Exam III – MATH 141 – Summer 2018

1. Find $\dfrac{dy}{dx}$ if $2xy + 3 = 0$.

   (a) $\dfrac{dy}{dx} = \dfrac{1}{2xy}$

   (b) $\dfrac{dy}{dx} = \dfrac{-y}{x}$

   (c) $\dfrac{dy}{dx} = -\dfrac{2y}{x}$

   (d) $\dfrac{dy}{dx} = 0$

   (e) None of the above

2. What is the slope of the curve $xy + y^2 - 2x = 0$ at the point $(1, -2)$? Note: y is a function of x.

   (a) $-\dfrac{3}{4}$

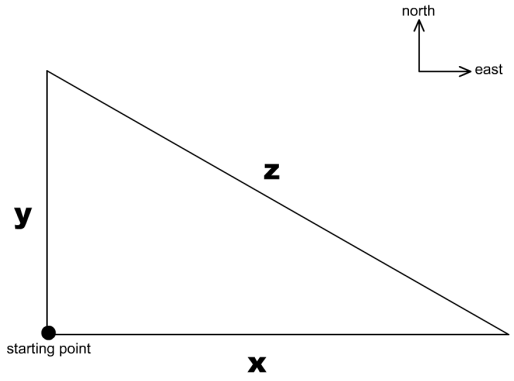
   (b) 0

   (c) $-\dfrac{4}{3}$

   (d) -4

   (e) None of the above.

3. Two cars start from the same point at the same time traveling at constant speeds. One travels north at 25 mph, and the other travels east at 60 mph. How fast is the distance between them increasing at the end of 2 hours? (Remember: $x^2 + y^2 = z^2$.) (Hint: you will need to differentiate implicitly with respect to time.)



   (a) 75 mph

   (b) 130 mph

   (c) 25 mph

   (d) 65 mph

   (e) None of the above.

4. Suppose that total revenue is $R(x) = 280x - 0.4x^2$ and total cost is $C(x) = 5000 + 0.6x^2$ (both given in dollars), where x represents the number of units of a product that are produced and sold. Find the rate of change of the profit with respect to time when $x = 200$ units and $dx/dt = 300$ units per day.

   (a) \$36,000
   (b) \$72,000
   (c) -\$36,000 per day
   (d) \$0 per day
   (e) None of the above.

5. Which of the following is **true**?

   (a) $\ln(e) = e$
   (b) $e = \lim_{h \to 0}(1 + h)^{\frac{1}{h}}$
   (c) $\dfrac{d}{dx}e = e$
   (d) $\dfrac{d}{dx}e^{x^2} = e^{x^2}$
   (e) $e = 2.718$

6. Find $\displaystyle\int_2^5 2x + 1 \, dx$

   (a) 36
   (b) 24
   (c) 6
   (d) -14
   (e) None of the above

7. Which of the following statements is **true**?

   (a) $\ln(AB) = \ln(A) + \ln(B)$
   (b) $\ln(A/B) = \ln(A)^B$
   (c) $\dfrac{d}{dx}\ln(x^2) = \dfrac{1}{x^2}$
   (d) $\ln(x + y) = \ln(x) + \ln(y)$
   (e) $\ln(e^x) = e^x$

For problems 8 and 9, use the following information.
Let $N(a)$ be the number of units sold, and a the amount spent on advertising, in thousands of dollars. Suppose we have the model

$$N(a) = 1000 + 200 \ln a \qquad a \geq 1$$

8. How many units were sold after spending \$1,000 on advertising?

   (a) 2000
   (b) 1000
   (c) 1500
   (d) 1200

9. Find $N'(10)$

   (a) 0
   (b) 20
   (c) 200
   (d) 1000
   (e) $200 \ln 10$

10. If $y = 2e^3$, then dy/dx is

    (a) e
    (b) $2e^3$
    (c) $6e$
    (d) 0
    (e) None of the above.

11. Find $f'(x)$ for $f(x) = 3e^{2x^2+\frac{1}{x}} + \ln(3)$.

    (a) $3\left(4x - \dfrac{1}{x^2}\right) e^{2x^2+\frac{1}{x}}$
    (b) $3\left(4x - \dfrac{1}{x^2}\right) e^{2x^2+\frac{1}{x}}$
    (c) $e^{4x-\frac{1}{x^2}}$
    (d) $3\left(2x^2 + \dfrac{1}{x}\right) e^{2x^2+\frac{1}{x}}$
    (e) None of the above.

12. Find $g'(x)$ for $g(x) = xe^{-x^2}$.

(a) $-e^{-x^2}$

(b) $e^{-x^2} - 2x^2e^{-x^2}$

(c) $e^{-x^2} - 2x^2e^{-2x}$

(d) $e^{-x^2} - 2e^{-x^2}$

(e) None of the above.

13. Differentiate $y = \ln(e^{2x} + 3x)$.

(a) $(2e^{2x} + 3)\ln(e^{2x} + 3x)$

(b) $\dfrac{1}{2e^{2x} + 3}$

(c) $\dfrac{2e^{2x} + 3}{e^{2x} + 3x}$

(d) $\dfrac{1}{e^{2x} + 3x}$

(e) None of the above.

14. Differentiate $y = \ln\left(\dfrac{x^7 + 8}{x^{10}}\right)$ (*Hint*: Logarithm properties might make this easier).

(a) $\ln(x^7 + 8) - 10\ln x$

(b) $\dfrac{1}{\frac{x^7+8}{x^{10}}}$

(c) $\dfrac{7x^6}{x^7 + 8} - \dfrac{10}{x}$

(d) None of the above.

15. Differentiate $y = \ln(e^{4x} + 8) + 7^{x^3+1}$.

(a) $\dfrac{4e^{4x}}{e^{4x} + 8} + 3x^2\ln(7)7^{x^3+1}$

(b) $\dfrac{4e^{4x}}{e^{4x} + 8} + 7^{3x^2}$

(c) $\dfrac{1}{e^{4x} + 8} + 7^{3x^2}$

(d) $\dfrac{1}{e^{4x} + 8} + 3x^2\ln(7)7^{3x^2+1}$

(e) None of the above.

16. What initial investment, P_0, should one make in order to have (exactly) $\$25,000$ after 20 years, when interest is compounded continuously at 5%?

(a) $P_0 = \dfrac{25,000}{e^2} \approx \$3,383.38$

(b) $P_0 = \dfrac{25,000}{2e} \approx \$4,598.49$

(c) $P_0 = \dfrac{500,000}{2e^3} \approx \$12,446.77$

(d) $P_0 = \dfrac{25,000}{e} \approx \$9,196.97$

(e) $P_0 = \$5,000$

Use the following information to answer questions 17 and 18.
Suppose that $\mathbf{P_0 = \$1,500}$ is invested in the Grothendieck Bond Fund for which interest is compounded continuously at 5.2% per year. That is, the balance P grows at the rate given by

$$\mathbf{\frac{dP}{dt} = 0.052P}.$$

17. What is the balance after 3 years?

(a) $ 4,243.82

(b) $1,664.40

(c) $1,753.23

(d) $1,578

(e) All the above

18. In about how many years will an investment of $1,500 double itself?

(a) 11.75 years

(b) 13.33 years

(c) 1.22 years

(d) 2 years

19. Integrate $\int \left(3x^2 + 7 + \frac{2}{x^2}\right) dx$.

(a) $x^3 + 7 + 2x + C$

(b) $\frac{x^3}{3} + 7x + \frac{2}{x} + C$

(c) $x^3 + 7x2\ln|x^2| + C$

(d) $x^3 + 7x - \frac{2}{x} + C$

(e) None of the above.

20. Integrate $\int 7e^{3x} dx$.

(a) $\frac{7}{3}e^{3x} + C$

(b) $\frac{e^{3x}}{3} + C$

(c) $21e^x + C$

(d) $7e^{3x} + C$

(e) None of the above.

21. If $f'(x) = \sqrt{x^5} dx$ and $f(4) = 12$, find $f(x)$.

(a) $\frac{1}{6}\sqrt{x^6} + c$

(b) $\frac{2}{7}x^{7/2} - \frac{172}{7}$

(c) $\frac{2}{6}x^{2/6} + 12$

(d) $\frac{1}{6}\sqrt{x^6} + \frac{4}{3}$

(e) None of the above.

22. Find the area under $f(x) = 3x^2 + x + 2$ over the interval $[-1, 2]$

(a) 13

(b) 16.5

(c) 0

(d) 10.5