Math 155
Final Exam: Version A
Spring 2018

NAME: $\qquad$

SECTION: $\qquad$ TIME: $\qquad$

INSTRUCTOR: $\qquad$

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. You can ask for scratch paper from a proctor. Turn in any scratch paper that you use during the exam. You will have two hours to work on the exam.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 17 |  |
| 2 | 11 |  |
| 3 | 21 |  |
| 4 | 17 |  |
| 5 | 8 |  |
| 6 | 16 |  |
| 7 | 14 |  |
| 8 | 10 |  |
| MC1 | 6 |  |
| MC2 | 6 |  |
| MC3 | 6 |  |
| MC4 | 6 |  |
| MC5 | 6 |  |
| MC6 | 6 |  |
|  |  |  |
| Bonus1 | 3 |  |
| Bonus2 | 3 |  |
| Bonus3 | 3 |  |
| Total | 150 |  |

## CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person until the exams have been returned to us. You can pick up final exams after grades are posted in the front office of Weber, the math building.

1. (17 points) Suppose that the population $w_{t}$ of birds satisfies the discrete-time dynamical system

$$
w_{t+1}=\frac{2}{7} w_{t}\left(k-w_{t}\right),
$$

where $k>0$ is a positive parameter.
(a) Find all equilibria.
(b) For the equilibrium $w^{*}=0$, use the Stability Theorem/Criterion to determine the values of $k$ for which that equilibrium is stable. Use correct notation in your steps that show how you used the Stability Theorem/Criterion.
2. (11 points)

Consider the function $f(x)=x \mathrm{e}^{-2 x^{2}}$.
Find all values of $x$ where the rate of change of $f(x)$ is zero or undefined. These x values are called critical points of $f(x)$.
3. (21 points) Evaluate the following definite and indefinite integrals. Show all of your work.
(a) $\int_{1}^{3} x^{\frac{1}{2}} d x$
(b) $\int x \sin \left(x^{2}+1\right)+e^{2} d x$
(c) Use integration by parts to evaluate $\int 5 x \mathrm{e}^{2 x} d x$.
4. (17 points) $P(t)$ is the position (in meters) of a car at time $t$ (in seconds). The car's velocity is given by

$$
\frac{d P}{d t}=0.2 t^{2}
$$

a) Label both axes on the graph, below, of $y=\frac{d P}{d t}$.
b) Estimate the displacement of $P(t)$ between times $t=1$ and $t=3.5$ using a left-hand Riemann Sum with $\Delta t=0.5$. Draw your rectangles or step functions on the graph below.

c) Find the exact value of the displacement in car's position $P(t)$ between $t=1$ and $t=3.5$ using integration.
5. (8 points) Peanut the cat starts with a concentration of medicine in his bloodstream equal to 5 milligrams per liter ( $\mathrm{mg} / \mathrm{L}$ ). Each morning the cat has used up $13 \%$ of the medicine in his bloodstream. Each afternoon the cat gets enough medication to increase the concentration of medicine in his bloodstream by $3.5 \mathrm{mg} / \mathrm{L}$. Let $M_{t}=$ concentration of medicine on day $t$.

Write down a discrete-time dynamical system, together with an initial condition, that describes this situation.
6. (16 points) Let $P(t)$ equal the position of a toy car at time $t$ (in seconds). Suppose that

$$
\frac{d P}{d t}=0.7 \sin (4 t+\pi)
$$

(a) Use a definite integral to determine the displacement in the position of the car between times $t=\frac{\pi}{4}$ and $t=3 \pi$. Show work. You will only get partial credit for finding the final answer with your calculator if there is not work.
(b) Determine $P(t)$ if $P(0)=7$. (That is, find a solution to the differential equation $\frac{d P}{d t}=0.7 \sin (4 t+\pi)$ with initial condition $P(0)=7$.)
7. (14 points) A population $p_{t}$ of mice obeys the discrete-time dynamical system

$$
p_{t+1}=1.4 p_{t} .
$$

(a) Write down the solution to this discrete-time dynamical system if $p_{0}=642$. (In other words, find a function that gives population for any time $t$.)
(b) If $p_{0}=642$, at what time will the population reach size 2000 ?
8. (10 points) Peanut the cat gained too much weight and needs to go on a diet. Peanut's veterinarian says that Peanut should eat 200 calories per day. Taste of the Wild cat food has 27 calories per cubic inch of food. You want to measure Peanut's food with your US measuring cups. How many cups of food should Peanut get each day? You know the following unit conversions:

1 US cup $=236.46$ cubic centimeters
1 inch is 2.54 times as large as 1 centimeter

START OF MULTIPLE CHOICE. Please Bubble Version A on Scantron. Please bubble answers on scantron form starting at problem 1 on scantron. Scantron must have your student ID to be counted for a grade. Each multiple choice problem is worth 6 points.

MC 1. The graph of the mass of an object $y=M(x)$ is given below.


The value of the rate of change of mass at time $x=-1$ is:
A) Positive
B) Negative
C) Undefined
D) Zero
E) Not enough information.

MC 2. The graph below shows the rate of change of the mass of a bacterial culture. Where is the mass of the bacterial culture increasing on the interval $-3<x<2$ ?

A) $\quad$ I. $-3<x<-2$
B) II. $0<x<1$
C) III. $1<x<2$
D) All three intervals: I, II, and III
E) Both I and II

MC 3. Graph of $y=f^{\prime}(x)$


Graph of $y=g^{\prime}(x)$


Graph of $y=h^{\prime}(x)$


Which of the following is true?
A) $\int_{-2}^{2} f^{\prime}(x) d x<\int_{-2}^{2} g^{\prime}(x) d x<\int_{-2}^{2} h^{\prime}(x) d x$
B) $\int_{-2}^{2} g^{\prime}(x) d x<\int_{-2}^{2} h^{\prime}(x)<\int_{-2}^{2} f^{\prime}(x) d x$
C) $\int_{-2}^{2} h^{\prime}(x) d x<\int_{-2}^{2} f^{\prime}(x) d x<\int_{-2}^{2} g^{\prime}(x) d x$
D) $\int_{-2}^{2} f^{\prime}(x) d x<\int_{-2}^{2} h^{\prime}(x) d x<\int_{-2}^{2} g^{\prime}(x) d x$

MC 4. A centimeter is 10 times as long as a millimeter. We measured Mango the cat and found out she is $x$ millimeters long. If we measure her in centimeters how long will she be?
A) $10 x$ centimeters.
B) $\frac{1}{10} x$ centimeters.
C) $10 x$ millimeters.
D) $\frac{1}{10} x$ millimeters.
E) None of the above.

## MC 5.

How many cubic centimeters are equal to 716 cubic inches?
$(2.54 \mathrm{~cm}=1 \mathrm{inch})$
A) (716)(2.54) cubic centimeters
B) $(716)\left(2.54^{3}\right)$ cubic centimeters
C) $\frac{716}{2.54}$ cubic centimeters
D) $\frac{716}{2.54^{3}}$ cubic centimeters
E) $\frac{2.54^{3}}{716}$ cubic centimeters

MC 6. The graph below is $y=f(x)$


Which of the graphs below could be the graph of its derivative, $f^{\prime}(x)$ ?
A)

B)



MC7 Bonus. The graph below shows the rate of change of a population of fish with respect to time.


Where on the interval from $-2 \leq t \leq 1$ is the population the largest? In other words, where does population have a global maximum? (Label axes).
A) $t=-2$
B) $t=-1.8$
C) $t=-1.15$
D) $t=1.6$
E) $t=1$

MC 8 Bonus. Graph of $y=h^{\prime}(t)$.


The graph of a function $h^{\prime}(t)$ is shown above. Let

$$
h(x)=\int_{0}^{x} h^{\prime}(t) d t
$$

Which of the following is true?
A) $h(3)<h^{\prime}(3)<h^{\prime \prime}(3)$
B) $h(3)<h^{\prime \prime}(3)<h^{\prime}(3)$
C) $h^{\prime \prime}(3)<h^{\prime}(3)<h(3)$
D) $h^{\prime \prime}(3)<h(3)<h^{\prime}(3)$
E) $h^{\prime}(3)<h(3)<h^{\prime \prime}(3)$

## MC 9. Bonus

The table below gives information about functions $f$ and $g$. Let $h$ be defined as $h(x)=f(g(x))$. What is the rate of change of $h$ at $x=4$ ?

| $x=$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 20 | 23 | 18 | 14 |
| Rate of change of $f$ at $x$ | 7 | -3 | -7 | -2 |
| $g(x)$ | -10 | -11 | -4 | 2 |
| Rate of change of $g$ at $x$ | -0.5 | 2 | 4 | 3 |

a) -2
b) -3
c) -6
d) -7
e) -9

