NAME: _____

Instructor: _____

Time your class meets: _____

Math 160 Calculus for Physical Scientists I
Exam 2
October 16, 2014, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a calculator on this exam. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

_____
(Signature)

_____
(Date)

Please do not write in this space.

1-3. (17pts)	
4. (7pts)	
5. (15pts)	
6. (12pts)	
7. (15pts)	
8. (12pts)	
9. (8pts)	
10. (14pts)	
TOTAL	

Multiple Choice for #1-4. Circle only one answer for each problem unless it indicates otherwise.

1. (3pts) At the point $(0, 0)$, the graph of $f(x) = |x|$

   (a) has $y = 0$ as a tangent line.

   (b) has infinitely many tangent lines.

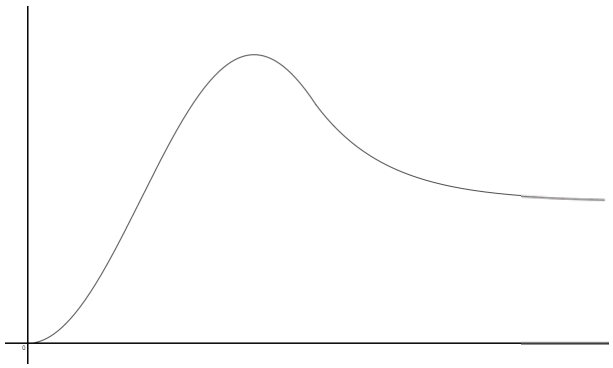
   (c) has no tangent line.

   (d) has $y = -x$ and $y = x$ as both of its tangent lines.

   (e) none of the above

2. (8pts) Below is the graph of a function that changes with respect to time. Which of the following statements are accurately modeled by the graph? (CIRCLE ALL CORRECT RESPONSES)

   For each response that you circle, fill in the blank with the function represented by the graph (position, velocity, or acceleration).



   (a) Olga climbs to the top of a mountain but quickly descends to a shelter halfway down when she sees a thunderstorm on the horizon.

   _____

   (b) Tatiana is jumping on a trampoline until her foot slips and she falls to the ground.

   _____

   (c) Alexie accelerates from a stop sign before reaching a school zone and needing to slow down to a legal speed.

   _____

   (d) Maria's plane reaches cruising altitude and stays there for the rest of the flight.

   _____

   (e) Anastasia is proud of her efforts during the Fort Collins 10K Race, since she never had to slow down. [Note: A 10K race is a distance of 10 kilometers (6.2 miles)]

   _____

3. (6pts) Which of the following statements are true? (CIRCLE ALL CORRECT RESPONSES)

   (a) If $f(x)$ is continuous at $x = 5$, then $f'(5)$ exists.

   (b) If $f(x)$ is continuous at $x = 5$, then $f'(5)$ does not exist.

   (c) If $f(x)$ is continuous at $x = 5$, then $f'(5)$ may or may not exist.

   (d) If $f(x)$ is not continuous at $x = 5$, then $f'(5)$ does not exist.

   (e) If $f(x)$ is not continuous at $x = 5$, then $f'(5)$ may or may not exist.

   (f) If $f'(5)$ does not exist, then $f(x)$ is not continuous at $x = 5$.

   (g) If $f'(5)$ exists, then $f(x)$ is continuous at $x = 5$.

4. **Circle the correct response and then explain your answer below**.

   (2pts) Two racers start a race at exactly the same moment and finish at exactly the same moment (they tied at the finish). Which of the following statements must be true. Explain how you know.

   (a) At some point during the race, the two racers were not tied.

   (b) The racer's speed at the end of the race was exactly the same.

   (c) The racers must have had the same speed at exactly the same time at some point in the race.

   (d) The racers had to have the same speed at some moment, but not necessarily at exactly the same time.

   Explain (5pts):

5. (15pts - 5pts each) Use the given information in the table to find the following derivatives:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	$\frac{3}{2}$	$-\pi$	-2	1
-2	3	$-\frac{1}{2}$	-1	0

(a) $\dfrac{d}{dx}\left(\dfrac{f(x)}{x^2}\right)\Bigg|_{x=1}$

(b) $\dfrac{d}{dx}\left(f(x)\cdot g(x)\right)\Bigg|_{x=-2}$

(c) $\dfrac{d}{dx}\left(f(g(x))\right)\Bigg|_{x=1}$

6. (12pts - 4pts each) Indicate whether each of the following statements is **True** or **False**. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample. (A counterexample is an example of a function for which the "if" part of the statement is true, but the "then" part is false.) A graph with an explanation can be used as a counterexample.

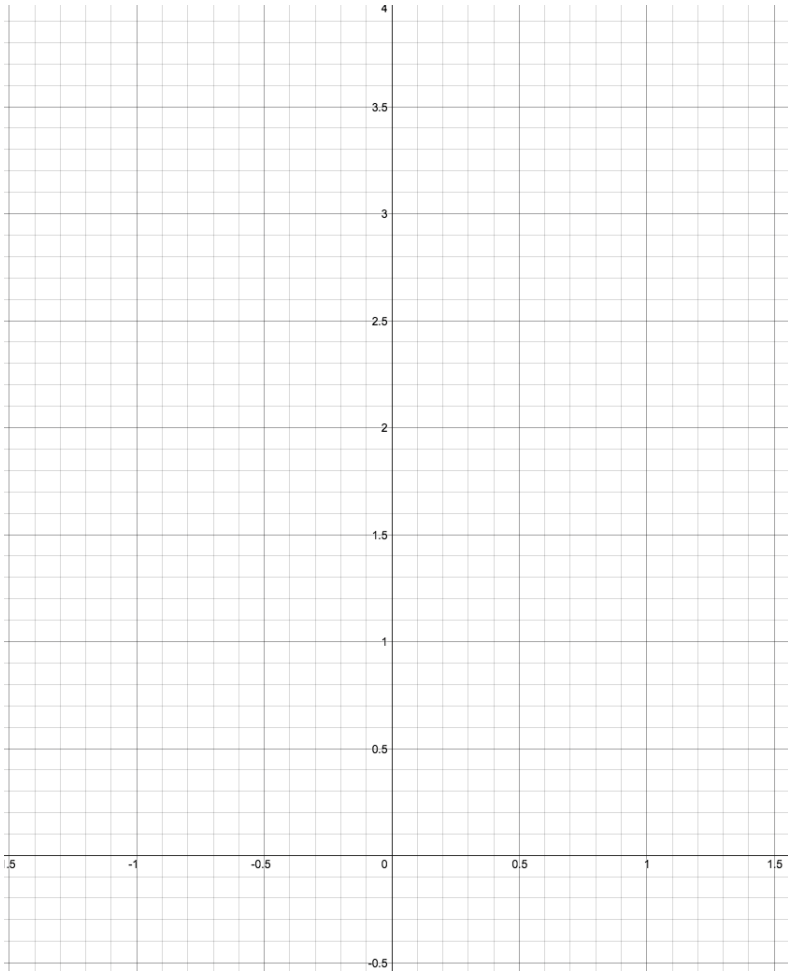
   (a) If $f(x)$ is defined on the interval $[-3, 3]$, then $f(x)$ must have a maximum on $[-3, 3]$.

   (b) Given that $f(1) = f(3) = 0$ and $f'(2) = 0$, then $f(x)$ must be continuous on the interval $[1, 3]$

   (c) Two different functions, $f(x)$ and $g(x)$, cannot have the same derivative functions unless both $f(x)$ and $g(x)$ are linear functions with the same slope.

7. (15pts) Use $f(x) = |x| + x^2$ to answer the following questions

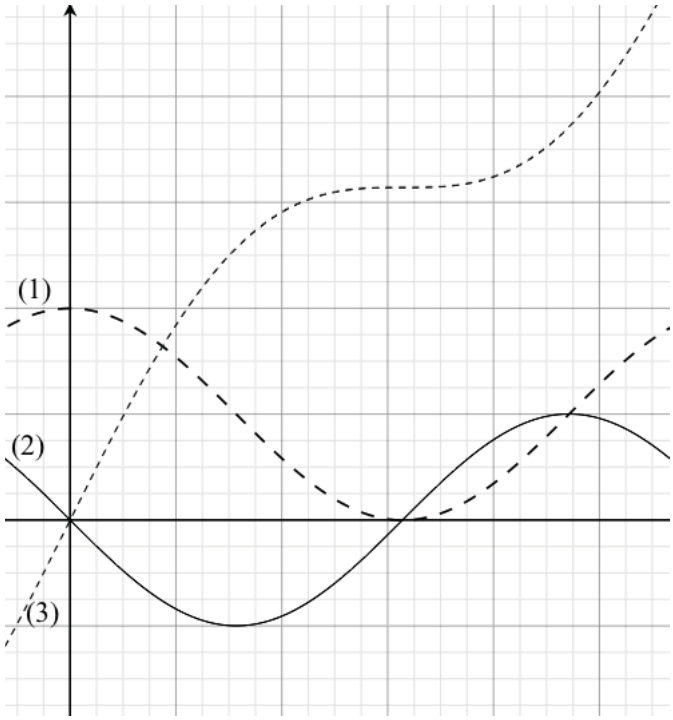
(a) (5pts) Sketch an accurate graph of $f(x)$ in the axes below. [*An accurate graph shows the function's domain, has the correct shape, and key points on the graph have the correct coordinates.*]



(b) (10pts) Using the definition of the derivative (as a limit), determine if $f'(0)$ exists.

$f'(0)$ does / does not (CIRCLE ONE) exist.

8. (12pts) Below are the graphs of a position function $s(t)$, a velocity function, $v(t)$, and an acceleration function $a(t)$ with respect to time, t.



Which graph is position? Which graph is velocity? Which graph is acceleration? Give reasons for your answers in sentences. Your explanation should include a discussion of slope with regard to each graph.

Graph 1 = position $s(t)$, velocity $v(t)$, acceleration $a(t)$ (CIRCLE ONE)

Graph 2 = position $s(t)$, velocity $v(t)$, acceleration $a(t)$ (CIRCLE ONE)

Graph 3 = position $s(t)$, velocity $v(t)$, acceleration $a(t)$ (CIRCLE ONE)

9. (8pts) In the axes provided, sketch the graph of a function that has the following properties. You MUST label each property of your graph with the corresponding letter.

   (a) At point $x = 0$, $f'(0)$ does not exist, but $\lim_{x \to 0} f(x) = f(0)$

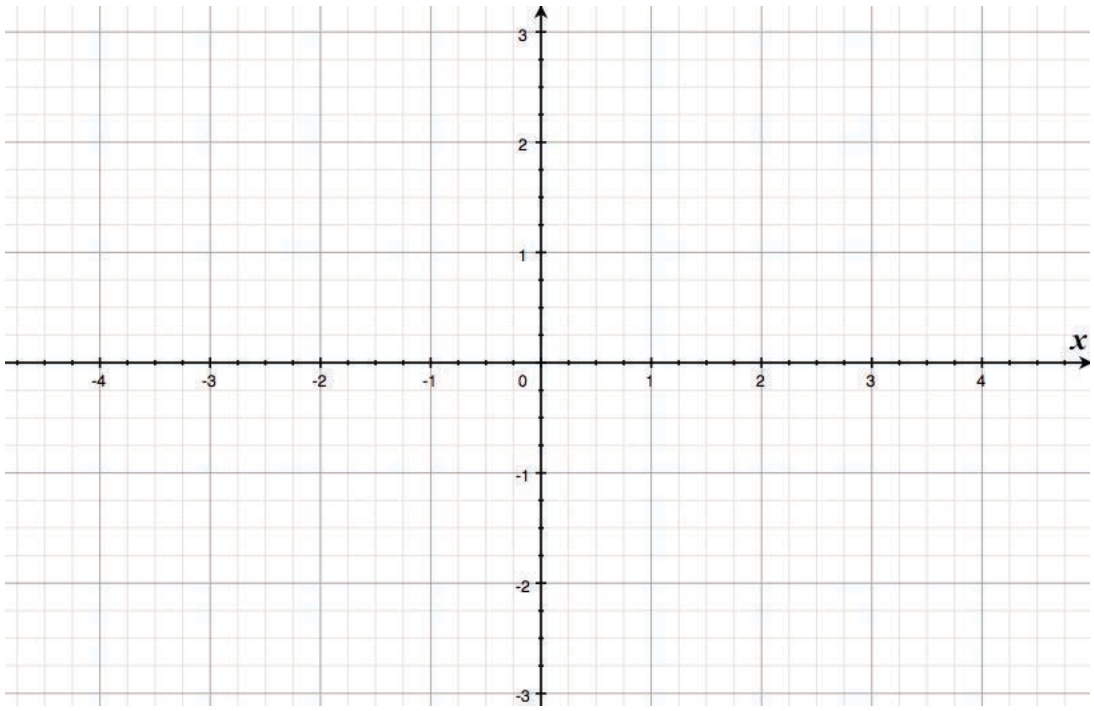
   (b) $\lim_{x \to -2^+} f(x) = 0$

   (c) $\lim_{x \to -2^-} f(x) = -\infty$

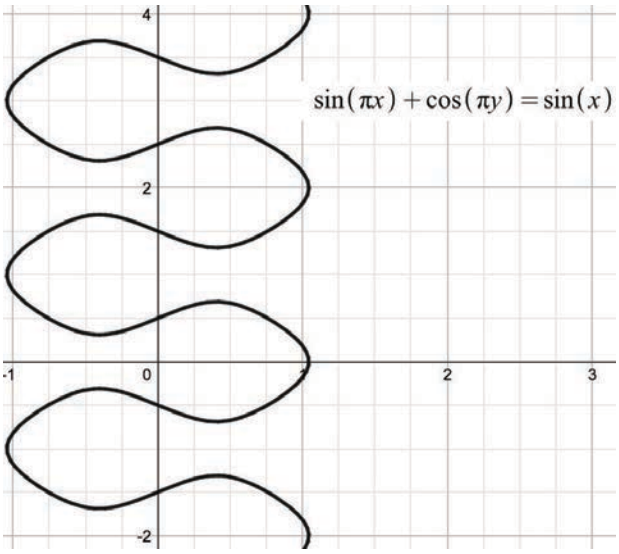
   (d) $\lim_{x \to -\infty} f(x) = 1$

   (e) $\lim_{x \to \infty} f(x) = 0$

   (f) A local maximum at $x = 1$



10. Use the curve of $\sin(\pi x) + \cos(\pi y) = \sin(x)$ below to answer the following questions:
LEAVE ALL ANSWERS IN EXACT FORM. DO NOT USE DECIMALS.



(a) (2pts) Draw the line tangent to the curve at the point $\left(0, \frac{1}{2}\right)$.

(b) (8pts) Find $\frac{dy}{dx}$ using implicit differentiation. Show all work.

(c) (4pts) Find the equation of the tangent line you drew in (a).
(i.e. find the equation of the line tangent to the curve at the point $\left(0, \frac{1}{2}\right)$) .
LEAVE ALL ANSWERS IN EXACT FORM. DO NOT USE DECIMALS.