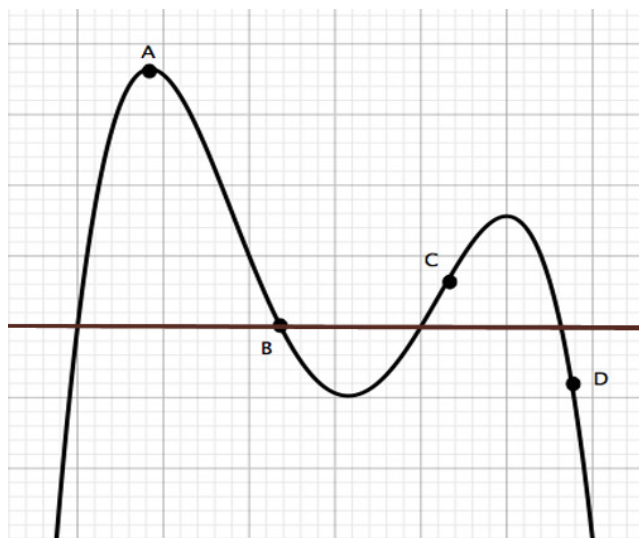


Exam II – MATH 141 – Summer 2018

Answer Key

Use the following graph of the function $y=f(x)$ to answer questions 1 through 3.
Assume that none of the points A, B, C, and D are points of inflection.



1. At point A, which of the following statements is true?
 - (a) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is negative.
 - (b) $f(x)$ is positive, $f'(x)$ is positive, and $f''(x)$ is positive.
 - (c) $f(x)$ is positive, $f'(x)$ is zero, and $f''(x)$ is negative.
 - (d) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is positive.
 - (e) none of the above

2. At point B, which of the following statements is true?
 - (a) $f(x)$ is zero, $f'(x)$ is positive, and $f''(x)$ is negative.
 - (b) $f(x)$ is negative, $f'(x)$ is negative, and $f''(x)$ is negative.
 - (c) $f(x)$ is zero, $f'(x)$ is positive, and $f''(x)$ is positive.
 - (d) $f(x)$ is positive, $f'(x)$ is zero, and $f''(x)$ is positive.
 - (e) none of the above

3. Which of the following statements is true?
 - (a) There is a relative maximum at **D**.
 - (b) There is an absolute minimum between **B** and **C**.
 - (c) Between **B** and **C**, $f(x)$ is concave down.
 - (d) Between **C** and **D**, $f(x)$ is concave down.
 - (e) none of the above

4. Find the absolute minimum and maximum of $f(x) = x^2 + 2x + 4$ on $[-4, 1]$

- (a) The absolute maximum is 7 . The absolute minimum is 3.
- (b) The absolute maximum is 7 . The absolute minimum is 12.
- (c) The absolute maximum is 12 . The absolute minimum is 3.
- (d) The absolute maximum is 7 . The absolute minimum is -1 .
- (e) There is no absolute minimum or maximum

5. For $f(x) = x^4 + \frac{1}{x^2}$ find $f^{(3)}(x)$.

- (a) $\frac{12x^2+6x}{x^5}$
- (b) $\frac{24}{x^4}$
- (c) $12x^2 + \frac{6}{x^4}$
- (d) $24x - \frac{24}{x^5}$
- (e) none of the above

6. Where is $f(x) = \frac{8}{x^2 - 4}$ concave down?

- (a) $(-2, 2)$
- (b) $(\infty, -2)$ and $(2, \infty)$
- (c) $f(x)$ is never concave down
- (d) $(-\infty, \infty)$
- (e) none of the above

7. $f(x) = 3x + \frac{1}{x}$ on $(-\infty, 0)$ has an absolute maximum of

- (a) $\frac{-3}{\sqrt{3}} - \sqrt{3}$
- (b) 2
- (c) 0
- (d) -3.464
- (e) All of the above

8. If $f(x) = x^5 + \sqrt{x}$, then $f''(x) =$
- (a) $5x^4 - \frac{20x^3}{4x^{3/2}}$
 - (b) $5x - \frac{1}{4x^{3/2}}$
 - (c) $5x^4 + \frac{1}{2\sqrt{x}}$
 - (d) $20x^3 - \frac{1}{4x^{3/2}}$
 - (e) $-\frac{20x^3}{4x^{3/2}}$
9. What are the critical values of $g(x) = \sqrt[5]{x^2 - 6x}$?
- (a) $x = 3$ only
 - (b) $x = 2$ only
 - (c) $x = 0$ and $x = 6$
 - (d) $x = 0$, $x = 3$ and $x = 6$
 - (e) none of the above
10. Which of the following is true?
- (a) If $f(c)$ is a relative minimum, then c is a critical value.
 - (b) If $f(x)$ is a continuous function over an open interval (a, b) , then $f(x)$ has an absolute maximum and an absolute minimum over (a, b) .
 - (c) If $f'(c) = 0$, then $f(c)$ is a relative maximum.
 - (d) none of the above
11. Find the coordinates of the relative extrema of $f(x) = (x - x^2)^{0.8}$
- (a) $(0, 0)$ and $(0, 1)$
 - (b) $(0, 0)$, $\left(\frac{1}{2}, \left(\frac{1}{4}\right)^{0.8}\right)$ and $(1, 0)$
 - (c) $\left(\frac{1}{2}, \left(\frac{1}{4}\right)^{0.8}\right)$ only
 - (d) $(0, 0)$ only
 - (e) $(1, 0)$ only

12. If the revenue $R(x)$ (in dollars), and cost $C(x)$ (in dollars) for the production and sale of x units of a product are given by

$$R(x) = 50x - x^2, \quad C(x) = 2x + 5,$$

then the profit $P(x)$ (in dollars) equals

- (a) $-x^2 + 48x - 5$
- (b) $-x^2 - 52x - 5$
- (c) $x^2 - 52x + 5$
- (d) $-x^2 + 48x + 5$
- (e) none of the above

Use the function $P(x)$ from problem 12 to answer questions 13 through 15.

13. Which of the following is the best interpretation of $P(10)$?
- (a) The exact profit from producing and selling the 10th unit is \$375.
 - (b) The exact profit from producing and selling the first 10 units is \$375.
 - (c) The approximate profit from producing and selling the 10th unit is \$375.
 - (d) The approximate profit from producing and selling the first 10 units is \$375.
 - (e) none of the above
14. Which of the following is the best interpretation of $P'(10)$?
- (a) The exact profit from producing and selling the first 10 units is \$28.
 - (b) The exact profit from producing and selling the first 11 units is \$28.
 - (c) The approximate profit from producing and selling the 10th unit is \$28.
 - (d) The approximate profit from producing and selling the 11th unit is \$28.
 - (e) none of the above
15. Given that $P'(10) = 28$ dollars per unit, which of the following statements is **false**?
- (a) $R'(10) - C'(10) = 28$.
 - (b) When $x = 10$, the derivative of the profit function is positive.
 - (c) When 10 units are produced and sold, the marginal profit is 28 dollars per unit.
 - (d) When $x = 10$, $P(x)$ is decreasing.
 - (e) none of the above

16. Suppose that $y = f(x)$ is differentiable over all real numbers and has only one critical value at $x = 1$. If $f'(0)$ is negative and $f'(3)$ is positive, then there is
- (a) a relative minimum at $x = 1$
 - (b) a point of inflection at $x = 1$
 - (c) a relative maximum at $x = 1$
 - (d) none of the above
17. Given $f(4) = 3$, $f'(4) = 0$, and $f''(4) = 10$, there is
- (a) an inflection point at $x = 4$
 - (b) a relative maximum at $x = 4$
 - (c) a relative minimum at $x = 4$
 - (d) neither a relative maximum nor a relative minimum at $x = 4$
18. Alice and Bob are selling widgets and determine that to sell x widgets, the price must be

$$p(x) = 280 - 0.4x$$

They have also found that the cost of producing x widgets is

$$C(x) = 5000 + 0.6x^2$$

What price per widget should Alice and Bob charge to maximize their profit?

Hint: Find the revenue function, $R(x)$, and recall that

$$R(x) = (\text{price per unit}) \cdot (\text{number of units sold})$$

- (a) \$280
 - (b) \$14,600
 - (c) \$140
 - (d) \$224
 - (e) \$0
19. Find the absolute extrema of $f(x) = \frac{2x}{x^2 + 9}$ over the interval $[-1, 5]$.
- (a) The absolute maximum is $\frac{1}{3}$ and the absolute minimum is 0.
 - (b) The absolute maximum is 3 and the absolute minimum is $-\frac{1}{5}$.
 - (c) The absolute maximum is 3 and the absolute minimum is 0.
 - (d) The absolute maximum is $\frac{1}{3}$ and the absolute minimum is $-\frac{1}{5}$.
 - (e) There is neither an absolute maximum nor an absolute minimum.

20. Sound Software estimates that it will sell N units of a program after spending a thousand dollars on advertising, where

$$N(a) = -2a^2 + 400a + 12, \quad 0 \leq a \leq 300.$$

Find the amount that must be spent on advertising in order to achieve the maximum number of units that can be sold.

- (a) \$22,506
 - (b) \$300,000
 - (c) \$100,000
 - (d) \$12,000
 - (e) \$0
21. Of all the numbers whose sum is 50, find the maximum product.
- (a) 0
 - (b) 625
 - (c) 1000
 - (d) 50
 - (e) none of the above
22. A carpenter is building a rectangular shed with a fixed perimeter of 54 ft. What are the dimensions of the shed with largest area that can be built?
- (a) Width is 30 ft, height is 24 ft.
 - (b) Width is 10 ft, height is 17 ft.
 - (c) Width is 13.5 ft, height is 13.5 ft.
 - (d) Width is 2 ft, height is 25 ft.
 - (e) None of the above