NAME: Practice to Post in Calc Center

Instructor: ______________________

Time your class meets: ______________

Math 160 Calculus for Physical Scientists I
Exam 1
February 11, 2016, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

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(Signature)

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(Date)

Please do not write in this space.

1-3. (10pts)	
4. (24pts)	
5. (20pts)	
6. (20pts)	
7. (6pts)	
8. (20pts)	
TOTAL	

Multiple Choice (10pts):

1. Which of the following is the correct statement for the ***mathematical definition*** of continuity of a function $f(x)$ at the point $x = 5$?

   (a) You can draw the graph without picking up your pencil.

   (b) $f(5)$ exists and $\lim_{x\to 5} f(x)$ exists.

   (c) $f(5)$ exists or $\lim_{x\to 5} f(x)$ exists.

   (d) $f(5)$ exists and $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5} f(x) = f(5)$.

   (e) None of the above

2. The statement: "$f(x)$ is defined at $x = 3$" implies that **(Circle all that apply)**

   (a) $f(x)$ *must* be continuous at $x = 3$.

   (b) $\lim_{x\to 3} f(x)$ *must* exist.

   (c) $\lim_{x\to 3} f(x)$ *cannot* exist.

   (d) $f(3)$ *must* exist.

   (e) There *must* be a point with y-coordinate 3 on the graph of $f(x)$.

   (f) There *must* be a point with x-coordinate 3 on the graph of $f(x)$.

   (g) The graph of $f(x)$ *must* cross the x-axis at the point $(3, 0)$.

   (h) The graph of $f(x)$ *must* cross the y-axis at the point $(0, 3)$.

3. If $f(4)$ is undefined, then **(Circle all that apply)**

   (a) $\lim_{x\to 4} f(x)$ *cannot* exist

   (b) $\lim_{x\to 4} f(x)$ *could* be equal to 3

   (c) $\lim_{x\to 4} f(x)$ *could* be equal to ∞

   (d) $\lim_{x\to 4} f(x) = f(4)$

   (e) There is no point with y-coordinate 4 on the graph of $f(x)$.

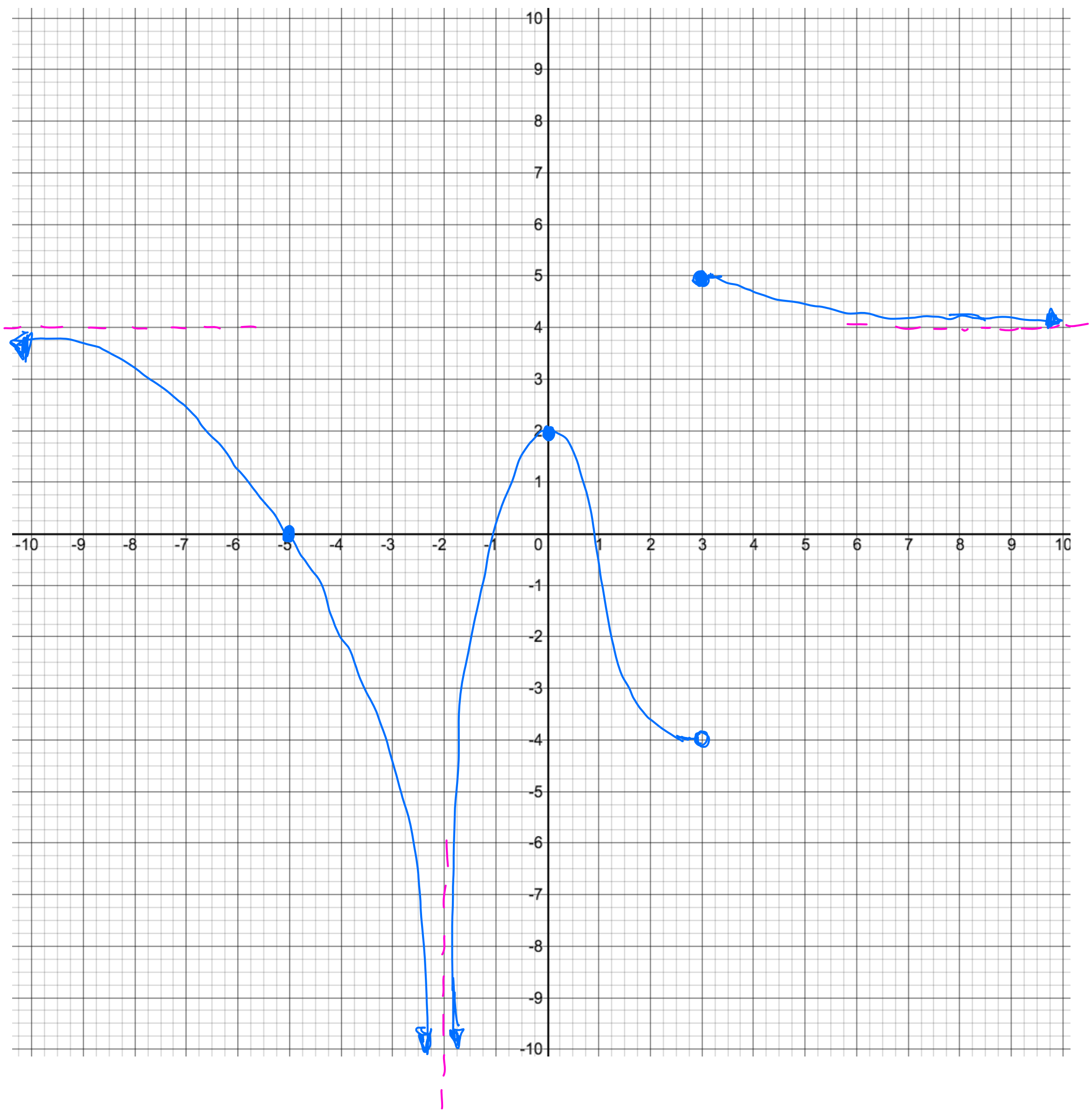
   (f) There is no point with x-coordinate 4 on the graph of $f(x)$.

   (g) The graph of $f(x)$ *must* have a horizontal asymptote at $y = 4$.

   (h) The graph of $f(x)$ *must* have a vertical asymptote at $x = 4$.

4. (24pts) The graph of a **function**, $f(x)$, that has the following properties. Sketch the graph of $f(x)$ and then answer the following questions about $f(x)$.

- $f(x)$ is continuous for $-2 < x < 3$
- $f(x)$ is continuous for $x > 3$
- $f(-5) = 0$
- $\lim_{x \to -\infty} f(x) = 4$
- $\lim_{x \to -2} f(x) = -\infty$
- $f(0) = 2$
- $\lim_{x \to 3^-} f(x) = -4$
- $f(3) = 5 = \lim_{x \to 3^+} f(x)$
- $\lim_{x \to \infty} f(x) = 4$



Multiple Choice

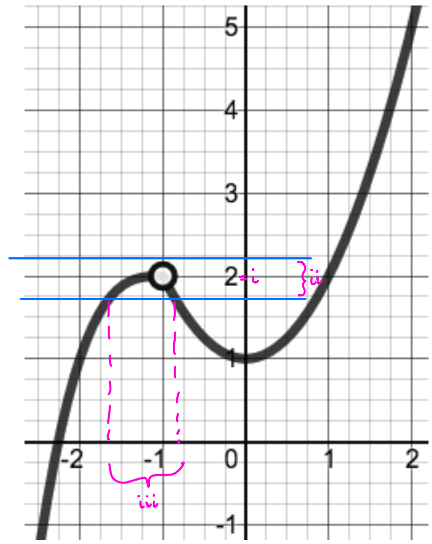
(a) The graph of $f(x)$

i. has a vertical asymptote at $x = -2$.
ii. has a vertical asymptote at $x = 3$.
iii. has a vertical asymptote at $x = 4$.
iv. Does not have a vertical asymptote.
v. None of the above.

(b) The graph of $f(x)$

i. has a horizontal asymptote at $y = -2$.
ii. has a horizontal asymptote at $y = 3$.
iii. has a horizontal asymptote at $y = 4$.
iv. Does not have a horizontal asymptote.
v. None of the above.

5. (20pts) Let $h(x) = \begin{cases} (x+1)^3 + 2 & \text{if} \quad x < -1 \\ x^2 + 1 & \text{if} \quad x > -1 \end{cases}$



A portion of the graph of $h(x)$ is shown to the right.

(a) Show **algebraically** that $h(x)$ has a limit L at $x_0 = -1$ and find this limit.

$$\lim_{x \to -1^-} (x+1)^3 + 2 = 2$$

$$\lim_{x \to -1^+} (x^2+1) = 2$$

$$\lim_{x \to -1} h(x) = 2$$

$L = \lim\limits_{x \to -1} h(x) =$ 2

(b) Suppose we will allow a tolerance of 0.25 within L for the function values (i.e. $\epsilon = 0.25$). **Algebraically** find the interval of x-values that will ensure the function values lie within this tolerance window. Write any decimals to **3** places.

$-1.6299605 \quad -1 \quad -.866025$

Round in

$-1.630 < x < -0.866$

(c) On the graph of $h(x)$ above, label the following:

i. The limit, L, that you found in (a). Your label should be placed on the y-axis and labeled (i).

ii. The tolerance window around L. Your label should be placed on the y-axis and labeled (ii).

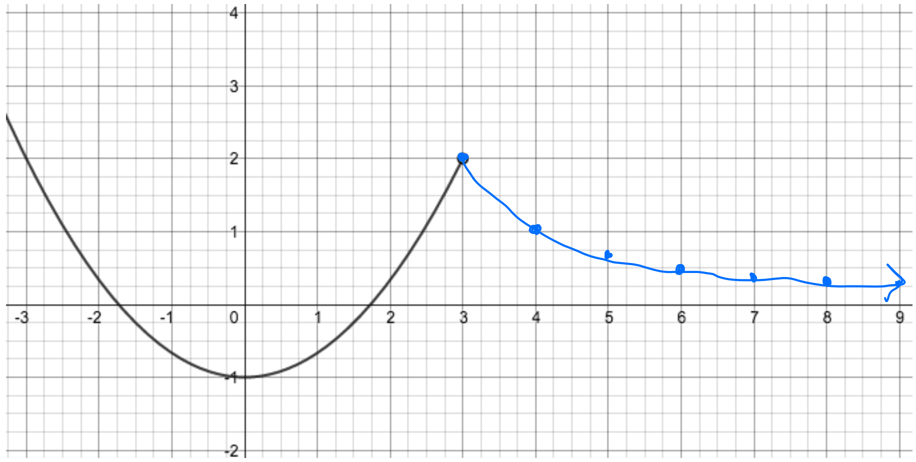
iii. The window about $x_0 = -1$ that you found in (b). Your label should be placed on the x-axis and labeled (iii).

(d) What is the maximum amount of error that can occur on either side of $x_0 = -1$ so that the function values still lie within 0.25 of L? (i.e. what is the δ value?)

Max. Error= 0.134

[This value should be a positive number.]

6. (20pts) The graph of $f(x) = \frac{1}{3}x^2 - 1$, for $x < 3$ is provided below.



Build a new function $G(x) = \begin{cases} f(x) & \text{if} \quad x < 3 \\ \frac{2}{x-M} & \text{if} \quad x \geq 3 \end{cases}$ Where M is a constant.

(a) What is the value of $\lim_{x \to 3^-} G(x)$?

$$\lim_{x \to 3^-} G(x) = 2$$

(b) What is the value of $\lim_{x \to 3^+} G(x)$? (Hint: Leave your answer in terms of M)

$$\lim_{x \to 3^+} G(x) = \frac{2}{3-M}$$

(c) What is the value of $G(3)$? (Hint: Leave your answer in terms of M)

$$G(3) = \frac{2}{3-M}$$

(d) Determine the value of M so that $G(x)$ is continuous at $x = 3$.

$$\lim_{x \to 3^-} G(x) = \lim_{x \to 3^+} G(x) = G(3)$$

$$2 = \frac{2}{3-M}$$

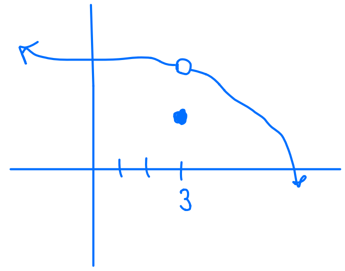
$$M = 2$$

(e) Using the value you got for M, complete the drawing of the graph of $G(x)$, for $x \geq 3$, in the picture above.

7. (6pts) Is the following statement true or false? If the statement is true, explain how you know it's true. If it is false, give a counterexample and explain why it is a counterexample.

$$\text{If } g(3) \text{ is } \underline{\text{defined}}, \text{ then } \lim_{x\to 3} g(x) = g(3)$$

False



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8. (20pts) Directions for Limits: Evaluate the following limits algebraically. (Not numerically, or graphically, or by using l'Hôpital's Rule. "Algebraically" means by manipulating the expression so that you can use the limit theorems. Allow the possibility that the limit does not exist or may be infinite. If the limit does not exist or is infinite, explain how you know. Answers must be accompanied by supporting work. No partial credit will be given for answers with no supporting work. Points will be taken off for incorrect notation. All trigonometric functions should be evaluated. DO NOT simply write an answer with no justification)

(a) $\lim\limits_{b\to 1} \dfrac{\frac{1}{b}-1}{b^2-1} = -1/2$

"Common denominators"
simplify expression

(b) $\lim\limits_{x\to 5^-} \dfrac{x^2-25}{|x-5|} = -10$

Rewrite as a piecewise function

(c) $\lim\limits_{\theta\to 0} \dfrac{\sin(\pi\theta)}{\theta} = \pi$

Special Trig Limits

(d) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}+2}{x-4} = 0$

Limits at infinite