NAME: ______________________________

Instructor: ____________________________

Time your class meets: __________________

Math 160 Calculus for Physical Scientists I
Exam 1
February 12, 2015, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a calculator on this exam. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE

I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____

(Signature)

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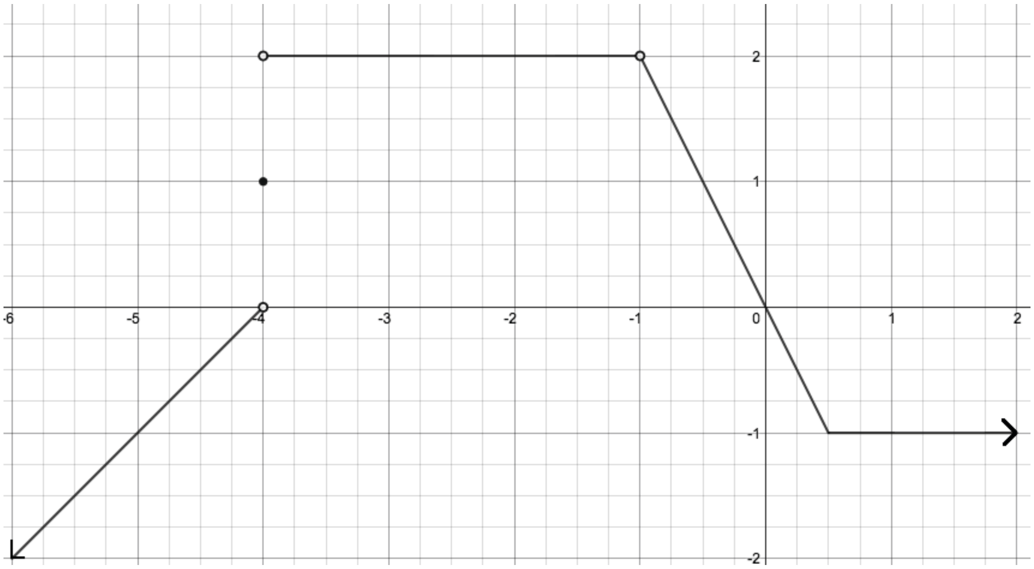
(Date)

Please do not write in this space.

1-6. (18pts)	
7. (16pts)	
8. (28pts)	
9. (13pts)	
10. (9pts)	
11. (16pts)	
TOTAL	

Use the graph of the function, $f(x)$, below to answer the following multiple choice questions. Circle only one answer for each question. These will be graded as right or wrong. No partial credit will be given.



#1-4 - 2 pts each

1. $\lim\limits_{x \to -\infty} f(x) =$
   (a) ∞
   (b) $-\infty$
   (c) -2
   (d) Does not exist
   (e) None of the above.

2. $\lim\limits_{x \to -4} f(x) =$
   (a) 0
   (b) 1
   (c) 2
   (d) Does not exist
   (e) None of the above.

3. $\lim\limits_{x \to -1} f(x) =$
   (a) 0
   (b) 1
   (c) 2
   (d) Does not exist
   (e) None of the above.

4. $\lim\limits_{x \to \infty} f(x) =$
   (a) $-\infty$
   (b) ∞
   (c) -1
   (d) Does not exist
   (e) None of the above.

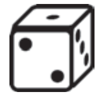
5. (5pts) At $x = -4$, $f(x)$ IS / IS NOT (circle one) continuous.

   Using the mathematical definition of continuity, explain why you selected your answer.

6. (5pts) At $x = -1$, $f(x)$ IS / IS NOT (circle one) continuous.

   Using the mathematical definition of continuity, explain why you selected your answer.

7. (16pts) You are 3D printing six-sided dice for a *Yahtzee* tournament. After some research, you find out that most dice have a volume of 4 cm^3 each. An example of a single *Yahtzee* dice is illustrated below. Note that each side length, s, has the same measure.



(a) In the blank below, write the function that relates the volume of a single dice, V, and the dice's side length, s.

$V(s) =$__________________

(b) What is the *perfect* side length? i.e. What side length will result in a volume of 4 cm^3? Your answer should be written to **6 decimal places**.

s_0=_________________________

(c) Due to the nature of your 3D printer, the volume of each dice will be within 0.01 cm^3 of 4 cm^3. Algebraically determine the interval around s_0 that will ensure that corresponding output values are within 0.01 cm^3 of 4 cm^3. If you use decimals, your answer should be written to **6 decimal places**.

____________ $< s_0 <$ ____________

(d) What is the maximum amount of error that can occur on either side of s_0 so that the volume values still lie within 0.01 of $4cm^3$? (i.e. what is the δ value?) If you use decimals, your answer should be written to **6 decimal places**.

Max. Error=__________________

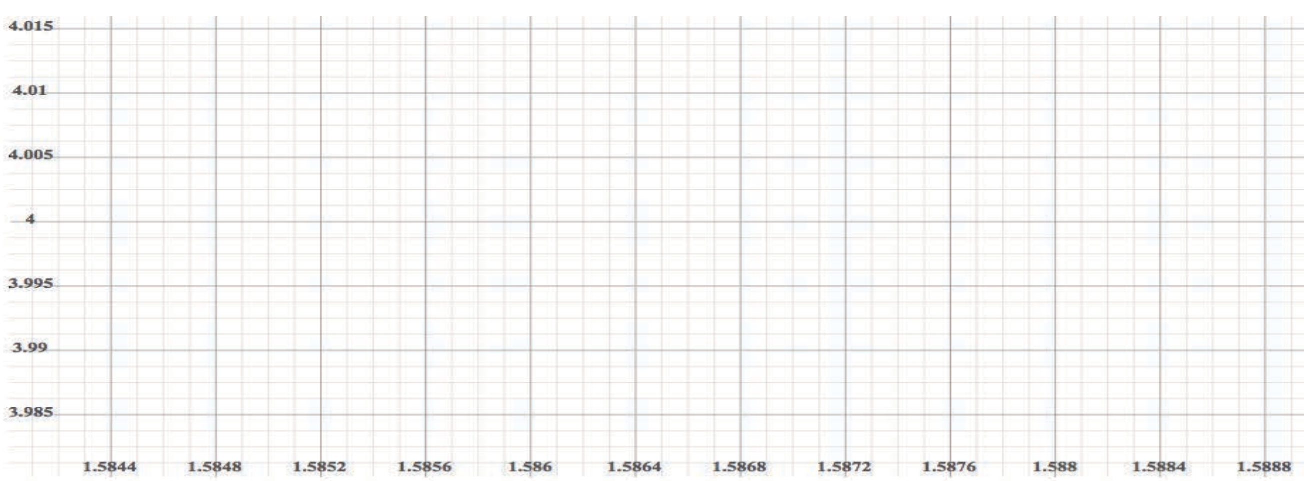
(e) Below is the portion of the graph of $V(s)$ relevant for this problem. Using the graph, label the following:

(i) $V(s) = 4$ cm^3

(ii) The interval of ± 0.01 cm^3 around $4cm^3$.

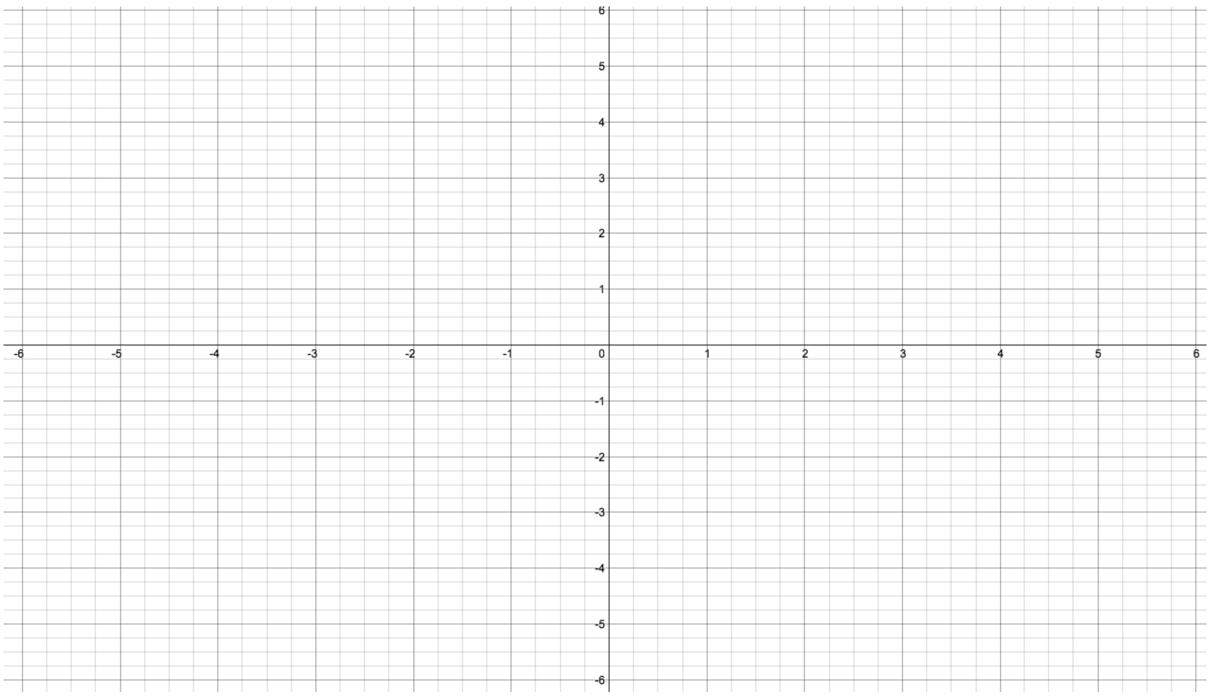
(iii) $s = s_0$

(iv) The interval you found in part (c).



8. (10pts) Sketch the graph of **a function**, $F(x)$, that has the following properties:

- $\lim\limits_{x \to -\infty} F(x) = 1$
- $\lim\limits_{x \to -2^-} F(x) = \infty$
- $\lim\limits_{x \to -2^+} F(x) = 3$
- $F(-2) = 3$
- $\lim\limits_{x \to 1} F(x) = -1$
- $F(1) = 3$
- $\lim\limits_{x \to 4^-} F(x) = 3$
- $\lim\limits_{x \to 4^+} F(x) = 4$
- $\lim\limits_{x \to \infty} F(x) = 5$



(3pts) Fill-In-The-Blanks: Using the information given above, we are guaranteed that $F(x)$ has a vertical asymptote at $x =$__________ because

$$\lim_{x \to \underline{\hspace{1cm}}} F(x) = \underline{\hspace{2cm}}$$

(6pts) Fill-In-The-Blanks: Using the information given above, we are guaranteed that $F(x)$ has horizontal asymptotes at $y =$__________ and $y =$__________ because

$$\lim_{x \to \underline{\hspace{1cm}}} F(x) = \underline{\hspace{2cm}} \qquad \textit{and} \qquad \lim_{x \to \underline{\hspace{1cm}}} F(x) = \underline{\hspace{2cm}}$$

(9pts) Using the information given above, all of the known discontinuities of $F(x)$ occur at $x =$_____________ (list ALL x-values, there is more than one).

For **each** x-value you listed in the blank above, state why $F(x)$ is not continuous. Be sure to use the mathematical definition of continuity.

9. (9pts) Dave computed the limit $\lim_{t\to -1} \frac{t^2+4}{|t+2|}$.

Below is Dave's work, which is incorrect.

$$\lim_{t\to -1} \frac{t^2+4}{|t+2|} = \lim_{t\to -1} \frac{(t+2)(t+2)}{|t+2|} = \lim_{t\to -1} \frac{\cancel{(t+2)}(t+2)}{\cancel{|t+2|}} = t+2 = 1$$

Identify *at least* three of his errors.

Error 1:

Error 2:

Error 3:

(4pts) Algebraically determine the correct value of the limit. Be sure to show all of your work.

$\lim_{t\to -1} \frac{t^2+4}{|t+2|} =$

10. The following questions have to do with the limit

$$\lim_{x \to 0} \cos\left(\frac{\pi}{x}\right)$$

(a) (2pts) Below is a table of values for $f(x) = \cos\left(\frac{\pi}{x}\right)$.

x	0.1	0.01	0.001	0.0001	0.00001	0.000001
$f(x)$	1	1	1	1	1	1

Based **solely** on information from this table, what would you predict as the value of $\lim_{x \to 0} \cos\left(\frac{\pi}{x}\right)$? ____________

(b) (2pts) Below is a table of values for $f(x) = \cos\left(\frac{\pi}{x}\right)$.

x	0.15	0.015	0.0015	0.00015	0.000015	0.0000015
$f(x)$	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5

Based **solely** on information from this table, what would you predict as the value of $\lim_{x \to 0} \cos\left(\frac{\pi}{x}\right)$? ____________

(c) (5pts) From the above results, what do you conclude about $\lim_{x \to 0} \cos\left(\frac{\pi}{x}\right)$? Explain in terms of what it means for a function to have a limit (or not to have a limit) how the tables in parts (a) and (b) support your conclusion.

11. (16pts - 4pts each) Indicate whether each of the following statements is **True** or **False**. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample. (A counterexample is an example of a function for which the "if" part of the statement is true, but the "then" part is false.) A graph with an explanation can be used as a counterexample.

If you use a term or phrase such as *continuity*, be sure to state the definition of the term or phrase that you used.

(a) If $\lim_{x \to 3^+} f(x) = 51 = \lim_{x \to 3^-} f(x)$, then $f(x)$ is continuous at $x = 3$.

(b) If $\lim_{x \to 17^-} g(x) = \infty$ and $g(17) = 4$, then g has a vertical asymptote at $x = 17$.

(c) If $f(x) = \dfrac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$, then $f(x)$ is equal to $g(x)$.

(d) If $\lim_{x \to 3} r(x)$ does not exist, then $r(3)$ does not exist.