NAME: ______________________________

Instructor: ___________________________

Time your class meets: __________________

Math 160 Calculus for Physical Scientists I
Exam 3 - Version 1
November 10, 2016, 5:00-6:50 pm

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____
(Signature)

____
(Date)

Please do not write in this space.

1-5. (15pts)	
6. (9pts)	
7. (8pts)	
8. (12pts)	
9. (20pts)	
10. (6pts)	
11. (15pts)	
12. (15pts)	
TOTAL	

Algebra Mistakes:	
Trigonometry Mistakes:	

For Multiple Choice questions, circle one answer, unless the question indicates otherwise.

1. (3pts) If $f(x)$ is a continuous and differentiable function on its entire domain, and $f'(x) = 0$ only at $x = c$, then $x = c$ is a local max if: **(circle all that apply)**
   (a) $f'(x)$ changes from negative to positive at $x = c$ (as x increases).
   (b) $f'(x)$ changes from positive to negative at $x = c$ (as x increases).
   (c) $f''(x)$ changes from positive to negative at $x = c$ (as x increases).
   (d) $f''(c) < 0$.
   (e) $f''(c) > 0$.

2. (3pts) Suppose that $H'(x)$ is differentiable, increasing, and negative on $[0, 1]$. Then $H(x)$ is
   (a) increasing and concave down on $(0, 1)$.
   (b) decreasing and concave down on $(0, 1)$.
   (c) increasing and concave up on $(0, 1)$.
   (d) decreasing and concave up on $(0, 1)$.
   (e) None of the above

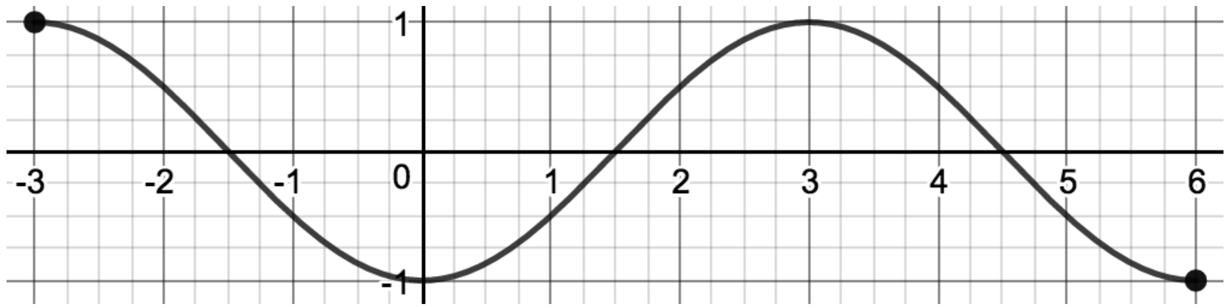
3. (3pts) Given that $\int_1^5 D(x)\,dx = -4$, what is the value of $\int_5^1 (1 - D(x))\,dx$?
   (a) 0
   (b) -8
   (c) 8
   (d) -9
   (e) 9
   (f) Cannot be determined.

4. (3pts) The function $g(x) = x^3 - 3x^2 + 4$ is concave up for all x in the interval(s)
   (a) $(-\infty, 0) \cup (2, \infty)$
   (b) $(0, 2)$
   (c) $(-\infty, 1)$
   (d) $(1, \infty)$

5. (3pts) For the function $p(x) = (x - 1)^4$ on $(-\infty, \infty)$,
   (a) $x = 1$ is a critical point.
   (b) $x = 1$ is an inflection point.
   (c) $x = 1$ is both a critical point and an inflection point.
   (d) None of the above.

6. Use the graph of $f(x)$ below to answer the following.

Note that on $[-3, 6]$, $f(x)$ is continuous.



(a) (2pts) Using right endpoints and $n = 6$ subintervals of equal length, draw the rectangles needed to compute the Riemann sum in the graph above.

(b) (2pts) What is value of Δx?

i. $\Delta x = 6$

ii. $\Delta x = 1$

iii. $\Delta x = 0.5$

iv. $\Delta x = 1.5$

(c) (3pts) Using the rectangles you drew in (a), estimate the value of $\int_{-3}^{6} f(x)\, dx$.

i. 0

ii. $\frac{3}{2}$

iii. $-\frac{3}{2}$

iv. Cannot be determined with the given information.

(d) (2pts) Which of the following expressions will give the exact value of the total area enclosed by the x-axis and $f(x)$? (i.e. total area)

i. $\int_{-3}^{6} f(x)\, dx$

ii. $\left| \int_{-3}^{6} f(x)\, dx \right|$

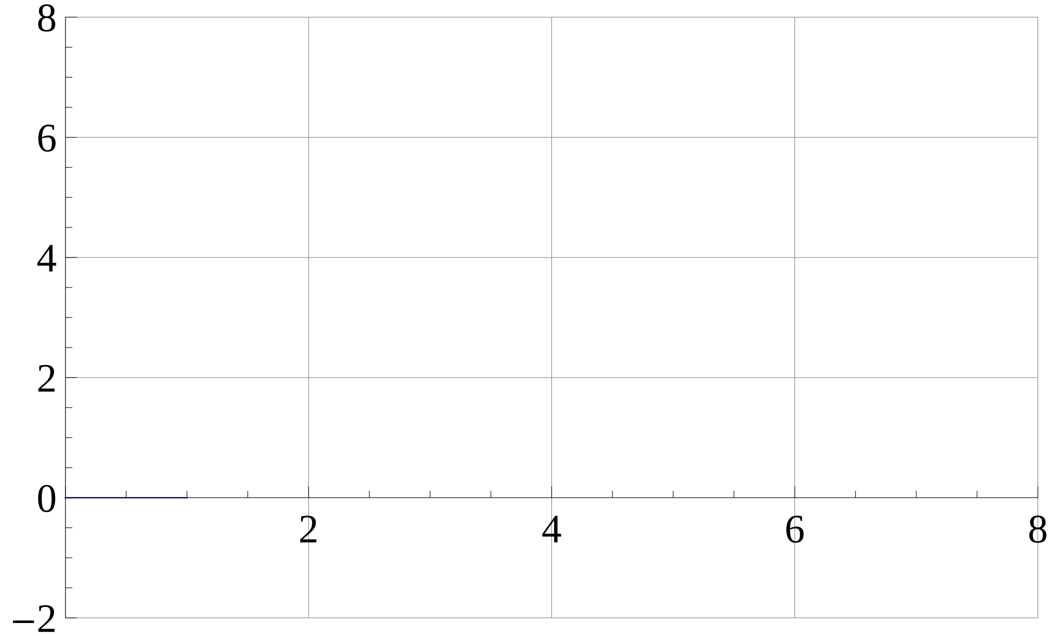
iii. $\int_{-3}^{6} |f(x)|\, dx$

iv. $2 \int_{-3}^{1.5} f(x)\, dx$

v. None of the above.

7. (8pts) Sketch the graph of a function defined on $[0, 8]$ that has all of the following properties:

- $f'(x) > 0$ for $0 < x < 4$
- $f'(x) < 0$ for $x > 4$
- $f''(x) > 0$ for $4 < x < 6$
- $f''(x) < 0$ for $x > 6$
- $f(x)$ is continuous on $0 \leq x < 8$
- $f'(x)$ is undefined at $x = 4$
- $f(4) = 4$
- $f(6) = 2$



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8. (12pts) Find the function that satisfies the following properties:

- the second derivative is $f''(x) = -6x$
- the graph of $f(x)$ has a horizontal tangent line at $x = 1$ and passes through the point $(2, 1)$.

$f(x) =$ ____________________

9. Use $f(x) = 3\sqrt[3]{x} - x$ and *calculus* to answer the following:

(a) (4pts) Find all critical points of f.

(b) (5pts) Identify all intervals on which f is increasing or decreasing.

(c) (5pts) Identify all intervals on which f is concave up or concave down.

(d) (6pts) Fill-in-the-Blank. Identify and classify all local (also called relative) extrema and where they occur:

The local minimum is ______ at $x =$______.

The local maximum is ______ at $x =$______.

There is an inflection point located at (______, ______)

10. (6pts - 3pts each) Indicate whether each of the following statements is **True** or **False**. If the statement is true, explain how you know it is true. If it is false, give a counterexample **and** explain why it is a counterexample. A graph with an explanation in words can be used as a counterexample.

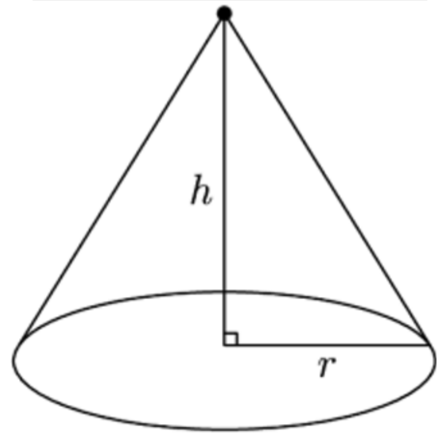
(a) If a is a positive real number, then $\int_{-a}^{a} (|x| - 1)\, dx > 0$.

(b) If a function, $f(x)$, is continuous, positive, and decreasing on $[-1, 4]$, then $\int_{-1}^{4} f(x)\, dx > 0$.

11. (15pts) You are in charge of designing the crew cabin for a proposed mission to Mars. Assume the cabin is shaped like a right circular cone, which has a volume of $V = \frac{\pi}{3}r^2h$. The base of the cabin must have an area no bigger than 49π square feet, else the cabin won't fit on top of the liquid fuel rocket boosters. Also, the cabin must have a volume of exactly 490π cubic feet, so that there is enough oxygen in the cabin for the astronauts. Our goal will be to minimize the height of the cone, so that the shuttle is as compact as possible.

(a) Write an equation for the height, h, of the cone in terms of the radius (r):

Picture of cone-shaped cabin:



$h(r) =$ ____________________

State the domain of the function:

__________ $< r \leq$ __________

(b) Find the derivative of your function $h(r)$, and use it to find all of the critical points of $h(r)$. (If there are none, explain).

(c) What value of the radius, r, will result in the minimum height of the cabin? Explain how you know. Be sure to use either the first or second derivative test as part your explanation.

(d) What is the minimum height, h, of the cabin?

Minimum height: $h =$ ____________________

12. (15pts - 5pts each) Evaluate the following integrals. Answers must be accompanied by supporting work that shows how you evaluated the integral - this may include a geometric argument. Do not expect partial credit for incorrect answers or answers with no supporting work.

(a) $\displaystyle\int \left(\frac{1}{x^3} + \sec^2(x) - \pi\right) dx$

(b) $\displaystyle\int \left(t^2 \cdot \sqrt{t}\right) dt$

(c) $\displaystyle\int_{-2}^{0} \sqrt{4 - x^2}\, dx$