NAME: Key for Calculus Center

Instructor: ______________________

Time your class meets: ______________

Math 160 Calculus for Physical Scientists I
Exam 1 - Version 1
September 15, 2016, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

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(Signature)

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(Date)

Please do not write in this space.

1-15. (39pts)	
16. (11pts)	
17. (24pts)	
18. (10pts)	
19. (16pts)	
TOTAL	

Algebra Mistakes:	
Trigonometry Mistakes:	

Problems 1-15 are Multiple Choice: 39pts

1. (2pts) The value of $\lim_{x \to 0} 5$ is

   (a) 0.

   (b) 5.

   (c) ∞.

   (d) Does not exist.

2. (2pts) The value of $\lim_{\theta \to 0} \dfrac{\theta}{\cos(\theta)}$ is

   (a) -1.

   (b) 0.

   (c) 1.

   (d) ∞.

   (e) Does not exist.

3. (2pts) The value of $\lim_{x \to 3^-} \dfrac{x^2 - 9}{|x - 3|}$ is

   Hint: rewrite as a piecewise function before solving the limit.

   (a) 6.

   (b) -6.

   (c) 0.

   (d) ∞.

   (e) Does not exist.

4. (2pts) The value of $\lim_{t \to \infty} \sin(t)$ is

   (a) $-\infty$.

   (b) ∞.

   (c) 0.

   (d) -1 and 1.

   (e) Does not exist.

5. (3pts) Which of the following statements is true about horizontal asymptotes?
(circle only one correct answer)

(a) The graph of a function can have at most two horizontal asymptotes.

(b) The graph of a function can have at most one horizontal asymptote.

(c) The graph of a function can have infinitely many horizontal asymptotes.

(d) Graphs of functions cannot have horizontal asymptotes.

Defn. of Horizontal Asymptote

6. (3pts) Suppose that $\lim_{x\to 5^-} h(x) = -\infty$, $\lim_{x\to 5^+} h(x) = 2$, and $h(5)$ does not exist. Then, using the mathematical definition of vertical asymptote, which of the following statements is true?
(circle only one correct answer)

(a) $x = 5$ is not a vertical asymptote of the graph of $h(x)$ because $\lim_{x\to 5^+} h(x) = 2$.

(b) $x = 5$ is a vertical asymptote of the graph of $h(x)$ because $h(5)$ does not exist.

(c) $x = 5$ is a vertical asymptote of the graph of $h(x)$ because $\lim_{x\to 5^-} h(x) = -\infty$.

(d) $x = 5$ is a vertical asymptote of the graph of $h(x)$ because $\lim_{x\to 5^-} h(x) \neq \lim_{x\to 5^+} h(x)$.

(e) $x = 5$ is not vertical asymptote of the graph of $h(x)$ because $\lim_{x\to 5^-} h(x) \neq \lim_{x\to 5^+} h(x)$.

Defn. of vertical asymptote

7. (3pts) The graph of $f(x) = \dfrac{|x|}{x^2+1}$ has

(a) no horizontal asymptotes.

(b) a horizontal asymptote at $y = 1$.

(c) a horizontal asymptote at $y = 0$.

8. (3pts) Which of the following statements are true about the function $f(x) = \dfrac{|x|}{x^2+1}$?

(a) $f(x)$ is continuous except at $x = -1$ and $x = 1$.

(b) $f(x)$ is continuous except at $x = -1$.

(c) $f(x)$ is continuous except at $x = 0$.

(d) $f(x)$ is continuous for all real numbers.

Use $p(x) = \begin{cases} \sin(\pi x), & x < 0 \\ \dfrac{x^2 - 1}{1 - x}, & x > 0 \end{cases}$ to answer questions 9 and 10.

9. (3pts) At $x = 0$, $p(x)$

   (a) is continuous.

   (b) has a jump discontinuity because $\lim_{x \to 0^-} p(x)$ exists and $\lim_{x \to 0^+} p(x)$ exists, but $\lim_{x \to 0^-} p(x) \neq \lim_{x \to 0^+} p(x)$.

   (c) has a removable discontinuity because $\lim_{x \to 0} p(x)$ exists, but $p(0)$ does not exist.

   (d) None of the above.

10. (3pts) At $x = 1$, $p(x)$

   (a) is continuous.

   (b) has a jump discontinuity because $\lim_{x \to 1^-} p(x)$ exists and $\lim_{x \to 1^+} p(x)$ exists, but $\lim_{x \to 1^-} p(x) \neq \lim_{x \to 1^+} p(x)$.

   (c) has a removable discontinuity because $\lim_{x \to 1} p(x)$ exists, but $p(1)$ does not exist.

   (d) None of the above.

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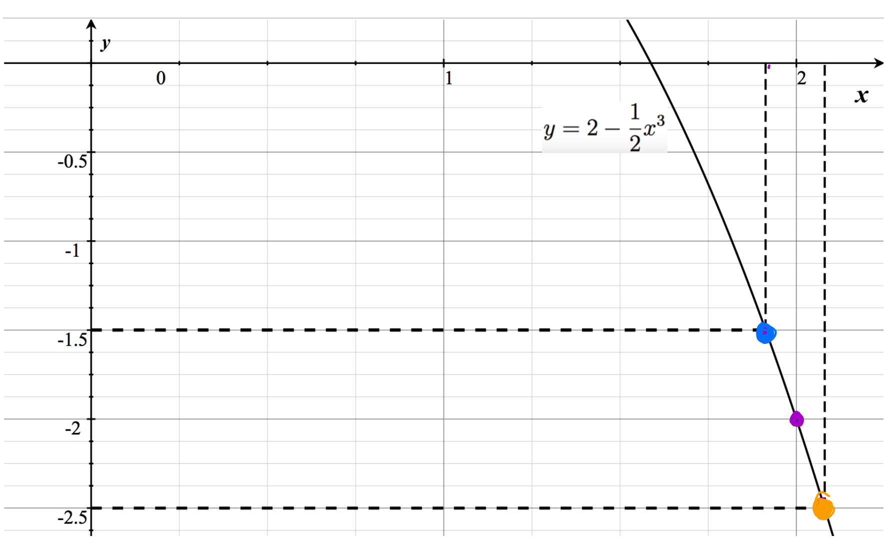
11. (3pts) Consider the function $y = 2 - \frac{1}{2}x^3$, whose graph is provided below.

   We can see that

$$\lim_{x \to 2} \left(2 - \frac{1}{2}x^3 \right) = -2$$

   Suppose we will allow a tolerance of 0.5 within $L = -2$ for the function values (i.e. $\epsilon = 0.5$). What is the **maximum** amount of error that can occur on either side of $x_0 = 2$ so that the function values still lie within 0.5 of L? (i.e. find the maximum δ).

   (a) 0.125. (b) 0.087. (c) 0.080. (d) 0.250.



Use the function, $f(x)$, below to answer questions 12-15.

$$f(x) = \begin{cases} Cx+1, & x < -1 \\ D & x = -1 \\ \dfrac{x^2-2x-3}{x+1}, & x > -1 \end{cases}$$

12. (2pts) The value of $\lim_{x \to -1^-} f(x)$ is

    (your answer may be in terms of C or D).

    (a) $C + 1$.
    (b) $-C + 1$.
    (c) D.
    (d) $-D$.
    (e) cannot be determined.

13. (2pts) The value of $\lim_{x \to -1^+} f(x)$ is

    (a) -4.
    (b) -2.
    (c) 2.
    (d) -1.
    (e) 5.
    (f) cannot be determined.

14. (3pts) The value C must be in order for $f(x)$ to be continuous at $x = -1$ is

    (a) -4.
    (b) -3.
    (c) -1.
    (d) 5.
    (e) cannot be determined.

15. (3pts) The value D must be in order for $f(x)$ to be continuous at $x = -1$ is

    (a) -4.
    (b) -3.
    (c) -1.
    (d) 5.
    (e) cannot be determined.

16. (11pts) Farmer Catherine wants to construct a square animal pen (also called an enclosure or corral).

(a) If a side length is represented by x, write the function, $A(x)$, that represents the area of the animal pen:

$A(x) = \underline{x^2}$

(b) Farmer Catherine wants to enclose an area of 100 ft^2 to hold her baby goats. What is the "perfect" side length that achieves this area?

units!

$x = \underline{10 \text{ ft}}$

(c) Farmer Catherine's measurements are not perfect, but she is able to enclose an area within 0.5 ft^2 of the perfect area 100 ft^2 (i.e. $\epsilon = 0.5$). What corresponding side lengths satisfy this range in area? Write any decimals to **4** places.

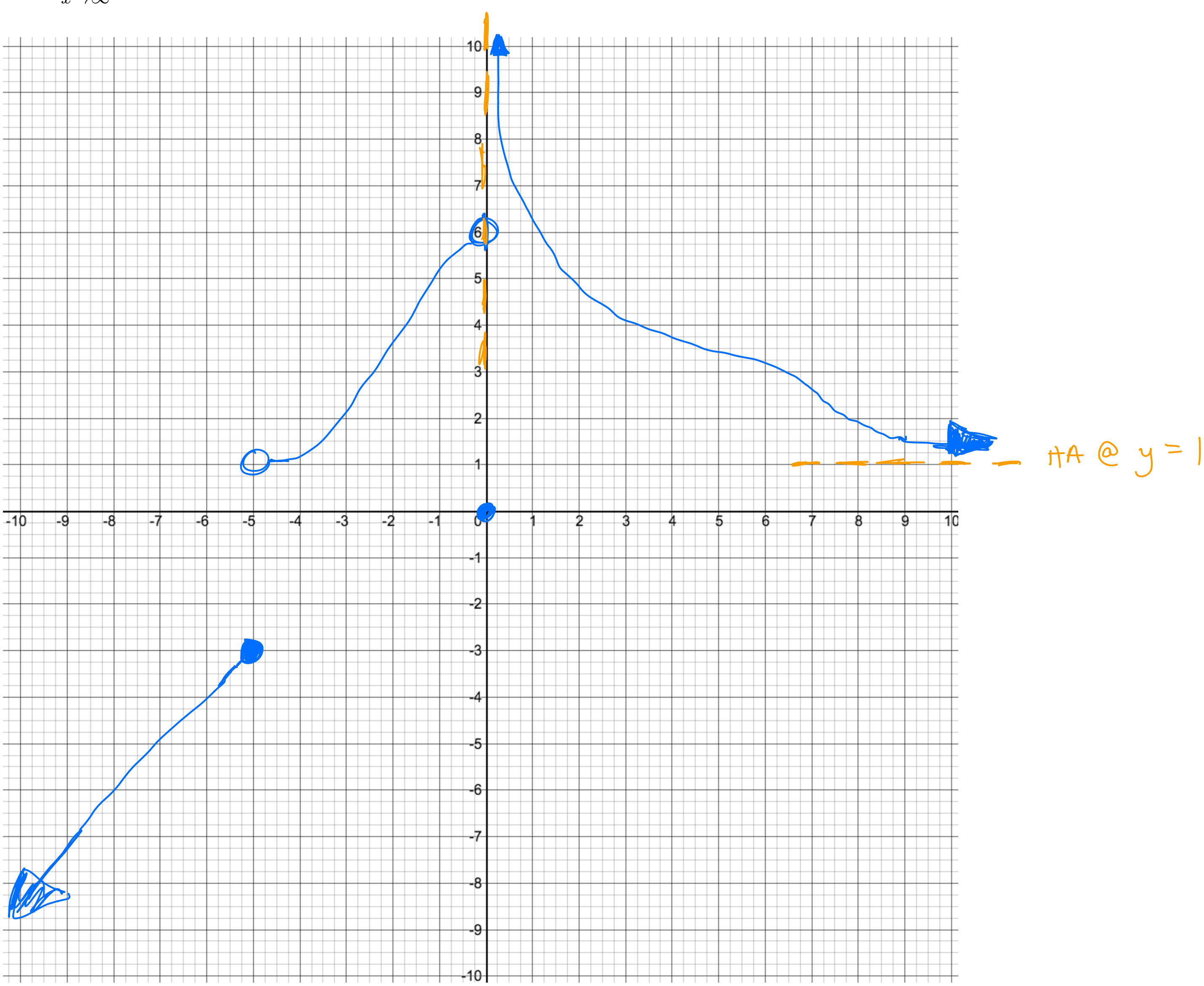
$x = 9.974968$ $\qquad$ $x = 10.024968$

Round lower value up (in) $\qquad$ Round upper # down (in)

$\underline{9.9750} < x < \underline{10.0249}$

17. (24pts) The graph of a **function**, $f(x)$, that has the following properties. Sketch the graph of $f(x)$ and then answer the following questions about $f(x)$.

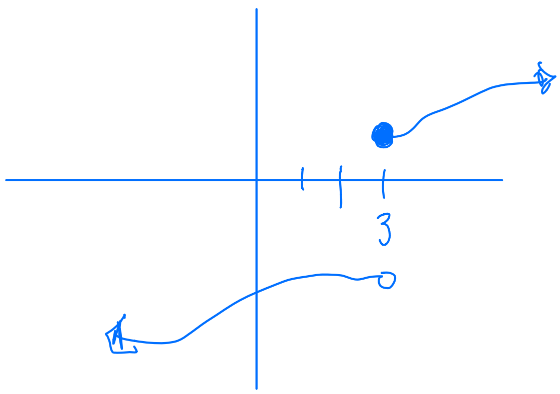
- $f(x)$ is continuous for $x < -5$
- $f(x)$ is continuous for $x > 0$
- $\lim_{x \to -\infty} f(x) = -\infty$
- $\lim_{x \to -5^-} f(x) = -3$
- $f(-5) = -3$
- $\lim_{x \to -5^+} f(x) = 1$
- $\lim_{x \to 0^-} f(x) = 6$
- $f(0) = 0$
- $\lim_{x \to 0^+} f(x) = \infty$
- $\lim_{x \to \infty} f(x) = 1$



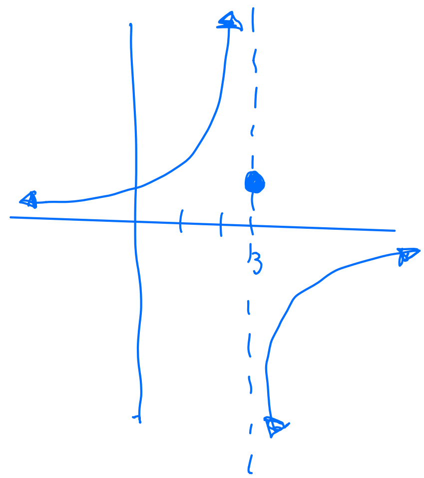
18. (10pts) Each of the statements below is <u>false</u>. For each of the false statements below, provide a counterexample and explain why, in sentences, it is a counterexample.

(a) If $g(3)$ is defined, then $g(x)$ is continuous at $x = 3$.

False

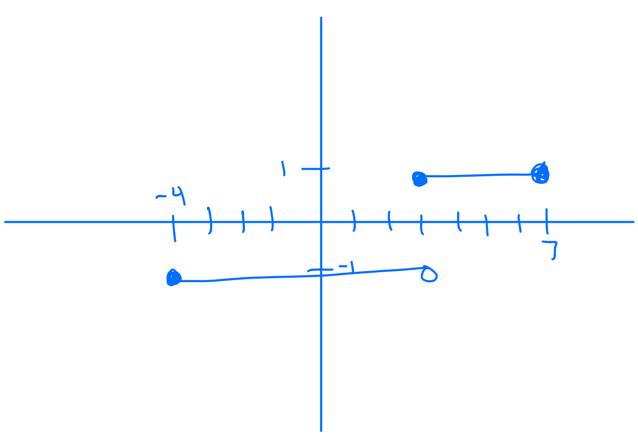


or



(b) Suppose that $h(t)$ is defined on $[-4, 7]$ and $h(-4) = -1$ and $h(7) = 1$. Then there is some c in the interval $(-4, 7)$ such that $h(c) = 0$.

False



Although $[-4, 7]$ is defined, $h(-4) = -1$ and $h(7) = 1$ the graph never touches the x-axis is $(h(c) \neq 0)$

19. Directions for Limits: Evaluate the following limits algebraically (manipulating the expression so that you can use limit theorems - not numerically, graphically, or with l'Hopital's rule.)

- If the limit does not exist or is infinite, explain how you know.
- Points will be taken off for incorrect notation.
- All trigonometric functions must be evaluated.
- No partial credit will be given for answers without supporting work.

(a) (5 points) $\lim_{t\to 1}\left(\frac{(t^2-4t+3)}{(t-1)}\cdot\frac{\tan(\pi t)}{\sin(\pi t)}\right)$ $= 2$

(b) (5 points) $\lim_{x\to\infty}\frac{\sqrt[3]{x}+2x^2-5}{3x^2+4-\pi}$ $= \frac{2}{3}$

(c) (6 points) Use the Sandwich Theorem (also called Squeeze Theorem) to show that $\lim_{x\to\infty}\frac{\sin(x)}{x}=0$

$-1 \le \sin x \le 1$

$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$

$\lim_{x\to\infty} -\frac{1}{x} = 0 \le \lim_{x\to\infty}\frac{\sin x}{x} \le \lim_{x\to\infty}\frac{1}{x} = 0$

Therefore, $\lim_{x\to\infty}\frac{\sin x}{x} = 0$ by the squeeze theorem.