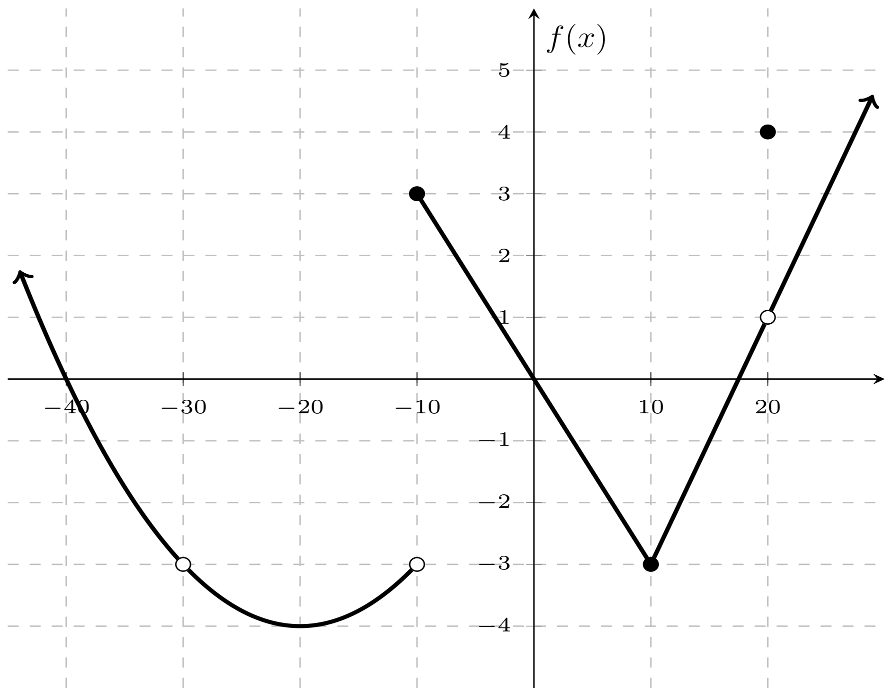
1. (20 points) Use the graph of $f(x)$ to answer each question below. Write "DNE" if the value does not exist.



(a) $\lim_{x\to -20} f(x) =$__________

(b) $\lim_{x\to 10^-} f(x) =$__________

(c) $\lim_{x\to 20} f(x) =$__________

(d) $\lim_{x\to 10} f(x) =$__________

(e) $\lim_{x\to 0} f(x) =$__________

(f) $f'(-20) =$__________

(g) $f'(5) =$__________

(h) $\lim_{h\to 0} \dfrac{f(15+h) - f(15)}{h} =$__________

(i) $f''(-15)$ is
- ◯ positive
- ◯ negative
- ◯ zero
- ◯ DNE

(j) Let $g(x) = f(5x^2)$,

i. compute $g'(x) =$____________
(Answer will be in terms of f and x.)

ii. evaluate $g'(1) =$____________
(Answer will be an exact number.)

(2) (20 points) Compute the derivative function of each function below using derivative rules.

(a) $f(x) = 3x^5 + x^{-3} + 10$

(b) $\ell(x) = sx^2 - \frac{1}{s}x + s^2$

(c) $h(t) = 2^t \cdot \cos(t)$

(d) $q(t) = \sin(\ln(t))$

(e) $j(x) = e^{\sqrt{x}+x}$

(f) $u(t) = \log_2(t)$
Hint: Use the inverse function rule.

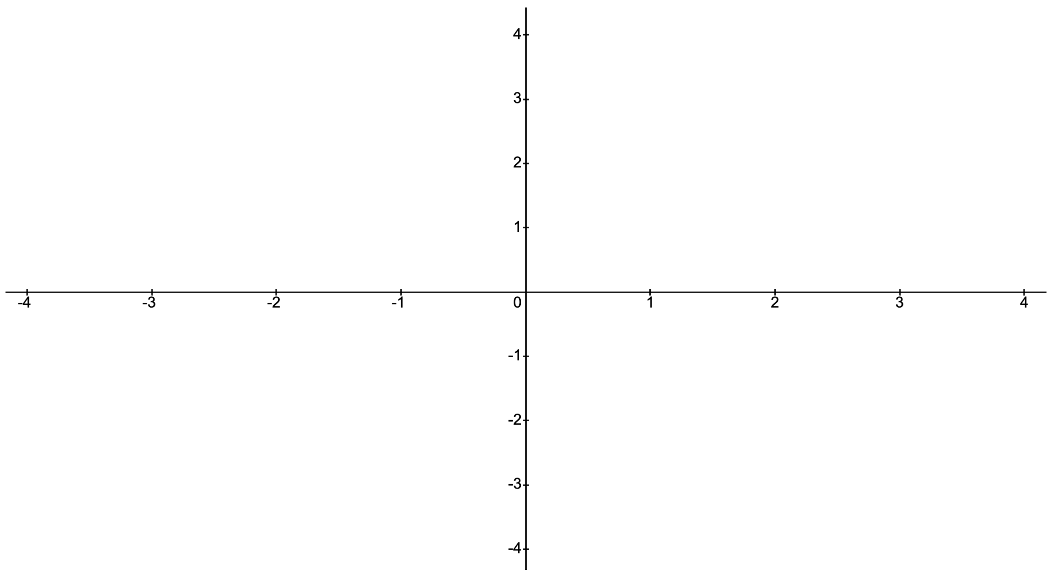
(g) $n(x) = \dfrac{p(x)}{e^x}$, where $p(x)$ is an unknown differentiable function

(3) (20 points) Let $h(x)$ be a function such that $h(-2) = -1$ and $h'(-2) = 3$.

(a) Write the equation for $L(x)$, the linear approximation of $h(x)$ centered at $a = -2$.

(b) Use $L(x)$ to approximate $h(-2.25)$.

(c) Suppose you also know that $h''(x) < 0$ for all x. Sketch a rough graph of $h(x)$ for x-values close to -2, including the graph of $L(x)$. Be sure to label both $h(x)$ and $L(x)$.



(d) Label your approximation from part (b) on your graph above. Is it an overestimate or an underestimate of $h(-2.25)$?

(4) (20 points) A population (in thousands) changes according to the updating function

$$p_{t+1} = 2(p_t)^4 - 8(p_t)^3 + 10(p_t)^2 - 3p_t.$$

The equilibrium points of this system are at $p^* = 0$, $p^* = 1$, and $p^* = 2$.

(a) Write the updating function rule as "$f(x) =$", then compute the derivative of the updating function rule.

(b) Use the Stability Theorem to classify each equilibrium point, and circle your conclusions below. You must show your work using the Stability Theorem for credit.

According to the Stability Theorem, $p^* = 0$ is

STABLE UNSTABLE INCONCLUSIVE

According to the Stability Theorem, $p^* = 1$ is

STABLE UNSTABLE INCONCLUSIVE

According to the Stability Theorem, $p^* = 2$ is

STABLE UNSTABLE INCONCLUSIVE

(5) (20 points) Consider the function

$$j(x) = 12x^{\frac{1}{3}} - x.$$

(a) Compute $j'(x)$.

(b) $x = 0$ is a critical number of $j(x)$. Explain why in one sentence.

(c) $x = 8$ and $x = -8$ are critical numbers of $j(x)$. Explain why in one sentence.

(d) Use the first derivative test to classify each of $x = 0, x = 8$, and $x = -8$ as a local maximum, local minimum, or neither of $j(x)$. Use a first derivative sign chart to justify your answers.

continue →

(e) Find the absolute maximum and minimum values of $j(x) = 12x^{\frac{1}{3}} - x$ on the domain $[-1, 27]$. Clearly show all of your work.

The absolute **maximum** of $j(x)$ is $y =$______________ and is achieved at $x =$______________.

The absolute **minimum** of $j(x)$ is $y =$______________ and is achieved at $x =$______________.