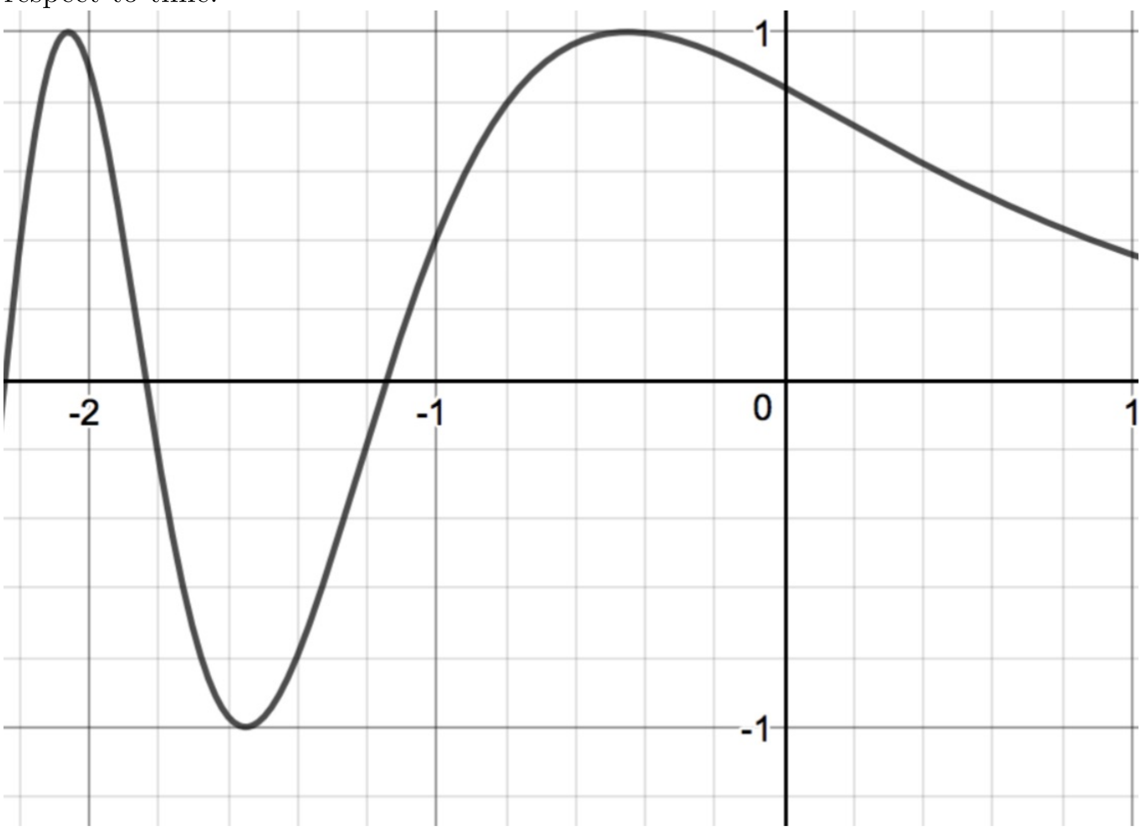
8. (Bonus) The variable a can be any number. The variable b can be any number except zero. Decide if the following statement is true or false for all numbers.

$$\frac{ab+b}{b} = a+b$$

A) True

B) False.

9. (Bonus) The graph below shows the **rate of change of a population** of fish with respect to time.



Where on the interval from $-2 \le t \le 1$ does the **population** have a relative minimum value? Relative minimums can occur at endpoints.

A) $t = -2$
B) $t = -1.15$ and $t = -2$
C) $t = -1.15$
D) $t = -1.6$
E) $t = 1$ and $t = -1.6$

10. (14 points) Suppose that the population w_t of birds satisfies the discrete-time dynamical system

$$w_{t+1} = 1.5w_t(1 - w_t),$$

(a) Find all equilibria.

(b) Use the Stability Theorem/Criterion to determine if the equilibrium $w^* = 0$ is stable. Use correct notation in your steps that show how you used the Stability Theorem/Criterion.

11. (11 points) Consider the function

$$f(x) = x\mathrm{e}^{(-x^3)}$$

Find *all* values of x where the rate of change of $f(x)$ is zero or undefined. These x values are called critical points of $f(x)$.

12. (15 points) Evaluate the following definite and indefinite integrals. Show work even if you can find answer on calculator.

    (a) $\int_2^3 x^{\frac{1}{3}} dx$

    (b) $\int x(x^2 + 3)^4 + 3\pi^2 \, dx$

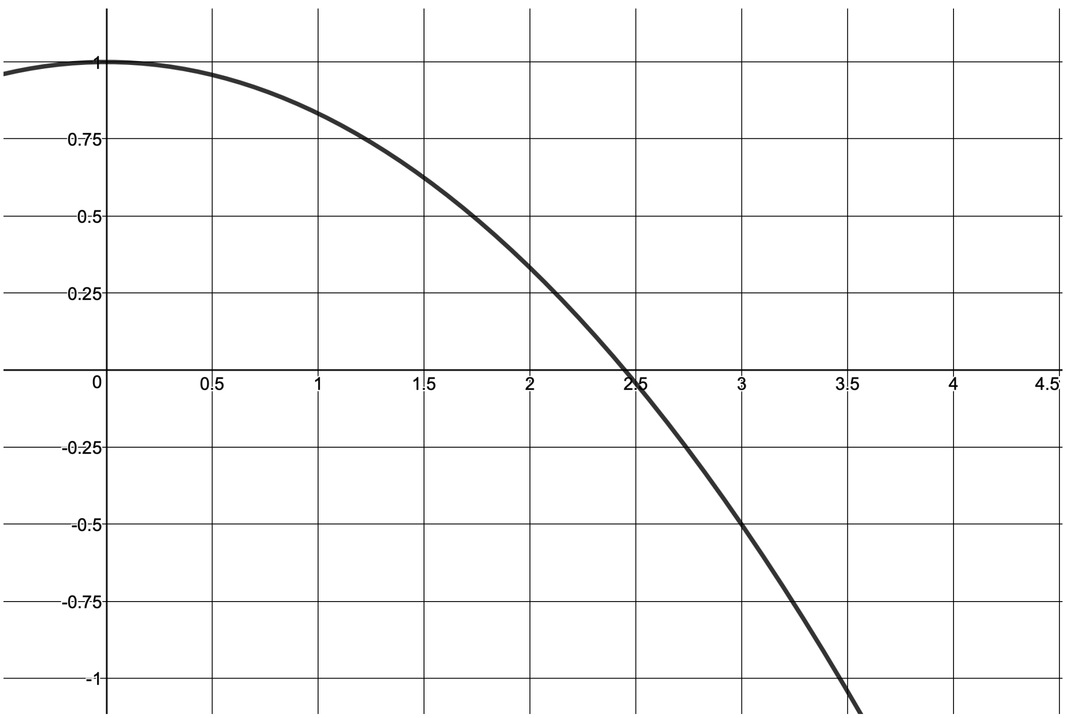
    (c) $\int e^{2x} dx$

13. (14 points) $P(t)$ is the position (in meters) of a car at time t (in seconds). The car's velocity is given by

$$\frac{dP}{dt} = -\frac{1}{6}t^2.$$

a) Label both axes on the graph, below, of $y = \frac{dP}{dt}$.

b) Estimate the displacement of $P(t)$ between times $t = 1.5$ and $t = 3$ using a left-hand Riemann Sum with $\Delta t = 0.5$. Draw your rectangles or step functions on the graph below. In other words, pretend that the rate of change $P'(t)$ is constant on intervals of size $\Delta t = 0.5$ and use those approximate rates of change to find the displacement of $P(t)$. Pretend that whatever the rate of change is on the left-side of the interval is the rate of change for the entire interval.



c) Find the exact value of the displacement in car's position $P(t)$ between $t = 1.5$ and $t = 3$ using integration. You do not have to show work on this problem, but may get partial credit if you show work and have wrong answer.

14. (16 points) Let $P(t)$ equal the position (in meters) of a toy car at time t (in seconds). Suppose that

$$\frac{dP}{dt} = \sin(2t).$$

(a) Use a definite integral to determine the displacement in the position of the car between times $t = \frac{\pi}{4}$ and $t = 3\pi$. Show work. You will only get partial credit if you only write down the final answer you find with your calculator.

(b) Determine $P(t)$ if $P(0) = 5$. (That is, find a solution to the differential equation $\frac{dP}{dt} = \sin(2t)$ with initial condition $P(0) = 5$.)

15. (14 points) A population p_t of insects obeys the discrete-time dynamical system

$$p_{t+1} = 1.3p_t.$$

(a) Write down the solution to this discrete-time dynamical system if $p_0 = 454$. (In other words, find a function that gives population for any time t.)

(b) If $p_0 = 454$, at what time will the population reach size 1000?

(c) Check your answer to part b using any method. If you use a graphing calculator you have to specify what you typed in and how you interpreted your answer.

16. (12 points) Dr. Byerley is fostering 8 kittens and is in charge of finding out how many calories each kitten eats per day. Taste of the Wild cat food has 27 calories per cubic inch of food. Dr. Byerley fed the kittens 4 cups early Wednesday morning and measured that 0.8 cups of food was remaining in their bowl at midnight on Wednesday. If each kitten is eating the same amount, how many calories did each individual kitten eat on Wednesday? Note that Taste of the Wild is a real cat food and the food measurements are reasonable for kittens.

Use the following unit conversions:
1 US cup = 236.46 cubic centimeters
1 inch is 2.54 times as large as 1 centimeter

17. (12 points) Olive the cat gets medicine every 24 hours at noon. In each 24 hour period the cat loses 67 % of the medicine in his blood. At noon the cat gets enough medication to increase the concentration of medicine in his bloodstream by 2.5 mg/L. At noon the cat starts with a concentration of medicine in his bloodstream equal to 7 milligrams per liter (mg/L). Let M_t = concentration of medicine after t, 24 hour periods have passed. $M_0 = 7$.

(a) Write down a discrete-time dynamical system, together with an initial condition, that describes this situation.

(b) How much medicine does Olive have in his bloodstream after two 24 hour periods. (If $M_0 = 7$, what is M_2?)

Scratch Paper. You can rip off the paper, but don't take it with you.