NAME: ______________________

Instructor: ______________________

Time your class meets: ______________

Math 160 Calculus for Physical Scientists I
Final Exam
Wednesday, December 17, 7:30am-9:30am

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?" *-Albert Einstein*

1. Turn off your cell phone and other devices (except your calculator).
2. Write your name on every page of the exam. Write your instructor's name on the cover sheet.
3. You may use a calculator on this exam. You must provide your own calculator; you may not use a laptop computer or smart phone.
4. No notes or other references, including calculator manuals or notes stored in calculator memory, may be used during this exam.
5. Use the back of the facing pages for scratch work and for extra space for solutions. Indicate clearly when you wish to have work on a facing page read as part of a solution to a problem.

HONOR PLEDGE
I have not given, received, or used any unauthorized assistance on this exam. Furthermore, I agree that I will not share any information about the questions on this exam with any other student before graded exams are returned.

____
(Signature)

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(Date)

Please do not write in this space.

1. (18pts)	
2-3. (12pts)	
4. (12pts)	
5. (13pts)	
6. (8pts)	
7. (12pts)	
8. (13pts)	
9. (12pts)	
TOTAL	

1. (18pts - 3pts each) Evaluate the following limits, derivatives, and integrals as instructed. L'Hopitals Rule is not allowed. If an answer is ∞ or does not exist explain how you know. **Answers will be graded as right or wrong.** You will only receive credit if your answer is fully correct with supporting work.

(a) $\displaystyle\lim_{x\to 3} \frac{x-3}{x^2+x-12}$

(b) $\displaystyle\lim_{b\to 1} \frac{b-1}{\frac{1}{b}-1}$

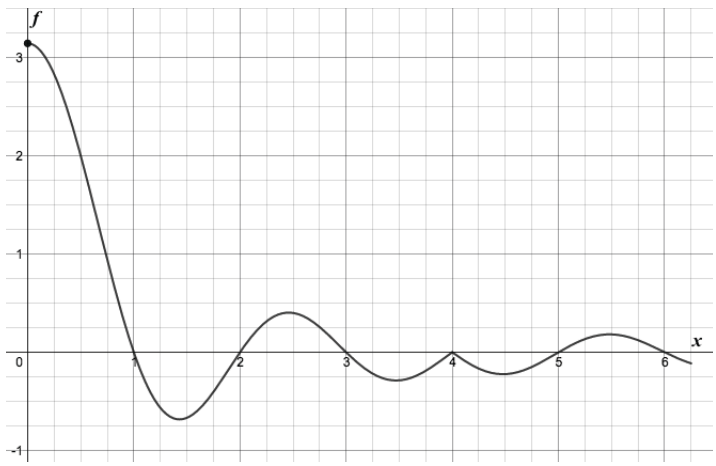
(c) Find $f'(x)$ given $f(x) = \sin(x)\tan(x^2)$

(d) Find $g'(t)$ given $g(t) = \dfrac{4-t}{3t^2+t^3}$

(e) $\displaystyle\int \left(-8x^4 - \frac{4}{x^2} + \sqrt[5]{x} + 1\right)\, dx$

(f) $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin(x)}{(\cos(x)+2)^2}\, dx$

Use the graph of $f(x)$ below to answer the following questions.



2. (6pts) Order the following from ***smallest*** to ***largest*** (fill in the blanks below):

$$\int_0^1 f(x)\,dx,\ \int_2^4 f(x)\,dx,\ \int_3^5 f(x)\,dx$$

____________________ $<$ ____________________ $<$ ____________________

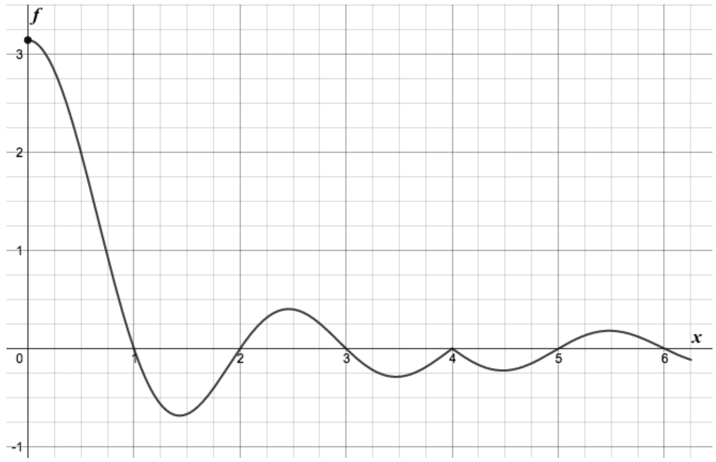
3. (6pts) The Riemann sum from using right-endpoints and 6 subintervals of equal length on $[0, 6]$ is a way of computing an approximate value for (circle one)

(a) $\int_1^6 f(x)\,dx$ (b) $\int_0^6 f(x)\,dx$ (c) $\int_1^5 f(x)\,dx$

and the value will be (circle one)

(a) Greater than zero. (b) Less than zero. (c) Equal to zero. (d) Cannot be determined.

4. (12pts) Let $g(x) = \displaystyle\int_0^x f(t)\,dt$ where f is graphed below and g is defined for $x \geq 0$:



(a) Does g have any local maxima within $(0, 6)$? If so, where are they located? (Explain how you know.)

(b) Does g have any local minima within $(0, 6)$? If so, where are they located? (Explain how you know.)

(c) At what value of x does g attain its absolute maximum on $(0, 6)$? (Explain how you know.)

5. (13pts) Consider the curve defined by the equation $x^2 + y^2 - xy = 1$

(a) Use implicit differentiation to find $\dfrac{dy}{dx}$.

(b) Verify that the points $(1, 1)$ and $\left(\dfrac{1}{\sqrt{3}}, -\dfrac{1}{\sqrt{3}}\right)$ are on the curve.

(c) Find the slope of the line tangent to the curve at each point.

Slope at $(1, 1)$ is ____________ Slope at $\left(\dfrac{1}{\sqrt{3}}, -\dfrac{1}{\sqrt{3}}\right)$ is ____________

The tangent lines are parallel / perpendicular / neither (circle one).

6. (8pts) Indicate whether each of the following statements is **True** or **False**. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample. (A counterexample is an example of a function for which the "if" part of the statement is true, but the "then" part is false.) A graph with an explanation in words can be used as a counterexample.

   (a) If a function, $f(x)$, is continuous at $x = -2$, then it is also differentiable at $x = -2$.

   (b) Suppose f is a function such that $f'(x) < 0$ for all x.
   Let $g(x) = f(f(x))$. Then g must be increasing for all x.

7. (12pts) Given two nonnegative real numbers whose sum is 9, find the maximum value of the product of one number with the square of the other number. Use **calculus** to justify your answer.

8. (13pts) In the axes provided, sketch the graph of a function $f(x)$ that has the following properties.

(a) $\lim_{x\to-\infty} f(x) = \infty$

(b) $f'(x) < 0$ and $f''(x) < 0$ for $x < -4$

(c) $\lim_{x\to-4^-} f(x) = -\infty$

(d) $\lim_{x\to-4^+} f(x) = \infty$

(e) $f'(x) < 0$ and $f''(x) > 0$ for $-4 < x < -2$

(f) $f'(x) > 0$ for $-2 < x < 0$

(g) $\lim_{x\to 0} f(x) = \infty$

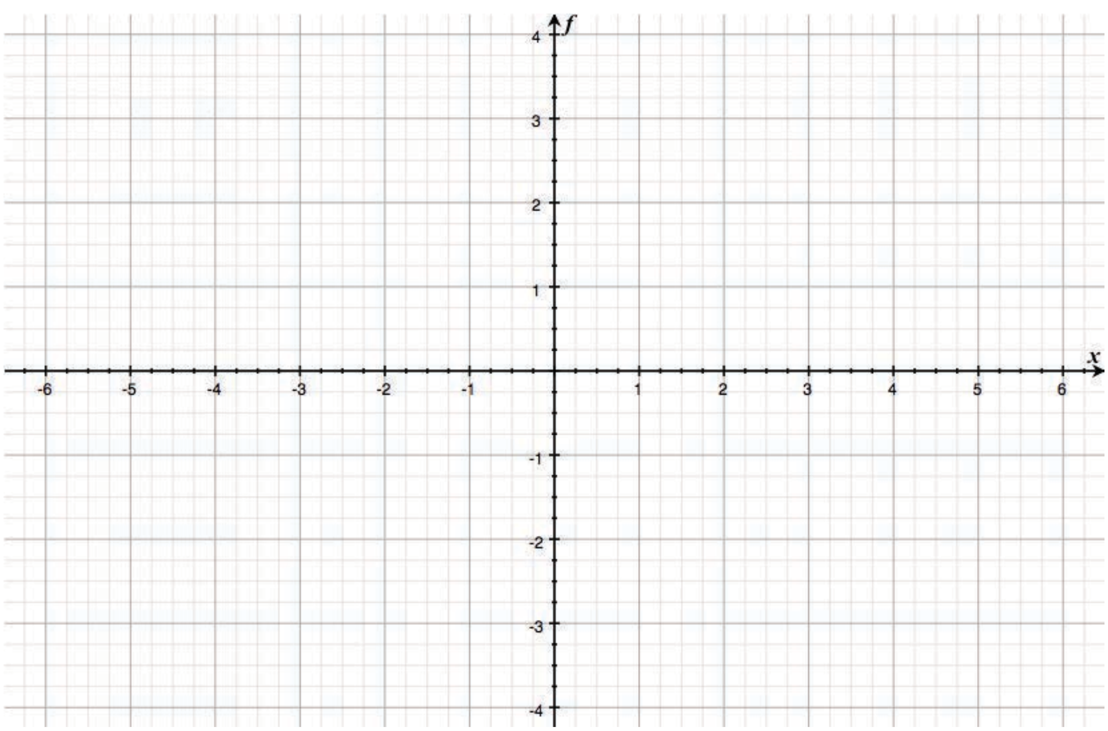
(h) $f'(x) < 0$ for $0 < x < 2$

(i) $f''(x) > 0$ for $0 < x < 3$

(j) $f'(x) > 0$ for $2 < x < 4$

(k) $f''(x) < 0$ for $x > 3$

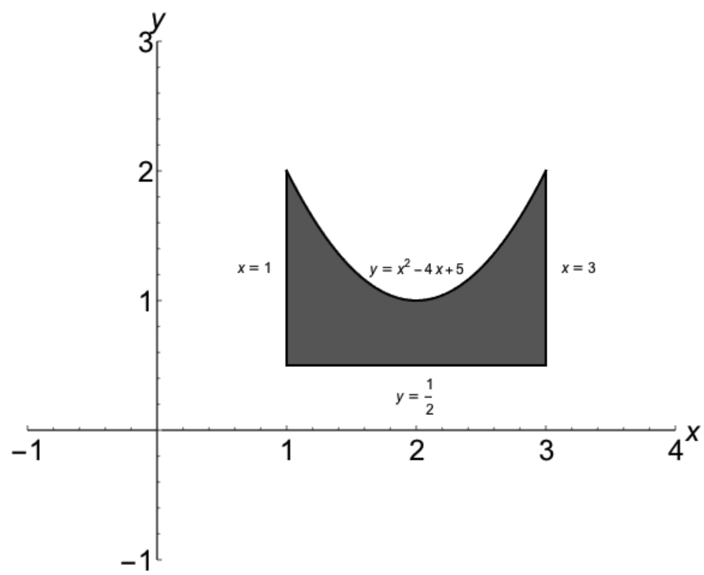
(l) $f'(x) < 0$ for $x > 4$



9. (12pts) The following is to be used for parts (a)-(c). Part (c) is on the next page.

Below is the graph of the region bounded by the curves:

$$f(x) = x^2 - 4x + 5, \quad g(x) = \frac{1}{2}, \quad x = 1, \quad x = 3$$



(a) Write out, but DO NOT evaluate the integral that will give the area of the region.

(b) Write out, but DO NOT evaluate the integral that will give the volume of the solid generated by revolving the region about the $x-$axis.

(c) A three-dimensional solid has a base that is the given region. The cross-sections of the solid are squares perpendicular to the x-axis (i.e. the length of a side of the square is the distance from $f(x)$ to $g(x)$). See the graphs below for reference.

Set up, but DO NOT evaluate the integral that gives the volume of the solid.