NAME: ________________________________

Instructor: ______________________________

Time your class meets: ___________________

Math 160 Calculus for Physical Scientists I

Exam 2 - Version 1

October 13, 2016, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____
(Signature)

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(Date)

Please do not write in this space.

1-7. (21pts)	
8. (22pts)	
9. (3pts)	
10. (12pts)	
11. (12pts)	
12. (8pts)	
13. (10pts)	
14. (12pts)	
TOTAL	

Algebra Mistakes:	
Trigonometry Mistakes:	

Problems 1-6 are Multiple Choice: 21pts

1. (3pts) What is the derivative of $y = x^2 \tan(3x)$?

   (a) $\dfrac{dy}{dx} = 2x \sec^2(3x)$

   (b) $\dfrac{dy}{dx} = 3x^2 \sec^2(3x) + 2x \tan(3x)$

   (c) $\dfrac{dy}{dx} = 3x^2 \sec^2(3x) + 6x \tan(3x)$

   (d) $\dfrac{dy}{dx} = 2x + 3\sec^2(3x)$

   (e) All of the above.

   (f) None of the above.

2. (3pts) What is the derivative of $f(x) = 6\sqrt[3]{\cos x}$?

   (a) $f'(x) = \dfrac{-6 \sin x}{3(\cos x)^{2/3}}$

   (b) $f'(x) = -2 \sin x (\cos x)^{-2/3}$

   (c) $f'(x) = -\dfrac{2 \sin x}{\sqrt[3]{(\cos x)^2}}$

   (d) $f'(x) = -\dfrac{2 \sin x}{(\cos x)^{2/3}}$

   (e) All of the above.

   (f) None of the above.

3. (3pts) The equation of the line tangent to the function $f(x) = 2$ is

   (a) $x = 0$.
   (b) $y = 2$.
   (c) $y = 0$.
   (d) None of the above.

4. (3pts) Suppose the position of a particle is given by $s(t) = -t^3 + \sin(\pi t)$.
What is the velocity of the particle at time $t = 1$?
   (a) -4
   (b) 2
   (c) $3 - \pi$
   (d) $-3 - \pi$
   (e) None of the above

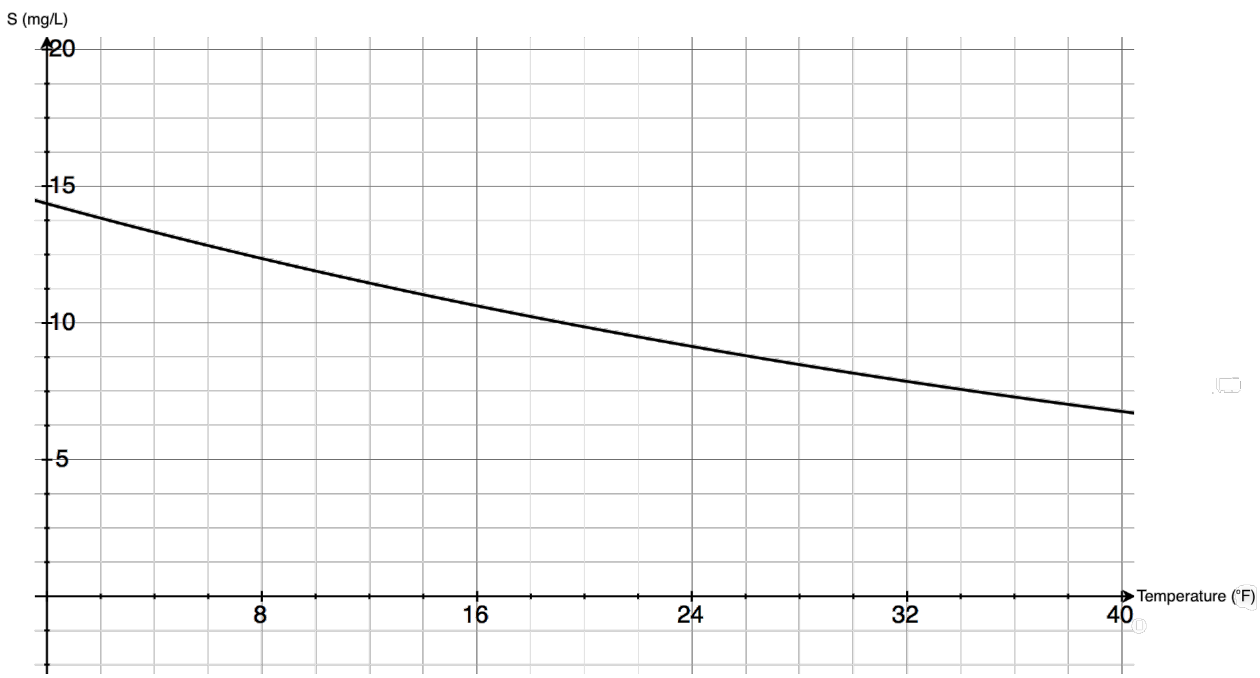
5. (3pts) The result of the limit $\lim_{h \to 0} \dfrac{5(2+h)^3 - 5(2)^3}{h}$ is the slope of the line tangent to the function

   (a) $f(x) = 5(2+h)^3$ (b) $f(x) = 5h^3$ (c) $f(x) = 5x^3$ (d) 40

   at the ordered pair

   (a) 2 (b) $(2, 2+h)$ (c) $(2+h, 2)$ (d) $(2, 40)$

The quantity of oxygen that can dissolve in water depends on the the temperature of the water (so thermal pollution influences the oxygen content of water). The graph below shows how oxygen solubility S (measured in $\frac{\text{mg}}{\text{L}}$) varies as a function of the water temperature T (measured in $^\circ F$).



6. (3pts) What is the meaning of the derivative $S'(T)$?
   (a) The rate at which the water temperature changes with respect to time.
   (b) The rate at which the solubility changes with respect to time.
   (c) The rate at which the water temperature changes with respect to the solubility.
   (d) The rate at which the solubility changes with respect to the water temperature.

7. (3pts) Determine all the critical points of the function $f(t) = t - 3\sqrt[3]{t}$.

(circle all correct responses)

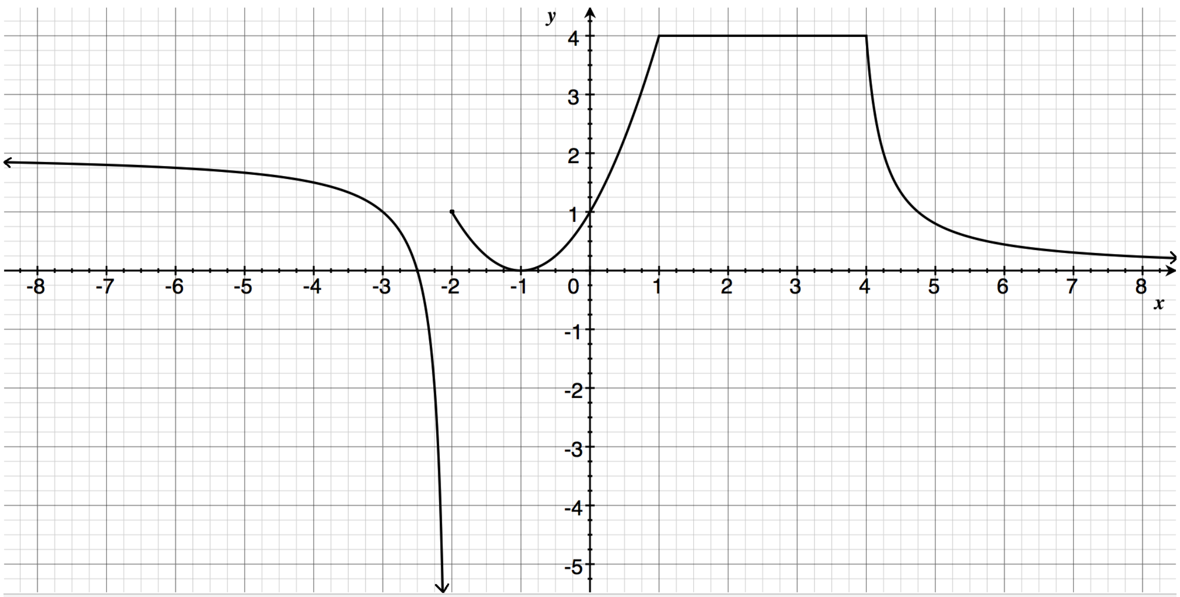
(a) $t = -1$

(b) $t = 1$

(c) $t = 0$

(d) None of the above.

8. (22pts) Consider the graph of a function, $f(x)$, below:



Of all the properties listed below, **circle all** the properties that are reflected in the graph of $f(x)$.

(a) $f(x)$ is not continuous at $x = -1$

(b) $f(x)$ is not continuous at $x = -2$

(c) $f(x)$ is not differentiable at $x = -2$

(d) $f(x)$ is not differentiable at $x = -1$

(e) $f(x)$ is not differentiable at $x = 1$

(f) $f(x)$ is not differentiable at $x = 4$

(g) $f'(x) < 0$ for $x < -2$

(h) $f'(x) < 0$ for $-2 < x < -1$

(i) $f'(x) < 0$ for $-1 < x < 1$

(j) $f'(x) < 0$ for $x > 4$

(k) $f'(x) > 0$ for $x < -2.5$

(l) $f'(x) > 0$ for $-2 < x < -1$

(m) $f'(x) > 0$ for $-1 < x < 1$

(n) $f'(x) > 0$ for $x > 4$

(o) $f'(x) = 0$ at $x = -2$

(p) $f'(x) = 0$ at $x = -1$

The absolute maximum of $f(x)$ is ____________. (if $f(x)$ does not have an absolute maximum value, draw ☺ in the blank)

The absolute minimum of $f(x)$ is ____________. (if $f(x)$ does not have an absolute minimum value, draw ☺ in the blank)

9. (3pts) Suppose that $f(x)$ is a continuous function. Given the limits

$$\lim_{h \to 0^-} \frac{f(-4+h) - f(-4)}{h} = 2 \quad \text{and} \quad \lim_{h \to 0^+} \frac{f(-4+h) - f(-4)}{h} = -1$$

which of the following statements is true?

(a) $f'(-4)$ exists.

(b) $f'(-4)$ is undefined because $f(x)$ has a vertical tangent line at $x = -4$.

(c) $f'(-4)$ is undefined because $f(x)$ forms a corner at $x = -4$.

(d) $f'(x)$ is continuous at $x = -4$.

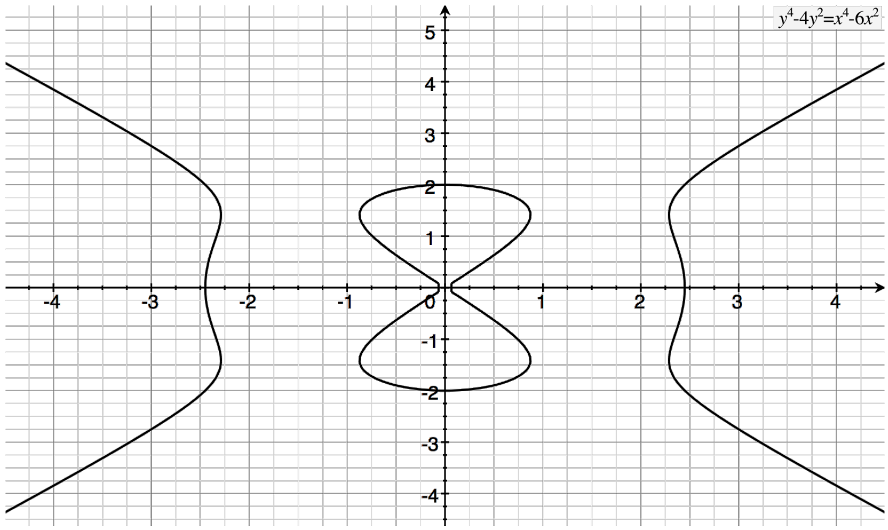
10. (12pts) Use the information provided in the table below to answer parts (a) - (b)

	$r(x)$	$r'(x)$	$s(x)$	$s'(x)$
$x = 2$	4	-1	1	3
$x = 4$	-10	7	2	-3

(a) Given that $F(x) = \dfrac{r(x)}{s(x)}$, find $F'(2)$.

(b) Given that $G(x) = \sqrt{r(x)}$, find $G'(2)$.

11. (12pts) Use the implicitly defined equation $y^4 - 4y^2 = x^4 - 6x^2$, to answer the following questions. The graph of the equation is provided below.



(a) Algebraically verify that the point $(0, -2)$ is on the graph of the curve.

(b) On the graph above, draw the tangent line to the curve at the point $(0, -2)$. The slope at this point is positive / negative / zero (circle one)

(c) Use implicit differentiation to find $\frac{dy}{dx}$.

(d) Use your result from part (c) to find the slope of the curve at the point $(0, -2)$.

(e) What is the equation of the line tangent to the curve at the point $(0, -2)$?

12. (8pts) Derek drove 140 miles in 2 hours to get to Sam's house. The speed limit was 65 miles per hour.

When Derek arrived at Sam's house, Derek stated:

"I had no way of measuring my speed, but I do not think that I was driving 70 miles per hour at any point in time."

Was Derek driving 70 miles per hour at any point in time? Explain your reasoning. Be sure to state any relevant definitions and theorems. For any theorems used, be sure to state why such theorems can be applied.

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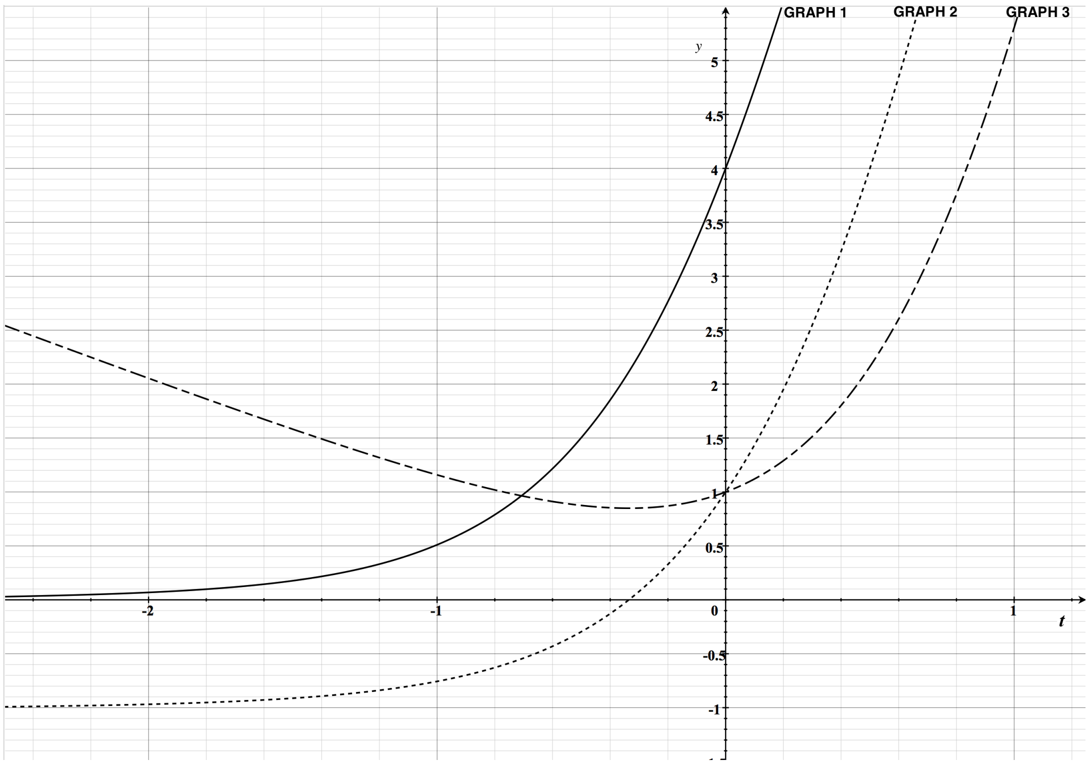
13. (10pts) Determine whether the statements are true or false. If a statement is true, explain why it is true. If it is false, provide a counter example and explain why it is a counterexample.

(a) If a function, $f(x)$, is defined for all x in its domain, then $f'(x)$ is defined for all x in the domain of $f(x)$.

(b) If the position of an object is always positive, then the velocity of that object must also be positive.

14. (12pts) Below are the graphs of a position function, $s(t)$, a velocity function, $v(t)$, and an acceleration function, $a(t)$, with respect to time, t.



Which graph is position? Which graph is velocity? Which graph is acceleration? Give reasons for your answers in sentences. Your explanation should include a discussion of slope with regard to each graph.

Graph 1 = <u>position $s(t)$, velocity $v(t)$, acceleration $a(t)$</u> (CIRCLE ONE)

Graph 2 = <u>position $s(t)$, velocity $v(t)$, acceleration $a(t)$</u> (CIRCLE ONE)

Graph 3 = <u>position $s(t)$, velocity $v(t)$, acceleration $a(t)$</u> (CIRCLE ONE)