NAME: _____

Instructor: _____

Time your class meets: _____

Math 160 Calculus for Physical Scientists I
Exam 3 - Version 1
April 14, 2016, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE

I HAVE NOT GIVEN, RECEIVED, OR USED ANY UNAUTHORIZED ASSISTANCE ON THIS EXAM. FURTHERMORE, I AGREE THAT I WILL NOT SHARE ANY INFORMATION ABOUT THE QUESTIONS ON THIS EXAM WITH ANY OTHER STUDENT BEFORE GRADED EXAMS ARE RETURNED.

_____

(SIGNATURE)

_____

(DATE)

Please do not write in this space.

1-10. (31pts)	
11-13. (9pts)	
14. (9pts)	
15. (20pts)	
16. (11pts)	
17. (10pts)	
18. (10pts)	
TOTAL	

Algebra Mistakes:	
Trigonometry Mistakes:	

Multiple Choice (31pts): for #1-10. Circle only one answer for each problem unless it indicates otherwise.

1. (3pts) Given that $\int_0^4 S(x)\,dx = \pi$, determine the value of $\int_4^0 (\sqrt{16-x^2} - S(x))\,dx$.

   (a) 3π

   (b) -3π

   (c) 5π

   (d) -5π

2. (3pts) Evaluate $\int_{-1}^0 (4x+2)\,dx$.

   (a) -1.5

   (b) 1.5

   (c) 3

   (d) 1

   (e) 0

3. (4pts) Circle the correct response for each.

   (a) If $f(x)$ is an **increasing** function, then approximating area using **right** endpoints will give an OVER / UNDER (CIRCLE ONE) estimate of the area under the curve.

   (b) If $f(x)$ is an **increasing** function, then approximating area using **left** endpoints will give an OVER / UNDER (CIRCLE ONE) estimate of the area under the curve.

   (c) If $f(x)$ is an **decreasing** function, then approximating area using **right** endpoints will give an OVER / UNDER (CIRCLE ONE) estimate of the area under the curve.

   (d) If $f(x)$ is an **decreasing** function, then approximating area using **left** endpoints will give an OVER / UNDER (CIRCLE ONE) estimate of the area under the curve.

4. (3pts) The function $f(x) = 3x^2 - 2x^3$ is concave down for all x in the interval

   (a) $(\frac{1}{2}, \infty)$
   (b) $(-\infty, \frac{1}{2})$
   (c) $(\frac{1}{4}, \infty)$
   (d) $(0, \frac{3}{2})$
   (e) None of the above.

5. (3pts) For the function $f(x) = (x+2)^4$ on $(-\infty, \infty)$,

   (a) $x = -2$ is both a critical point and an inflection point.
   (b) $x = -2$ is a critical point.
   (c) $x = -2$ is an inflection point.
   (d) None of the above.

6. (3pts) Let $f(x)$ be a differentiable function on the interval $[0, 5]$. Then **(circle all correct responses)**

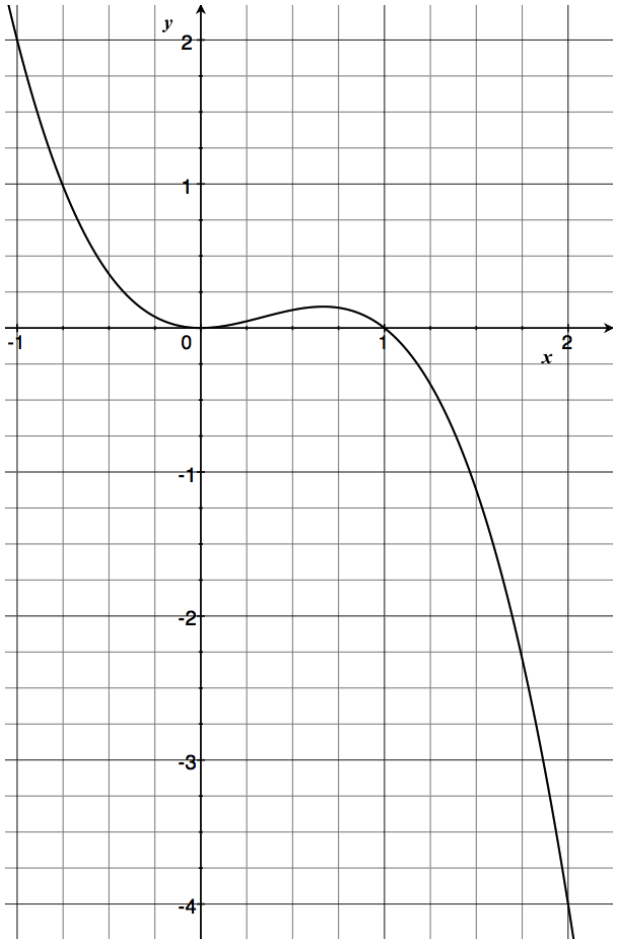
   (a) $f(x)$ will attain at least one local maximum and at least one local minimum on $[0, 5]$.
   (b) $f(x)$ will attain an absolute maximum and an absolute minimum on $[0, 5]$.
   (c) $f(x)$ is continuous on $[0, 5]$.
   (d) $f(x)$ will have an inflection point in $(0, 5)$.

7. (3pts) Suppose that the function $f(t)$ is continuous and always negative. If $G(t)$ is an antiderivative of $f(t)$, then we know that $G(t)$.

   (a) is continuous and always negative.
   (b) is continuous and always decreasing.
   (c) is continuous always concave down.
   (d) None of the above responses are correct.

Use $f(x) = x^2 - x^3$ on $[-1, 2]$ to answer the #8-9. The graph of $f(x)$ is provided below.

8. (3pts) Estimate the value of $\int_{-1}^{2} (x^2 - x^3)\, dx$ using $n = 6$ subintervals and left endpoints.



   (a) -2.3125
   (b) 0.25
   (c) 0.6875
   (d) 0.75
   (e) 1.8125
   (f) 2.8125
   (g) None of the above.

9. (3pts) Which of the following expressions will give the exact value of the total area enclosed by the x-axis and $f(x)$?

   (a) $\int_{-1}^{2} (x^2 - x^3)\, dx$

   (b) $\left| \int_{-1}^{2} (x^2 - x^3)\, dx \right|$

   (c) $\int_{-1}^{1} (x^2 - x^3)\, dx + \left| \int_{1}^{2} (x^2 - x^3)\, dx \right|$

   (d) $\left| \int_{-1}^{0} (x^2 - x^3)\, dx \right| + \int_{0}^{2} (x^2 - x^3)\, dx$

   (e) None of the above.

10. (3pts) Suppose that $H'(x)$ is continuous, increasing, and positive on $[0, 1]$. Then $H(x)$ is

    (a) increasing and concave down on $(0, 1)$.

    (b) decreasing and concave down on $(0, 1)$.

    (c) increasing and concave up on $(0, 1)$.

    (d) decreasing and concave up on $(0, 1)$.

    (e) None of the above.

11. (3pts) True/False (circle one):

    $f_1(x) = 2x + 3$ and $f_2(x) = 2x + 5$ are both antiderivatives of $F(x) = x^2$.

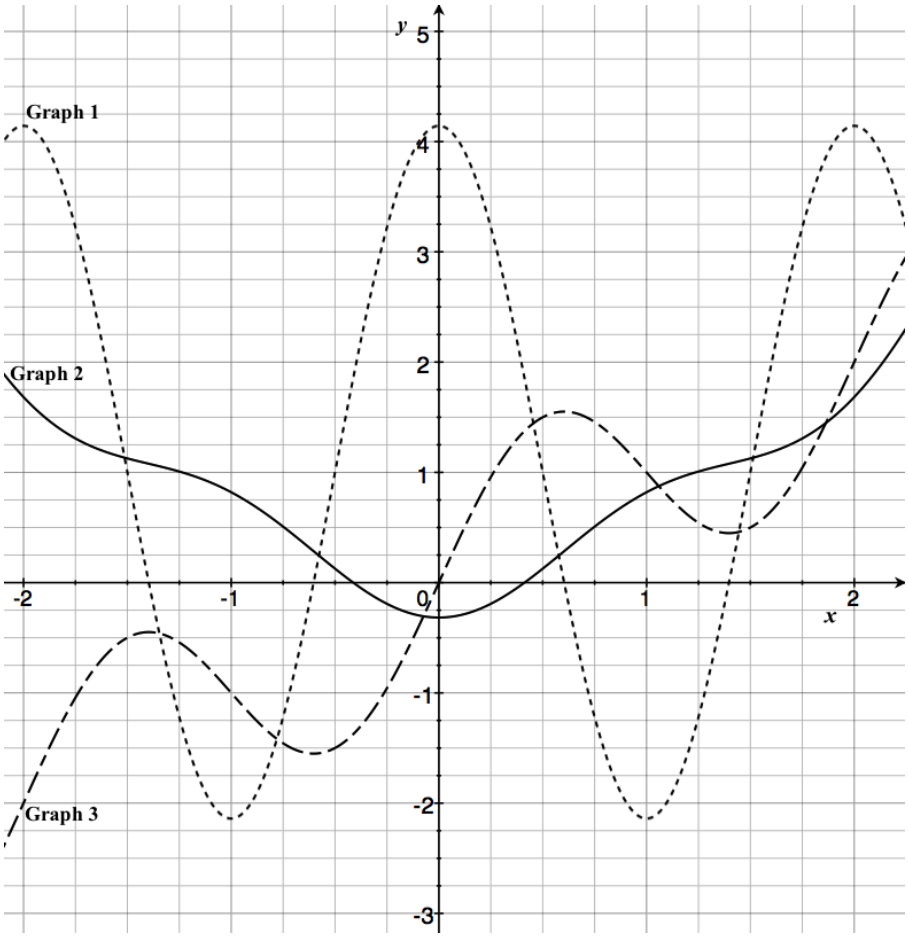
12. (3pts) True/False (circle one):

    Both $-\dfrac{1}{x+1}$ and $\dfrac{x}{x+1}$ are in the family of functions $\displaystyle\int \frac{1}{(x+1)^2}\,dx$.

13. (3pts) True/False (circle one):

    If $f''(5) < 0$, then $f(x)$ must have a local maximum at $x = 5$.

14. (9pts) Matching: Below are the graphs of function, $f(x)$, its derivative, $f'(x)$, and its second derivative, $f''(x)$. Which graph is $f(x)$? Which graph is $f'(x)$? Which graph is $f''(x)$?



Graph 1 = function $f(x)$, first derivative $f'(x)$, second derivative $f''(x)$ (CIRCLE ONE)

Graph 2 = function $f(x)$, first derivative $f'(x)$, second derivative $f''(x)$ (CIRCLE ONE)

Graph 3 = function $f(x)$, first derivative $f'(x)$, second derivative $f''(x)$ (CIRCLE ONE)

15. (20pts) The rate, P, (in mg of carbon per m^3 per hour) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{100I}{I^2 + I + 4}$$

Where I is the light intensity (**measured in thousands of foot-candles**).

Note that I cannot be negative.

For what light intensity is P a maximum? Use Calculus and either the first or second derivative test to support your work.

I used the first / second (circle one) derivative test to show that

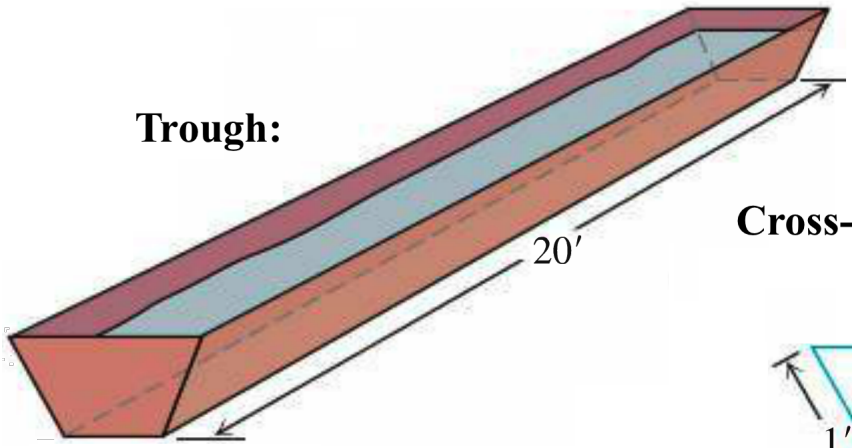
$I =$ _____ thousand foot-candles will result in a

maximum value of $P =$ _____.

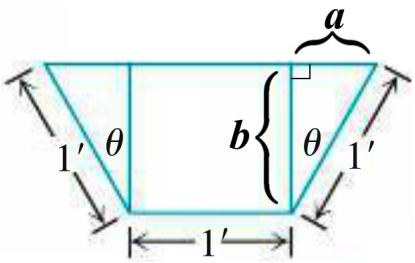
16. *Optimization Set up:* (11pts) The water trough in the figure below is made to be the dimensions shown in the figure. Only the angle θ can be varied. Both a and b are positive. (Note that $1' = 1$ foot)

[Retrieved from Thomas' Calculus. CSU Special Edition. p.223]

Trough:



Cross-Section of Trough:



(a) What is the area of the cross-section in terms of a and b?

area =_____

(b) What is the area in terms of the angle θ?

$A(\theta)$ =_____

(c) What is the domain of θ (i.e. for the context of this problem, what is the set of values allowed for θ)?

17. (10pts) Find the function that satisfies the following properties:

- the derivative is $f'(x) = \dfrac{1}{\sqrt{x}} + 3\pi\cos(\pi x)$
- the graph of $f(x)$ passes through the point $(4, 1)$

$f(x) =$_____

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18. (10pts - 5pts each) Find the most general anti-derivative for the following. Answers must be accompanied by supporting work. Do not expect partial credit for incorrect answers or answers with no supporting work.

(a) $\displaystyle\int \left(2x^6 + \frac{3}{x^2} + \sec(x)\tan(x) + \cos(2x) - 1\right) dx$

(b) $\displaystyle\int \left(t^3 \cdot \sqrt{t}\right) dt$