NAME: ______________________________

Instructor: ____________________________

Time your class meets: __________________

Math 160 Calculus for Physical Scientists I
Exam 2 - Version 1
March 9, 2017, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____
(Signature)

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(Date)

Please do not write in this space.

1-4. (15pts)	
5. (10pts)	
6. (11pts)	
7. (13pts)	
8. (12pts)	
9. (13pts)	
10. (11pts)	
11. (15pts)	
TOTAL	

1. (3pts) If the radius of a circle increases from $r = A$ to $r = B$, then the average rate of change of the area of the circle is

   (a) $\pi(B^2 - A^2)$.

   (b) $\pi(B - A)$.

   (c) $\pi\left(B^2 + A^2\right)$.

   (d) $\pi\,(B + A)$.

   (e) None of the above.

2. (3pts) We know that $f(2) = 4$ and $f'(2) = 3$, then the value of $\frac{d}{dx}\left(\frac{3x^2}{f(x)}\right)\Big|_{x=2}$

   (a) $\frac{21}{4}$

   (b) $\frac{4}{3}$

   (c) $\frac{3}{4}$

   (d) 4

   (e) None of the above.

3. (3pts) If $h'(t)$ is continuous on $(-\infty, \infty)$, then

   (a) $\lim_{t\to 100} h(t) = h'(100)$.

   (b) $\lim_{t\to 100} h(t) = h(100)$.

   (c) $\lim_{t\to 100} h(t)$ may not exist.

   (d) $\lim_{h\to 0} \frac{h(100 + h) - h(100)}{h}$ may not exist.

   (e) None of the above.

4. (6pts) The function $f(t)$ represents the distance of a vehicle moving along a straight road from its starting point at time t.

Below are three graphs of the derivative, $f'(t)$, of the distance function. Which graph best matches each of the following vehicle scenarios?

Scenario 1: A car in heavy traffic conditions.

Graph 1 / Graph 2 / Graph 3 (circle one)

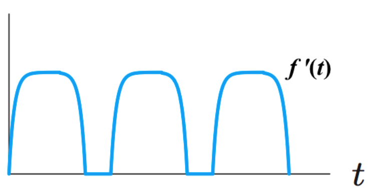
Scenario 2: A bus on a popular route, with no traffic.

Graph 1 / Graph 2 / Graph 3 (circle one)

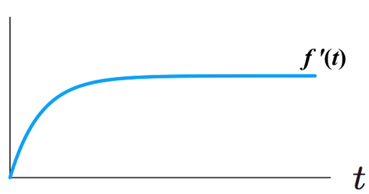
Scenario 3: A car with no traffic and all green lights.

Graph 1 / Graph 2 / Graph 3 (circle one)

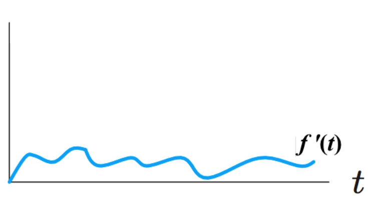
Graph 1:



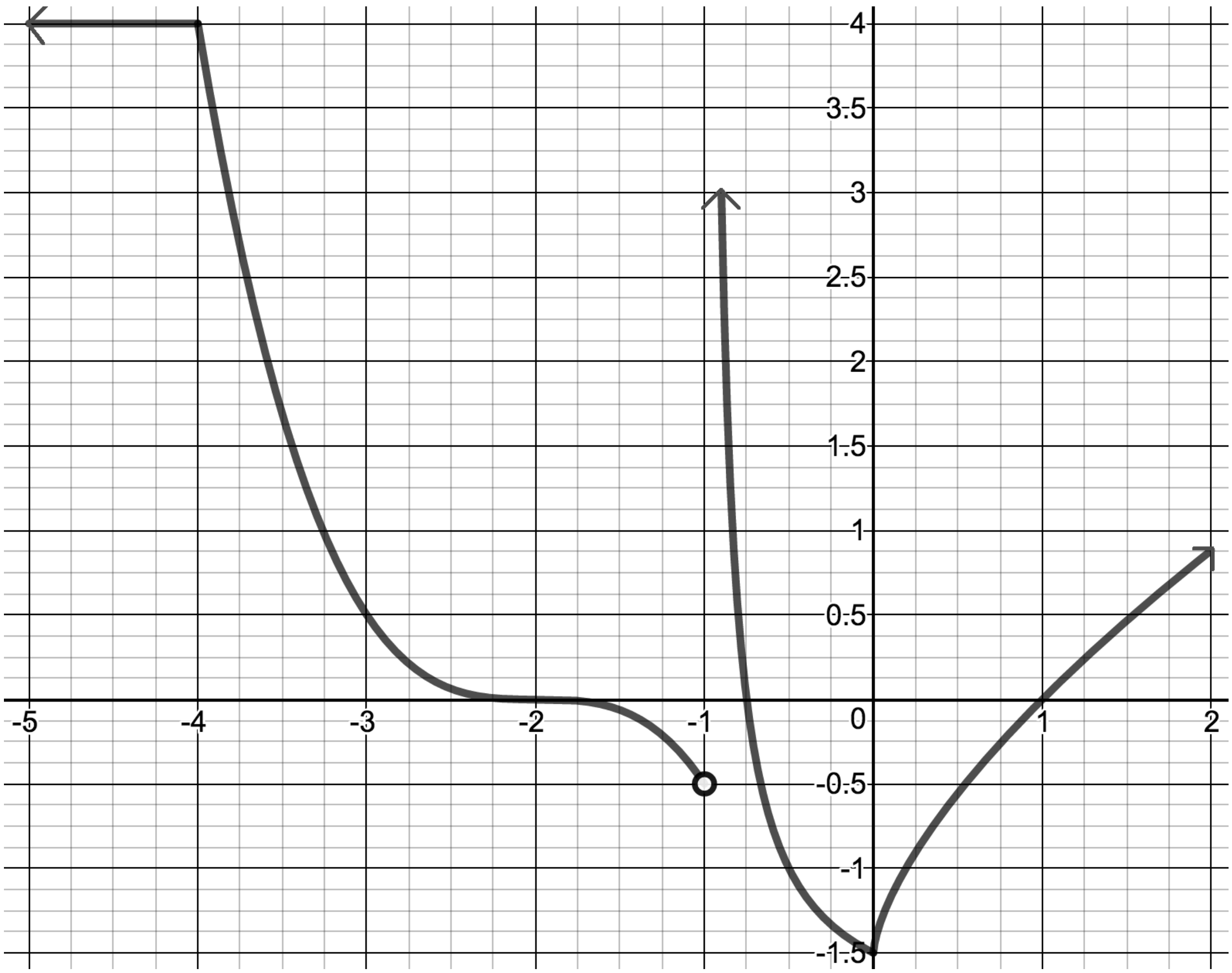
Graph 2:



Graph 3:



5. (10pts) Below is the graph of the function, $f(x)$.



Of all the properties listed below, circle all the properties that are reflected in the graph of $f(x)$.

(a) $f(x)$ is not continuous at $x = -4$

(b) $f(x)$ is not continuous at $x = -1$

(c) $f(x)$ is not continuous at $x = 0$

(d) $f(x)$ is not differentiable at $x = -4$

(e) $f(x)$ is not differentiable at $x = -1$

(f) $f(x)$ is not differentiable at $x = 0$

(g) $f'(x) = 0$ at $x = -2$

(h) $f'(x) = 0$ at $x = 0$

(i) $f'(x) < 0$ for $-4 < x < -2$

(j) $f'(x) < 0$ for $-2 < x < -1$

(k) $f'(x) < 0$ for $-1 < x < 0$

(l) $f'(x) < 0$ for $x > 0$

(m) $f'(x) > 0$ for $-4 < x < -2$

(n) $f'(x) > 0$ for $-2 < x < -1$

(o) $f'(x) > 0$ for $-1 < x < 0$

(p) $f'(x) > 0$ for $x > 0$

(q) $y = 4$ is an absolute maximum

(r) $y = 3$ is an absolute maximum

(s) $y = -0.5$ is an absolute minimum

(t) $y = -1.5$ is an absolute minimum

6. (11pts) Use $g(t) = \tan(f(t))$ to answer the following.

(a) Find the derivative of $g(t)$.

(b) Using your result from part (a) and the information provided in the table below, find the value of $g'(\pi)$. Circle the correct answer.

	$f(t)$	$f'(t)$
$t = 0$	2	$-\frac{1}{2}$
$t = \pi$	$\frac{\pi}{3}$	π

i. $g'(\pi) = 4\pi$

ii. $g'(\pi) = 4$

iii. $g'(\pi) = \pi$

iv. $g'(\pi) = 0$

v. $g'(\pi) = 1$

(c) Suppose that $f(t) = 3t$, then $g(t) = \tan(3t)$.

What is $g''(t)$? Circle the correct answer.

i. $3\sec^2(3t)$

ii. $18\sec^2(3t)\tan(3t)$

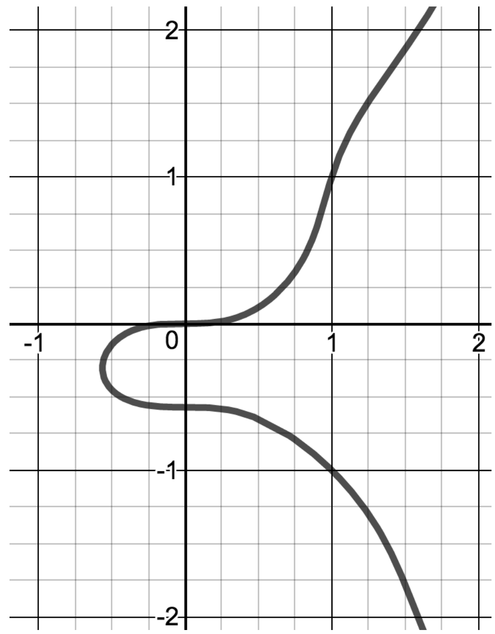
iii. $2\sec^2(3t)\tan(3t)$

iv. $6\sec^2(3t)\tan(3t)$

v. $2\sec(3t)$

7. (13pts) Below is the graph of the implicitly defined function $\sin(\pi y) + 3y^2 = 3x^3$. You may leave values in exact form. If you choose to use decimals, round to 3 decimal places.

(a) It can be seen in the picture that the point $(1, -1)$ is on the graph of $\sin(\pi y) + 3y^2 = 3x^3$. Algebraically verify this fact.



(b) Use calculus to find $\dfrac{dy}{dx}$

(c) Find the slope of the line tangent to $\sin(\pi y) + 3y^2 = 3x^3$ at the point $(1, -1)$. Keep your answer in EXACT form. DO NOT use decimals to approximate.

(d) Find the equation of the line tangent to $\sin(\pi y) + 3y^2 = 3x^3$ at the point $(1, -1)$. Keep your answer in EXACT form. DO NOT use decimals to approximate.

8. (12pts - 4pts each) Indicate whether each of the following statements is **True** or **False**.

If the statement is true, explain how you know it's true.

If it is false, give a counterexample **and** explain why it is a counterexample. (A counterexample is an example of a function for which the "if" part of the statement is true, but the "then" part is false.) A graph with an explanation can be used as a counterexample.

(a) If f is continuous at $x = 1$, then f is differentiable at $x = 1$

(b) Given that $f(x)$ is continuous on the interval $(1, 3)$, then $f(x)$ attains an absolute maximum on the interval $(1, 3)$.

(c) Two different functions, $f(x)$ and $g(x)$, cannot have the same derivative functions unless both $f(x)$ and $g(x)$ are linear functions with the same slope.

9. (13pts) Consider the function $f(x)$ given by

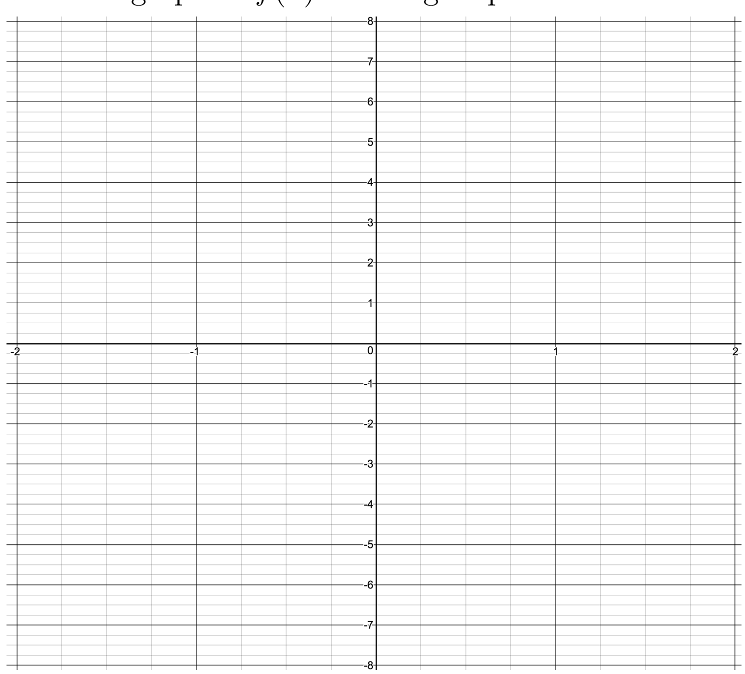
$$f(x) = \begin{cases} -x^3, & x \leq 0 \\ x^3, & x > 0 \end{cases}$$

(a) Use the **definition of the derivative** of a function at a point (as a limit) to determine whether $f(x)$ is differentiable at $x = 0$. Show details of how you made this determination. Evaluate any limits involved algebraically (without using a calculator).

If $f'(0)$ exists, fill in the blank with its value. If not, draw a smiley face in the blank.

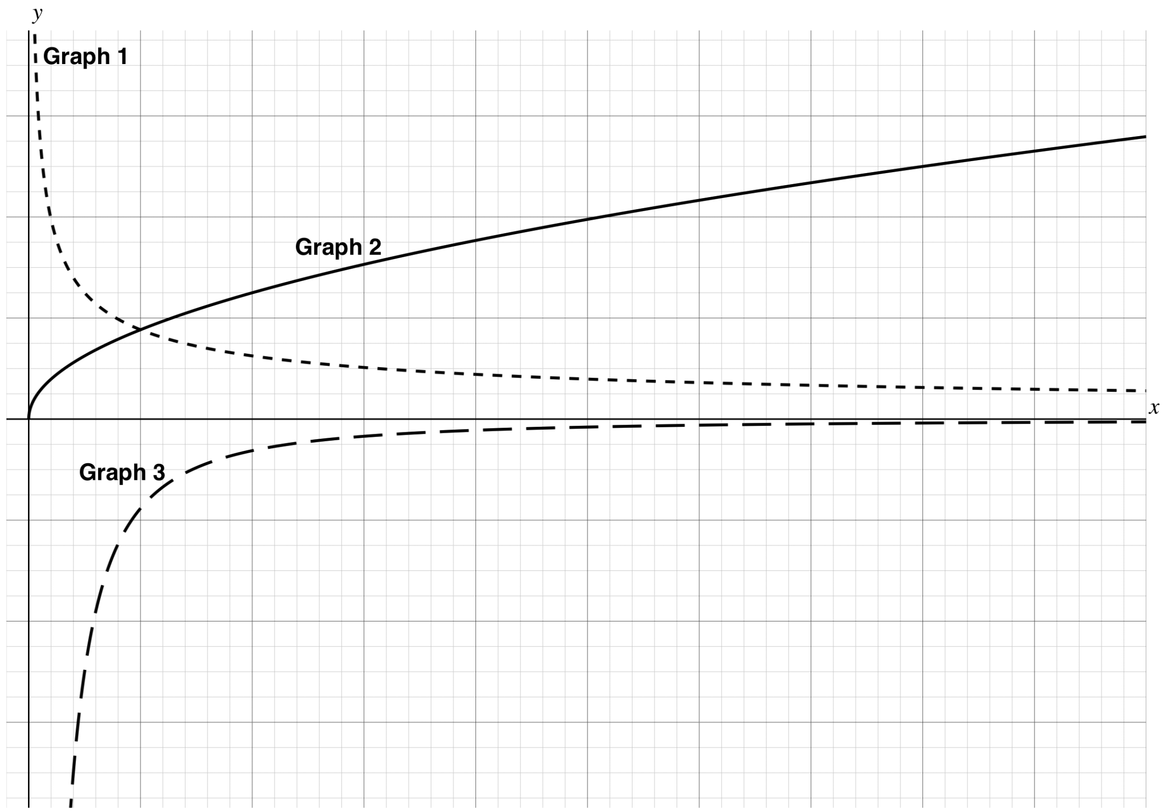
$f'(0) =$ _______________

(b) Sketch a graph of $f(x)$ on the grid provided.



(c) Describe how your graph in (b) supports the answer you found in (a).

10. (11pts) There are three graphs plotted in the coordinate system below ($f(x)$, $f'(x)$, and $f''(x)$).



Which graph is $f(x)$? Which graph is $f'(x)$? Which graph is $f''(x)$? Give reasons for your answers in sentences. Your explanation should include a discussion of slope with regard to each graph.

Graph 1 = $\underline{f(x),\ f'(x),\ f''(x)}$ (CIRCLE ONE)

Graph 2 = $\underline{f(x),\ f'(x),\ f''(x)}$ (CIRCLE ONE)

Graph 3 = $\underline{f(x),\ f'(x),\ f''(x)}$ (CIRCLE ONE)

In the space below, justify your answers using complete sentences. Your justification should include a discussion of slope with regard to each graph.

11. (15pts - 5pts each) Calculate the indicated derivatives by using the Differentiation Rules (Theorems). Answers must be accompanied by supporting work that shows how you calculated the derivative. **You do not need to simplify your answers on these problems.** If you do simplify an answer, you must simplify correctly.

(a) $\frac{d}{dx}\left(\frac{3}{x^3} + \cos(x) + \pi^4\right)$

(b) $\frac{d}{d\theta}\left[\frac{5\theta}{\sin(\theta) + 2}\right]$

(c) Find $g'(t)$ given $g(t) = (3t - t^{4/3})(4\sqrt{t} + 2t^3)$