NAME: Key for the Calc Center

Instructor: ______________________

Time your class meets: ______________

Math 160 Calculus for Physical Scientists I

Exam 1 - Version 1

February 9, 2017, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE

I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

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(Signature)

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(Date)

Please do not write in this space.

1-13. (43pts)	
14. (8pts)	
15. (19pts)	
16. (15pts)	
17. (15pts)	
TOTAL	

Algebra Mistakes:	
Trigonometry Mistakes:	

Problems 1-13 are Multiple Choice: 43pts

1. (3pts) The value of $\lim_{x\to 2}(-\pi)$ is

   (a) -2π. (b) $-\pi$. (c) -2. (d) $-\infty$. (e) Does not exist.

2. (3pts) The value of $\displaystyle\lim_{\theta\to\frac{\pi}{2}} \frac{\theta}{\sin(\theta)}$ is

   (a) 1. (b) 0. (c) $\frac{\pi}{2}$. (d) ∞. (e) Does not exist.

3. (4pts) The value of $\displaystyle\lim_{x\to 5^-} \frac{x^2-25}{|x-5|}$ is

   (a) -10. (b) 10. (c) 0. (d) ∞. (e) Does not exist.

4. (4pts) The value of $\displaystyle\lim_{x\to 0} \frac{1-\cos(x)}{2x}$ is

   (a) 0. (b) $\frac{1}{2}$. (c) 1. (d) Does not exist.

5. (3pts) The value of $\lim_{t\to\infty} \cos(t)$ is

(a) $-\infty$. (b) ∞. (c) 0. (d) -1 and 1. (e) Does not exist.

6. (3pts) The graph of $f(x) = \dfrac{|x|}{x^2+x+6}$ is

(a) continuous except at $x=-2$ and $x=3$.

(b) continuous except at $x=-3$ and $x=2$.

(c) continuous except at $x=0$.

(d) continuous for all real numbers.

7. (4pts) Which of the following statements is true about vertical asymptotes?
(circle only one correct answer)

(a) A function cannot have more than two vertical asymptotes.

(b) If $g(-3)$ does not exists, then $x=-3$ is a vertical asymptote of the graph of $g(x)$.

(c) If $\lim_{x\to-2^+} h(x) = \infty$, then $x=-2$ is a vertical asymptote of the graph of $h(x)$.

(d) Only rational functions have vertical asymptotes.

8. (4pts) Suppose that $\lim_{t\to\pi} G(t) = -3$. Then which of the following statements must also be true?
(Circle all that apply)

(a) $G(\pi)$ exists.

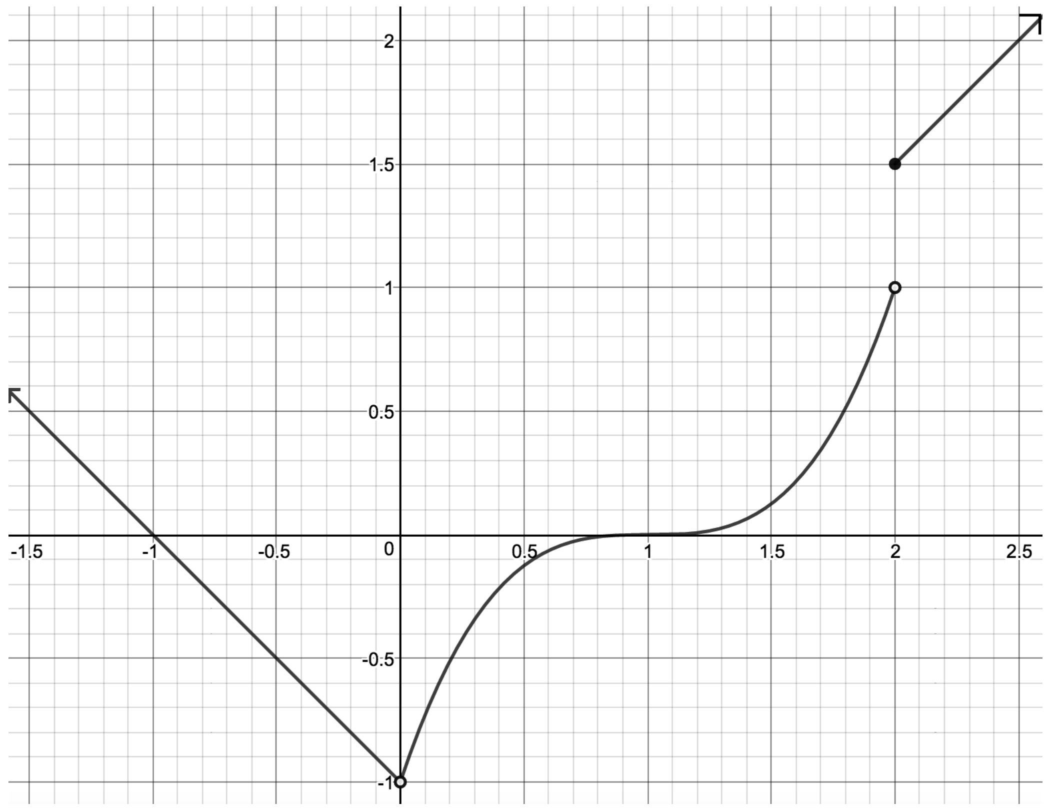
(b) $\lim_{t\to\pi^+} G(t) = -3$.

(c) $G(t)$ is continuous at $t=\pi$.

(d) $\lim_{t\to\pi^-} G(t) = -3$.

Use the function, $f(x)$, below to answer questions 9-10.

$$f(x) = \begin{cases} |x| - 1, & x < 0 \\ (x-1)^3 & 0 < x < 2 \\ |x-2| + \frac{3}{2}, & x \geq 2 \end{cases}$$



9. (3pts) The value of $\lim_{x \to 2^-} f(x)$ is

   (a) 2 (b) 1.5 (c) 1 (d) Does not exist.

10. (3pts) It can be seen from the graph that $\lim_{x \to 0} f(x) = -1$. Suppose we will allow a tolerance of 0.2 within $L = -1$ for the function values (i.e. $\epsilon = 0.2$). What is the **maximum** amount of error (to 3 decimal places) that can occur on either side of $x_0 = 0$ so that the function values still lie within 0.2 of $L = -1$ (i.e. what is the maximum δ value)?

    (a) 0.200
    (b) 0.071
    (c) 0.800
    (d) 0.928
    (e) None of the above.

Use the function, $g(t)$, below to answer questions 11-13.

$$g(t) = \begin{cases} \sin(\pi t), & t < 1 \\ Mt - 2 & t \geq 1 \end{cases}$$

11. (3pts) The value of $\lim_{t \to 1^-} g(t)$ is

   (a) $M - 2$
   (b) 0
   (c) -1
   (d) 1

12. (3pts) The value of $\lim_{t \to 1^+} g(t)$ is

   (a) $M - 2$
   (b) 0
   (c) -1
   (d) 1

13. (3pts) The value M must be in order for $g(t)$ to be continuous at $t = 1$ is

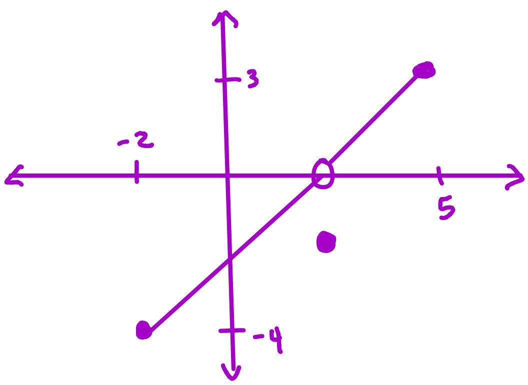
   (a) -1.
   (b) 0.
   (c) 1.
   (d) 2.

14. (8pts - 4pts each) Indicate whether each of the following statements is True or False. If the statement is true, explain how you know its true. If it is false, give a counterexample. (A counterexample is an example that shows the statement is false.)

(a) If $f(x)$ is defined on $[-2, 5]$, $f(-2) = -4$, and $f(5) = 3$, then there must be an x_1 in the interval $(-2, 5)$ such that $f(x_1) = 0$.

True / False (circle one)

Explanation:



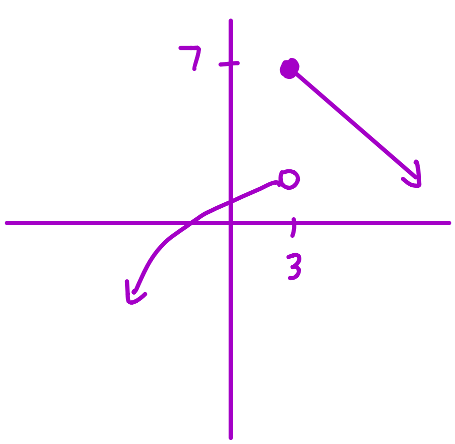
This example is defined everywhere on the interval $-2 \le x \le 5$

However, no x value in $-2 < x < 5$ has an output value of 0.

(b) If $G(3) = \lim_{x \to 3^+} G(x) = 7$, then $G(x)$ is continuous at $x = 3$.

True / False (circle one)

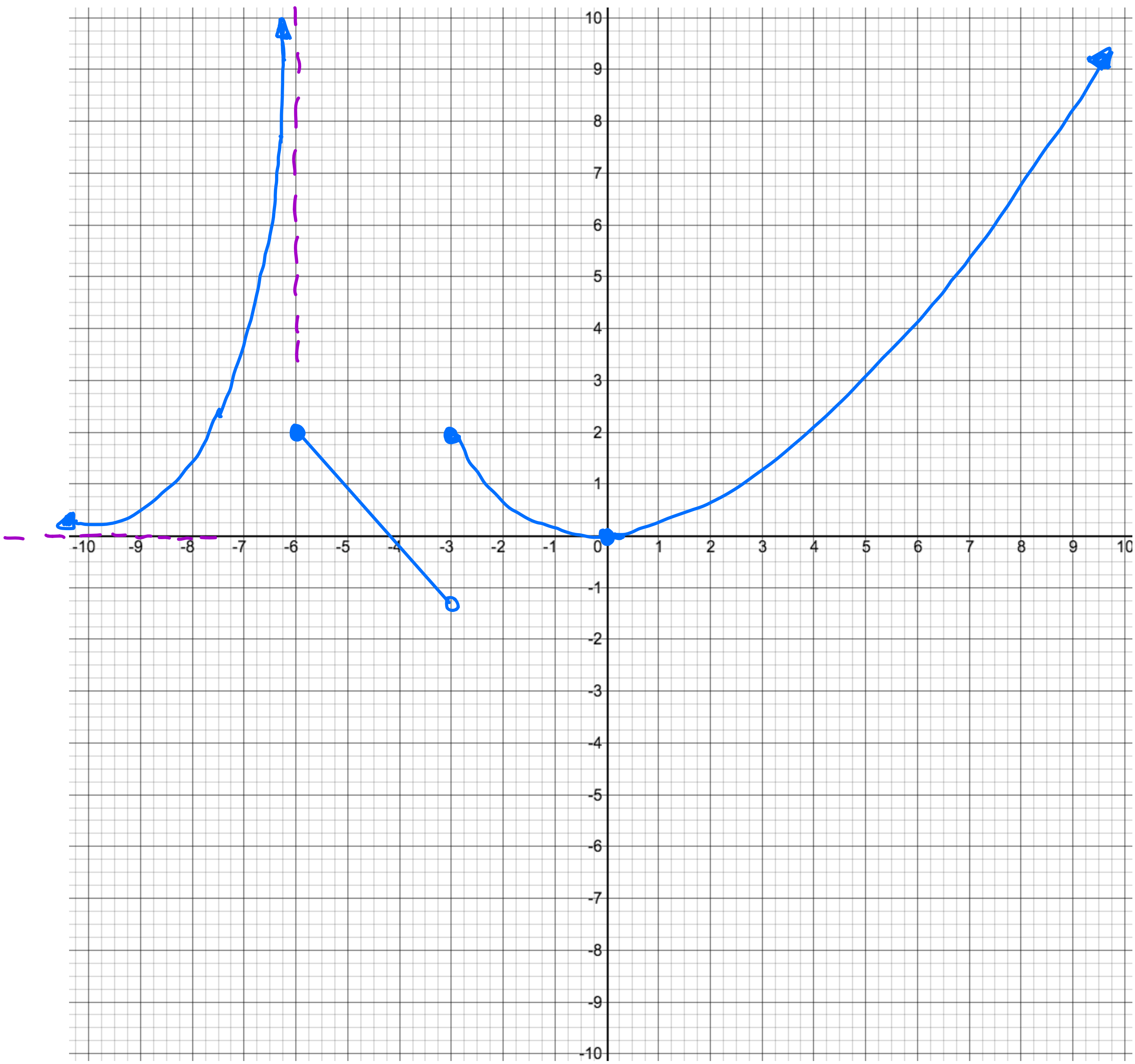
Explanation:



The definition of continuity requires the $\lim_{x \to 3^-} G(x) = 7$, as well but we don't know if this is true from the given info. There could be a jump discontinuity at $x=3$ like in the counterexample to the left.

15. (19pts) The graph of a **function**, $f(x)$, that has the following properties. Sketch the graph of $f(x)$.

- $f(x)$ is continuous for $x < -6$
- $f(x)$ is continuous for $x > 0$
- $\lim_{x \to -\infty} f(x) = 0$ HA @ y=0
- $\lim_{x \to -6^-} f(x) = \infty$ VA@ x=-6
- $\lim_{x \to -6^+} f(x) = 2$
- $f(-6) = 2$
- $f(x)$ is not continuous at $x = -3$
- $f(0) = 0$
- $\lim_{x \to 0^+} f(x) = 0$
- $\lim_{x \to \infty} f(x) = \infty$



16. (15pts) Sophia wants to construct a square pen (also called an enclosure or corral) for her pet rabbit, Stanley.

(a) If a side length is represented by x, write the function, $A(x)$, that represents the area of the animal pen:

$A(x) =$ x^2

(b) Sophia wants to enclose an area of 16 ft^2 to hold Stanley. What is the "perfect" side length that achieves this area?

$x =$ 4

(c) Sophia's measurements are not perfect, but she is able to enclose an area within 0.1 ft^2 of the perfect area 16 ft^2 (i.e. $\epsilon = 0.1$). What corresponding side lengths satisfy this range in area? Write any decimals to **4** places.

3.9875 $< x <$ 4.0125

17. (15pts - 5pts each) <u>Directions for Limits</u>: Evaluate the following limits algebraically (manipulating the expression so that you can use limit theorems - not numerically, graphically, or with l'Hopital's rule.)

- If the limit does not exist or is infinite, explain how you know.
- Points will be taken off for incorrect notation.
- All trigonometric functions must be evaluated.
- No partial credit will be given for answers without supporting work.

(a) $\lim_{t \to 0} \left(\frac{(t^2 - 4t)}{t} \cdot \frac{\sin(t)}{\tan(t)} \right) = -4$

(b) $\lim_{b \to 3} \frac{9 - b^2}{\frac{1}{b} - \frac{1}{3}} = 54$

(c) $\lim_{x \to \infty} \frac{\sqrt{x} - 6x^2 + \pi}{3x^2 + 4x - 1} = -2$