NAME: _____

Instructor: _____

Time your class meets: _____

Math 160 Calculus for Physical Scientists I
Exam 3
November 12, 2015, 5:00-6:50 pm

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?"   *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).

2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.

3. You may use a scientific calculator on this exam. No graphing or symbolic calculator is allowed. You must provide your own calculator; you may not use a laptop computer or smart phone.

4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.

5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE

I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

_____
(Signature)

_____
(Date)

Please do not write in this space.

|	
1-4. (12pts)	
5. (16pts)	
6. (10pts)	
7. (8pts)	
8. (24pts)	
9. (15pts)	
10. (15pts)	
TOTAL	

1. If $f(x)$ is a differentiable function on a closed interval and $x = 3$ is one of the endpoints of the interval, then

   (a) $f(x)$ must have an absolute maximum at $x = 3$.

   (b) $f(x)$ must have an absolute minimum at $x = 3$.

   (c) $f(x)$ could have an absolute maximum or minimum at $x = 3$.

   (d) $f(x)$ is not guaranteed to have any absolute extreme value.

2. If $g'(t)$ is continuous and positive on $[-3, 4]$, then $\int_{-3}^{4} g(t)\, dt$

   (a) is positive.

   (b) is negative.

   (c) could be positive or negative.

   (d) may not exist.

3. Suppose that $H'(x)$ is continuous, increasing, and negative on $[0, 1]$. Then $H(x)$ is

   (a) increasing and concave down on $(0, 1)$.

   (b) decreasing and concave down on $(0, 1)$.

   (c) increasing and concave up on $(0, 1)$.

   (d) decreasing and concave up on $(0, 1)$.

   (e) None of the above.

4. For the function $f(x) = x^4$ on $(-\infty, \infty)$,

   (a) $x = 0$ is both a critical point and an inflection point.

   (b) $x = 0$ is a critical point.

   (c) $x = 0$ is an inflection point.

   (d) None of the above.

5. Use the graph of $f(x)$ on $[0, 4.5]$ below to answer the following questions.
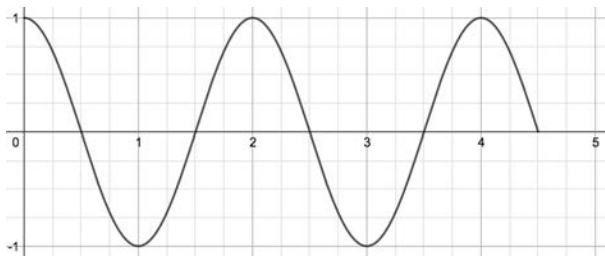
(a) (2pts) Using left endpoints and $n = 9$ subintervals of equal length, draw the rectangles needed to compute the Riemann sum in the graph above.

(b) (2pts) What is value of Δx? (Circle the correct response)
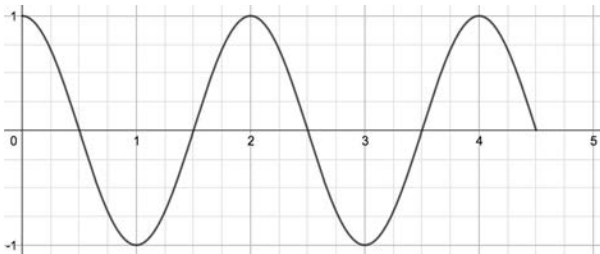
    i. $\Delta x = 9$ ii. $\Delta x = 1$ iii. $\Delta x = 0.5$ iv. $\Delta x = 4.5$

(c) (2pts) Using right endpoints and $n = 9$ subintervals of equal length, draw the rectangles needed to compute the Riemann sum in the graph below.

(d) (6pts) Four sums are provided below. One of them is the correct Riemann sum for the rectangles you drew in (a) and one of them is the correct Riemann sum for the rectangles you drew in (c).

(I) $\sum_{i=0}^{8} f(\Delta x) \cdot (\Delta x)$ (III) $\sum_{i=0}^{8} f(0 + i\Delta x) \cdot (\Delta x)$

(II) $\sum_{i=0}^{8} f(0.5\Delta x) \cdot (\Delta x)$ (IV) $\sum_{i=0}^{8} f(0.5 + i\Delta x) \cdot (\Delta x)$

The Riemann sum for the left endpoints used in part (a) is (I) / (II)/ (III) / (IV) (circle one)

The Riemann sum for the right endpoints used in part (c) is (I) / (II)/ (III) / (IV) (circle one)

(e) (2pts) The value of the Riemann sum for part (a) is greater than / less than / equal to (circle one) the sum for part (c).

(f) (2pts) The value of $\int_{0}^{4.5} f(x) \, dx$ is (circle one answer)

    i. positive.

    ii. negative.

    iii. zero.

    iv. None of the above.

6. (10pts - 5pts each) Indicate whether each of the following statements is **True** or **False**. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample. (A counterexample is an example of a function for which the "if" part of the statement is true, but the "then" part is false.) A graph with an explanation in words can be used as a counterexample.

(a) If a function, $f(x)$, is continuous and increasing on $[a, b]$, then $\int_a^b f(x)\, dx > 0$.

(b) It is not possible for $\int_a^b f(x)\, dx = 0$, unless $f(x) = 0$ for all x in $[a, b]$.

7. (8pts) The velocity of a particle moving back and forth along a number line is given by the equation: $v(t) = 4\pi \cos(2\pi t) - 8\pi \sin(2\pi t)$.

Determine the function that gives the position, $s(t)$, of the particle at time t if $s(0) = 0$. Evaluate all trigonometric functions.

$s(t) =$

8. (24pts) In the axes provided, sketch the graph of a function that has the following properties.

- $\lim\limits_{x\to-\infty} f(x) = 3$

- A vertical asymptote at $x = 2$

- $f'(x) < 0$ on the intervals $\quad (-\infty, -2) \cup (-1, 2) \cup (2, 5)$

- $f'(x) > 0$ on the intervals $\quad (-2, -1) \cup (5, \infty)$

- $f(-2) = 0$ and $f'(-2)$ is undefined

- $f(-1) = 1$ and $f'(-1) = 0$

- $f(5) = 1$ and $f'(5) = 0$

- $f''(x) < 0$ on the intervals $\quad (-\infty, -2) \cup (-2, 2)$

- $f''(x) > 0$ on the intervals $\quad (2, \infty)$

- $\lim\limits_{x\to\infty} f(x) = \infty$
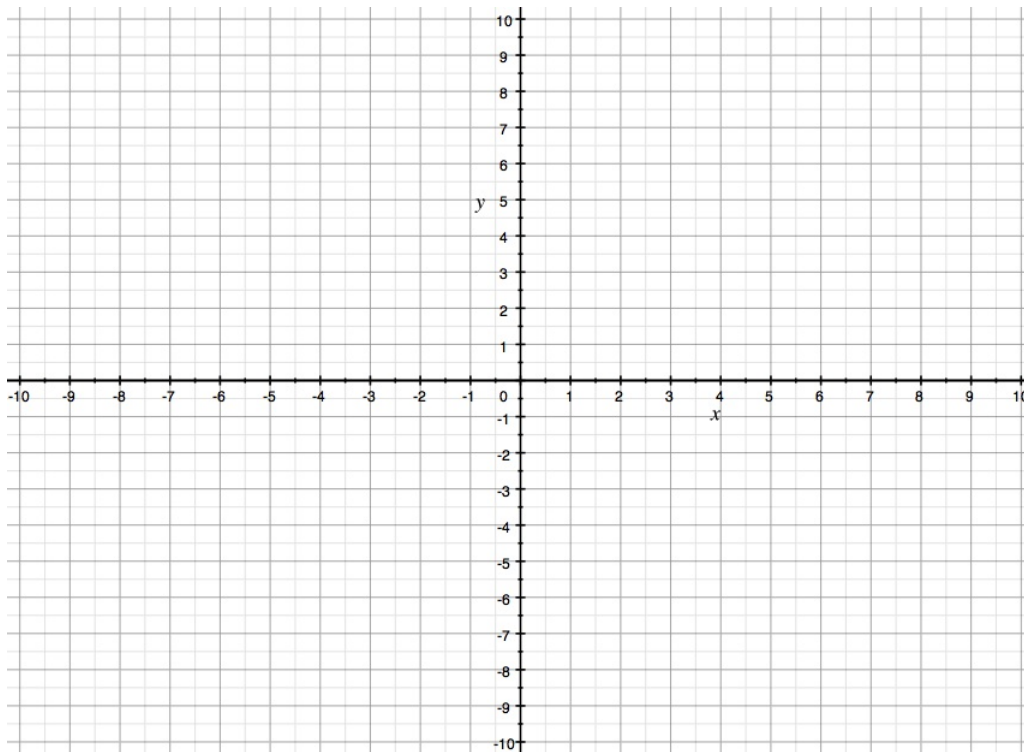
Fill-In-The-Blank:

The critical points for $f(x)$ are: $x =$ _____

The inflection points for $f(x)$ are: $x =$ _____

The relative maxima for $f(x)$ are: $f(x) =$ _____

The relative minima for $f(x)$ are: $f(x) =$ _____

9. (15pts) Your company can manufacture x grade A tires and y grade B tires a day, where x and y are measured in hundreds of tires.

The profit on a grade B tire is \$30. The profit on a grade A tire is $\frac{3}{5}$ the profit on a grade B tire.

Given that
$$y = \frac{40 - 10x}{6 - x} \qquad \text{on} \qquad 0 \le x \le 4$$

write the profit function in terms of grade A tires: $P(x) =$_____

However, due to a recent agreement with a seller, your company is required to manufacture at least two hundred grade A tires each day.

What is the "new" domain for the number of grade A tires, x? _____

Use **calculus** to determine the number of tires of each grade to make in order to maximize profit. Be sure to justify your work with a derivative test.

Number of grade A tires: _____

Number of grade B tires: _____

10. (15pts - 5pts each) Evaluate the following integrals. Answers must be accompanied by supporting work that shows how you evaluated the integral - this may include a geometric argument. Do not expect partial credit for incorrect answers or answers with no supporting work.

(a) $\int \left(\frac{1}{2}x^4 + \frac{2}{x^3} + \sec(x)\tan(x) + \cos(3x) - 1 \right) dx$

(b) $\int \left(t^3 \cdot \sqrt[3]{t} \right) dt$

(c) $\int_0^2 \sqrt{4 - x^2}\, dx$